ADDRESSING MATH ANXIETY IN TEACHING MATHEMATICS USING PHOTOGRAPHY AND GEOGEBRA

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Abstract

By inserting photographs into GeoGebra software and exploring various objectives related to the new Common Core Math Standards, the presenters will motivate students to learn math and minimize math anxiety. While covering the new Common Core Standards, the topics will explore the math that surrounds us in the real world thus creating a connection between the abstract math and the life experiences. When math has a purpose, then students are willing to spend time in exploring and understanding new concepts. Real-life photographs that are inserted into GeoGebra will provide the basis to observe relationships with different and similar shapes. The shapes will be given their mathematical names and explore similarities and differences. Those similar shapes will be classified as similar and/or congruent figures. Besides recognition of congruent figures in the photographs, the mathematical measurements of one-to-one ratios along with congruent angles are calculated by GeoGebra. The measurements of the similar figures will see congruent angles but proportional ratios of their sides. Technology like GeoGebra can help motivate young learners to enjoy learning mathematics while addressing math anxiety and attitudes. The presentation/paper will show educators how by importing photography into the GeoGebra software, teachers can explain math concepts and make the learning of math more real-world and relevant. In an age of STEM, it is critical that we motivate and turn young people onto math through technology. Online websites and resources for addressing math anxiety and attitudes will also be shared.

Key Words: Teaching Mathematics, GeoGebra, Photography, Geometry, Measurement, Technology, Common Core Mathematics Standards, Math Anxiety, Connections, Real-world

Introduction

By using photography and GeoGebra, we can better reach our students and show them how math surrounds us. In today's technologically-oriented world, students need to be proficient in Science, Technology, Engineering, and Mathematics (STEM) fields. As endorsed by the National Council of Teachers of Mathematics (NCTM, 2000) and stressed in the new Common Core State Standards(CCSS) in Mathematics, it is critical that we teach using technology, address attitudes and anxiety toward math, and make the math that students are learning relevant and meaningful. Often, it may be best to start teaching young people geometry first as opposed to numbers which are considered to be more abstract and difficult to learn. Geometry is one of the most concrete branches of mathematics and focusing on this first can benefit students' whole view of mathematics and their attitudes towards learning it. Today teachers also need to be cognizant and checking for attitudes and dispositions toward learning mathematics as math

anxiety is an issue in today's classrooms. This paper looks at ideas for teaching mathematics with the use of technology and photography using the free dynamic mathematics software, GeoGebra, to help teachers develop mathematically confident young people.

Math Anxiety: The Need to Check for Mathematical Dispositions

Math anxiety is a common problem in the classroom. Richardson and Suinn (1972) originally defined math anxiety as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (p. 551). Mathematics anxiety is the "irrational dread of mathematics that interferes with manipulating numbers and solving mathematical problems within a variety of everyday life and academic situations" (Buckley and Ribordy, 1982, p. 1).

Math anxiety research has been around since the 1970s (Richardson & Suinn, 1972). Math anxiety still continues to plague our society and affects our young peoples' success and achievement with the subject (Quander, 2013). Quander feels that elementary teachers need to help prepare students to be lifelong learners and develop a productive mathematical disposition so that they are prepared for future schooling and eventual careers. Math anxiety can impede not only mathematical performance but also interest and then career choice and many decisions in life. The idea of looking closely at math anxiety levels, motivation to learn mathematics, and using technology like GeoGebra to teach and motivate students is critical today in a world of STEM.

According to NCTM (1989), mathematics teachers need to assess students' mathematical disposition regarding:

- confidence in using math to solve problems, communicate ideas, and reason;
- flexibility in exploring mathematical ideas and trying a variety of methods when solving problems;
- willingness to persevere in mathematical tasks;
- interests, curiosity, and inventiveness in doing math;
- student ability to reflect and monitor their own thinking and performance while doing math;
- value and appreciate math for its real-life application, connections to other disciplines and cultures and as a tool and language.

