The Impact of GeoGebra in Math Teachers' Professional Development

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Abstract

The Mathematics Science Partnership (MSP) Institute at Florida Atlantic University has used *GeoGebra* to model effective teaching/learning strategies for the sake of lasting improvement and change in teacher's pedagogies. The paper will describe the Institute and then give the rationale for choosing *GeoGebra* as the preferred dynamic mathematics software. *GeoGebra* is a free, open source, multi-platform, translatable, dynamic mathematics software that has an extensive and very active international community of users who support each other with teaching materials and technical support in the use of the program. Examples of the use of GeoGebra will be shown where algebra and geometry are connected allowing users to explore multiple representations of math concepts. Some data that shows the impact that the institute has had in the in-service teachers will be shared, in the district as well as in the university.

Introduction

The President's Council of Advisors on Science and Technology (PCAST) released an executive report in November 2010 where specific recommendations to the administration are given to ensure that the United States is a leader in Science, Technology, Engineering, and Mathematics (STEM) education in the coming decades. One of the recommendations is to recruit and train 100,000 new STEM middle and high school teachers over the next decade that are able to prepare and inspire students and have strong majors in STEM fields and strong content-specific pedagogical preparation. PCAST regards teachers as the most important factor in ensuring excellence in STEM education. Despite the ongoing efforts to promote the use of technology in education (e.g., National Council of Teachers of Mathematics [NCTM], 2000; National Educational Technology Standards for Teachers [NETS.T], 2008), teachers' ineffective use of technology has been reported in the literature. One reason frequently cited is that teachers are not trained in utilizing technology in the classroom within context.

In this paper we look at how the MSP Institute has used *GeoGebra* to model effective teaching/learning strategies for the sake of lasting improvement and change in teacher's pedagogies here in South Florida. Process and types of problems are shared in the paper that were used in the program to motivate inservice teachers mathematically as well as some research on using *GeoGebra* for teaching mathematics. Technology is one of the principles for teaching mathematics set out by the National Council of Teachers of Mathematics Standards (NCTM) (2000).

The Institute

The institute "Standards Mapped Graduate Education and Mentoring for Middle Grade Math Teachers" (SMGEM) was funded in 2004 by a grant from the National Science Foundation (NSF) and it is a partnership between Florida Atlantic University (FAU) and Broward County Public Schools, the sixth largest public district in the nation. The ambitious goal of the Institute was to eliminate the many crucial gaps in content and pedagogy between the university-level approach to a teacher's math and science preparation and the daily requirements of a diverse standards-driven classroom. The Institute created and delivered a unique complete, standards-aware, and technologically literate curriculum for graduate-level middle grade teacher education as an extension of the Master in Science in Teaching (MST) degree program of the FAU Department of Mathematical Sciences. By the end of summer 2011, 66 mathematics teachers from Broward County have received their MST degree through this program. In comparison to matched non-participating teachers, these teachers made significant mathematics content gains in categories related to the Florida Sunshine State Standards (SSS); implemented a wider range of pedagogies in their classrooms including more effective use of specific technologies; grew in their intentional emphasis on and mastery of the state standards; increased in their personal confidence in the mathematical content they taught, and moved in disproportionate number into leadership positions in local school system administration, in regional professional organizations, and in national professional presentation settings.

GeoGebra: Dynamic Mathematics Software

A powerful teaching tool of the Institute has been the integration and use of technology, more specifically *GeoGebra*. We chose *GeoGebra* as the software to be utilized in the institute for several reasons. First of all, *GeoGebra* is a free, multi-platform, open-source dynamic mathematics software. Thus, because of its open-source nature there are no licensing issues associated with its use, allowing students and teachers freedom to use it both within the classroom and at home. Secondly, *GeoGebra* combines dynamic geometry, algebra, calculus, and spreadsheet features (which other packages treat separately) into a single easy-to-use package making it suitable for learning and teaching mathematics from elementary through university. Thirdly, *GeoGebra* has a large international user and developer community with users from 190 countries. The software is currently translated into 55 languages and attracts close to 300,000 downloads per month.

The most powerful feature of *GeoGebra* is the connection it makes between Geometry, Algebra, Calculus and Statistics. *GeoGebra* is a dynamic geometry system in which you work with points, vectors, segments, lines, and conic sections. *GeoGebra* is also a dynamic algebraic system, where equations and coordinates can be entered directly. Functions can be defined algebraically and then changed dynamically afterwards. *GeoGebra* has a simple CAS in the background, which has the ability to deal with variables for numbers, vectors and points, find derivatives and integrals of functions and offers commands like Root or Extremum. These two views are characteristic of *GeoGebra*: an expression in the algebra window corresponds to an object in the geometry window and vice versa. The spreadsheet view has been added recently making it possible to enter data in the spreadsheet and view graphs in the geometry window while maintaining its dynamic characteristic. Although *GeoGebra* has been primarily intended for mathematics instruction in secondary schools, it certainly has uses in Higher Education and even now being brought down to the elementary levels as well.

Theoretical Framework

The need to make teachers proficient in the use of technology in the classroom is increasing rapidly. However, knowledge of the technology does not guarantee good use of the technology in the classroom. The question of what teachers need to know in order to incorporate technology into their teaching has received a great deal of attention in the last decade (International Society for Technology in Education, 2000; National Council of Teachers of Mathematics, 2000). Mishra & Koeler (2006) have introduced the term 'Technological Pedagogical Content Knowledge' in order to describe a framework for the teacher's knowledge necessary to integrate technology in the classroom. Knowledge of technology cannot be isolated from the content, and good mathematics teaching requires an understanding on how technology is related to the pedagogy and mathematics (Hughes, 2005). In the institute, GeoGebra has been integrated in all courses interweaving all three key sources of knowledge: technology, pedagogy, and mathematics.

The following are examples that illustrate some of the concepts presented in our courses and how *GeoGebra* helped the inservice math teachers explore, discover, and understand some very abstract math concepts.

Continued Fractions

The theory of continued fractions goes as far back as to the time of Euclid; however, *GeoGebra* can bring this old concept to modern times. Using *GeoGebra* dynamic features, students can discover how some numbers can have an elegant representation using continued fractions and how easy the theory can be understood by looking at the geometric representation.

Euler showed in 1737 that every rational number could be expressed as a terminating simple continued fraction. GeoGebra allows our teachers to make the same discovery in a graphical and dynamical manner. With *GeoGebra* it is very easy to create rectangles that have rational lengths and widths. After creating rectangles, one can proceed to divide the rectangle into squares in a very systematic manner. After the rectangle is filled with the biggest possible squares that fit inside, the dimensions of the rectangle can be written as a proper fraction and changed into a continued fraction. The connection between the algebra and the geometric representation of the continued fraction is immediate and powerful. The geometric representation of finite continued fractions in *GeoGebra* offers teachers the opportunity to discover a representation of rational numbers that is directly connected to the Euclidean Algorithm. This elegant, but very abstract algorithm, of finding the greatest common denominator becomes transparent to teachers once they see the geometric representation in *GeoGebra*.