Jackson and Leffingwell (1999) found that in their study only seven percent of the population reported having positive experiences with mathematics from kindergarten through college. Their study cited that there are many covert (veiled or implied) and overt (apparent and definite) behaviors exhibited by the math instructor in creating math anxiety in students. Things like difficulty of material, hostile instructor behavior, gender bias, perceptions of uncaring teacher, angry behavior, unrealistic expectations, embarrassing students in front of peers, communication and language barriers, quality of instruction, and evaluation methods of the teacher. Math instructors' behaviors and teaching methods can be hurtful and negative to students learning math. Students often say: "I like the class because of the teacher" because the teacher knows how to present developmentally the subject matter, creates a learning environment conducive to learning with compassion, has high expectations for all students without regard to gender, race,

or language barriers, and uses a variety of assessment methods and teaching styles to better reach all students.

How to prevent math anxiety:

1. Using "Best Practice" in mathematics such as: manipulatives, cooperative groups, discussion of math, questioning and making conjectures, justification of thinking, writing about math, problem-solving approach to instruction, content integration, technology, assessment as an integral part of instruction, etc.
2. Incorporating the NCTM and State/Common Core Math Standards into the curriculum and instruction.
3. Discussing feelings, attitudes, and appreciation of mathematics with students (Furner, 2007).

How to reduce math anxiety:

1. Psychological Techniques like anxiety management, desensitization, counseling, support groups, bibliotherapy, and discussions.
2. Once a student feels less fearful about math, he/she may build their confidence by taking more mathematics classes.
3. Most research shows that until a person with math anxiety has confronted this anxiety by some form of discussion/counseling no "best practices" in math will help to overcome this fear (Furner, 2007).

It is also beneficial to provide students with a math attitude surveys at the beginning of a school year or course and also to read the book, *Math Curse* (Scieszka & Smith, 1995), to get students to talk about their true feelings toward math, surveys and bibliotherapy are both effective forms of starting the process of opening up and getting inner feelings out young people may have about mathematics or unpleasant past experiences. It is recommended that mathematics teachers survey their students at the beginning of a school year to check for their students' dispositions toward mathematics. There are two good online surveys that test for math anxiety and may be useful to classroom teachers as follows:

- Mathpower (http://mathpower.com/anxtest.htm)
- Mathipedia (http://www.mathipedia.com/student-math-anxiety-test.html)

While teaching mathematics, an advantageous instructional method called **Concrete-Representational-Abstract (CRA)** Model is encouraged and described as follows:

1. Start with the **C**oncrete using hands-on manipulatives like Geoboards
2. Move to **R**epresentational models in diagrams (or use Virtual Manipulatives like NLVM at: http://nlvm.usu.edu/)
3. Lastly, connect to the **A**bstract symbolism where student understand and function at an abstract level completely (GeoGebra software works well at: http://www.geogebra.org/cms/en/)

This model is really the bases for the best practices pedagogy for teaching mathematics starting with young people.

Connections need to be made when we teach math. Munakata and Vaidya (2012) based on their research found that students do not consider mathematics and science to be creative endeavors,

although the traditional artistic disciplines rank high in this regard. To address this problem in perception, the authors used photography as a means to encourage students to find the deep-rooted connections between science and mathematics and the arts. The photography project was used in a formal classroom setting as well as an outside activity, i.e. in a more informal setting. The project found student interest and motivation were peaked when photography was part of the instructional strategies to teach new material while making meaningful connections to the math using the photography. Jones (2012) also in her book, *Visualizing Mathematics*, discusses how teachers need to help students visualize and create representations of their math understanding so to turn them on to the subject.

Common Core Standards as They Relate to using GeoGebra

Today, most schools and states are adhering to the new Common Core Math Standards (National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO), 2010) found at: http://www.corestandards.org/

See Appendix B for examples on using GeoGebra and photography to create meaning and understanding of geometry for the students.

Examples of Common Core Math Standards we can teach using GeoGebra:

Grade 3

3.G.1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Grades 6

6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

By relating and understanding real-world problems through the use of dynamic technology like GeoGebra and connecting them to photography to make important connections in math, our learners recognize that geometry and shapes/mathematics surround us! The above Common Core Math Standards can all be taught using GeoGebra and Photographs. See Appendix B for more ideas.

Technology use in the Teaching of Mathematics

The use of technological tools is critical in today's world. Our students need to learn to excel at higher levels of generalization, model and solve complex problems, and focus on decision-making and reasoning (National Council of Teachers of Mathematics (NCTM) 1989, 2000, 2006). NCTM believes that mathematical power can arise from technology that includes: increased opportunity for learning, increased opportunities for real-life social contexts, and orientation to the future. The President's Council of Advisors on Science and Technology (PCAST) (Holdren, Lander, & Varmus, 2010) released an executive report in November 2010 where specific recommendations to the administration are given to ensure that the United States is a leader in Science, Technology, Engineering, and Mathematics (STEM) education in the coming decades. One recommendation is to recruit and train 100,000 new STEM middle and high school teachers over the next decade that are able to prepare and inspire students to have strong majors in STEM fields and strong content-specific pedagogical preparation. PCAST regards teachers as the most important factor in ensuring excellence in STEM education. Despite the ongoing efforts to promote the use of technology in education (e.g., National Council of Teachers of Mathematics [NCTM], 2000; National Educational Technology Standards for Teachers [NETS.T], 2008), teachers' ineffective use of technology has been reported in the literature. One reason frequently cited is that teachers are not trained in utilizing technology in the classroom within the subject context. Hwang, Su, Huang, & Dong, (2009) found that by combining virtual manipulatives and software like GeoGebra along with whiteboard, teachers can better model problems, help students understand and solve the problems while reaching higher levels in the teaching of many mathematical ideas in the curriculum.

GeoGebra

GeoGebra is a multi-platform dynamic mathematics software for all levels of education from elementary through university that joins dynamically geometry, algebra, tables, graphing, spreadsheets, statistics and calculus in one easy-to-use package (Hewson, 2009; Hohenwarter, Hohenwarter, & Lavicza, 2009). This open-source dynamic mathematics software can be downloaded for free and accessed at: http://www.geogebra.org/cms/en/info. There are no licensing issues associated with its use, allowing students and teachers freedom to use it both within the classroom and at home. GeoGebra has a large international user and developer community with users from 190 countries is currently translated into 55 languages.

In research by Fahlberg-Stojanovska, & Stojanovski (2009), they discovered that using GeoGebra is motivating for students and helps them learn at a higher level while exploring and conjecturing as they draw and measure. Rosen & Hoffman (2009) established the importance to integrate both concrete and virtual manipulatives into the math classroom, such as representational models like GeoGebra. Furner & Marinas (2007) found that young people can easily transition from the concrete when using manipulatives like geoboards to the abstract when using geometry sketching software like GeoGebra. Although GeoGebra has been primarily intended for mathematics instruction in secondary schools, it certainly has uses in higher education and even now introduced in the elementary math levels. The Appendix A provides online websites on resources related to GeoGebra.

GeoGebra can be used to show how mathematical equations can be applied to everyday objects. Aydin & Monaghan (2011) in their research feel that math teachers need to explore the potential for students to "see" mathematics in the real world through "marking" mathematical features of digital images using a dynamic geometry system like GeoGebra. Mathematics teachers may find the following videos (Mathematics and Multimedia, n.d.) of basic training for GeoGebra at: http://mathandmultimedia.com/2011/01/01/geogebra-essentials-series/ useful as they provide great resources for how to quickly use GeoGebra in their classrooms.

GeoGebra was described as raising the enthusiasm for the effective and wise application of technology to the teaching/learning enterprise (Fahlberg-Stojanovska and Stojanovski, 2009; Hewson, 2009). Observations of participants in schools and during the summer workshops are also cited as evidence. GeoGebra was also credited with changing teacher habits. Two features were specifically referenced as causing this change: 1) that it is an award winning software system, and therefore has admirable features, and 2) that it provides an effective pedagogical model for teachers.

As described by Mishra and Koehler (2006) Technological Pedagogical Content Knowledge (TPCK) is the basis of good teaching with technology and requires not only content knowledge or pedagogical knowledge but an understanding of the representation of concepts using technologies, how to teach these math concepts using technology, knowledge on the challenges their students will face when presented with this new pedagogy, and how technology can be used to build on existing knowledge and develop new knowledge. Scandrett (2008) feels that math teachers need to always start by using concrete models in geometry using manipulatives like geoboards which provide a concrete model of understanding. Rosen & Hoffman (2009) have found that teachers need to connect students understanding from the concrete to abstract and using virtual manipulatives and software like GeoGebra better help make those connections to representational models connecting the concrete using geoboards to something oven more abstract in understanding. With the availability of GeoGebra, teachers are able to make graphical representations of math concepts. As the concepts are introduced with pictorial representations, teachers and their students are able to make the connections between the pictures, the math concepts, and the symbolic representation. When presented with a new concept, students need to think, visualize and explore relationships and patterns. This is consistent with the CRA (Concrete, Representational, and Abstract) Model for teaching mathematics currently in better reaching students as they learn and understand mathematical concepts. Technology makes all of this possible for them in a short amount of time.

Summary

So why should math teachers use GeoGebra? Some reasons are because it is:

- free to download and use from GeoGebra.org
- up and coming dynamic teaching tool in our schools today
- user-friendly for students and teachers
- a connection from the hands-on Geoboards to virtual Geoboards to something even more abstract
- a software that provides many resources and teaching tools at its wiki for educators at: http://www.GeoGebra.org/en/wiki/index.php/Main_Page

- used for primary-aged students through college
- fun, easy to use, and students learn a lot about geometry, algebra, measurement and beyond by using this dynamic tool.

Why is it important to make connections and excite students about learning math?

- Show a purpose for math
- Develop relationships
- Show practical applications to math in life
- Employ innovative teaching in the classroom
- Stimulate through photography/Modeling
- Employ emerging technologies in math with the real world
- Address math anxiety so students feel confident for any STEM field when they graduate from school

Additional help with math anxiety and its research can be found at:

- Professor Freedman Provides Math Help at: http://www.mathpower.com/
- *Mathitudes Online* website at: http://www.coe.fau.edu/centersandprograms/mathitudes/

A famous quote from W. V. Williams (1988) is a reminder of how critical it is to teach for understanding making things as hands-on and real-world as possible.

> *"Tell me mathematics, and I will forget; show me mathematics and I may remember; involve me...and I will understand mathematics. If I understand mathematics, I will be less likely to have math anxiety. And if I become a teacher of mathematics, I can thus begin a cycle that will produce less math-anxious students for generations to come."*

Furner (1999) also made these related observations:

> *"If math teachers do something about helping their students to develop their confidence and ability to do math, we can impact their lives in a positive way forever."*

> *"Our students' careers and ultimately many of their decisions they will make in life could rest upon how we decide to teach math. We must make the difference for the future of our kids in an ever growing, high-tech, competitive, global world which depends so heavily on mathematics."*

Young learners intrigued by technology will construct and investigate geometric shapes and many math ideas with GeoGebra and will start enjoying math and have less math anxiety in our STEM World that we now live in! By using technology like GeoGebra and photographs, our young learners who are often intrigued by technology will construct and investigate geometric shapes with GeoGebra and start liking and enjoying math more! Some PowerPoints and Data Files for GeoGebra as they relate to this paper can be accessed at: http://matharoundus.com/. Many free resources for math teachers Grades K-12 to download are in Appendix A.

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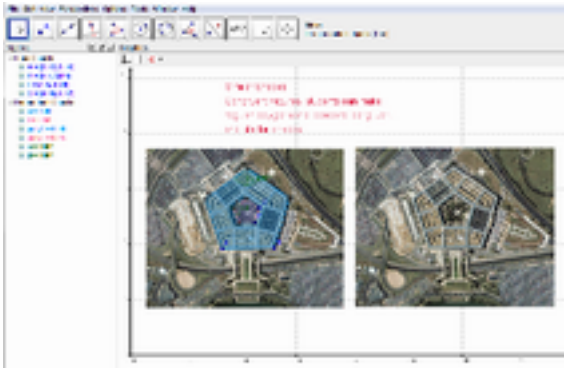
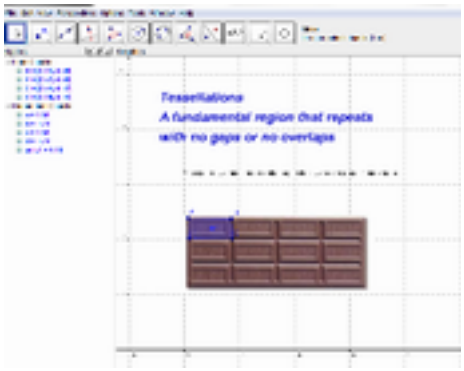
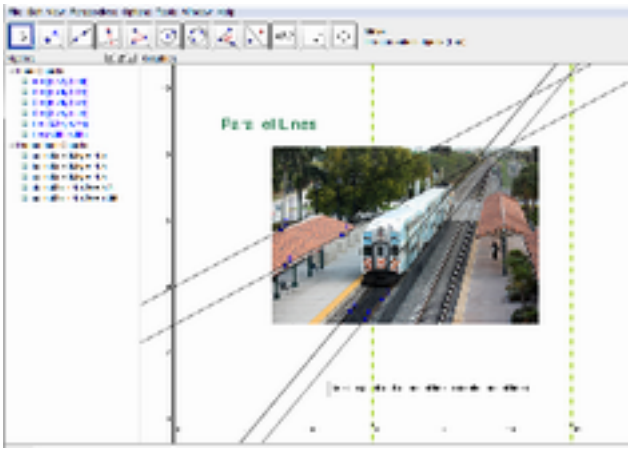
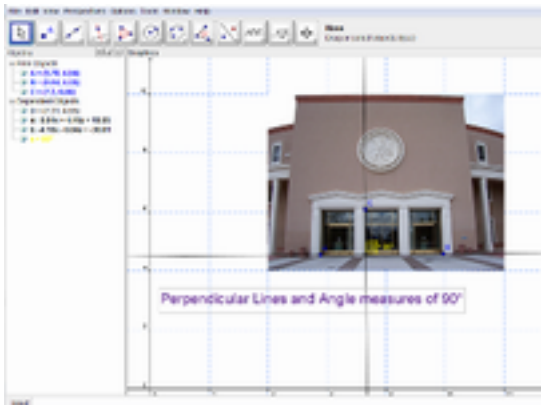
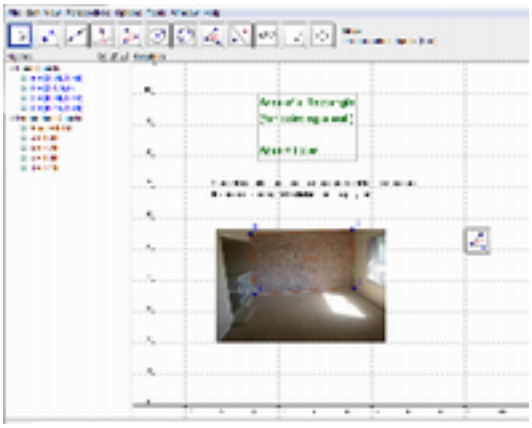
Appendix A: GeoGebra Websites and Resources for the Classroom

Geoboard Resources	http://msteacher.org/epubs/math/QuickTakes/geoBoard.aspx
GeoGebra	http://GeoGebra.org
GeoGebra Wiki Forum	http://www.GeoGebra.org/en/wiki/index.php/Main_Page
GeoGebra Data Files	http://matharoundus.com
Math Academy	http://www.mathacademy.com/pr/minitext/anxiety/
Mathitudes Online	http://www.coe.fau.edu/mathitudes/

Appendix B: K-6 Math Topics

Similar Shapes 	**Tessellations** 
Parallel Lines 	**Perpendicular Lines** 
Area 	**Angles and Measures**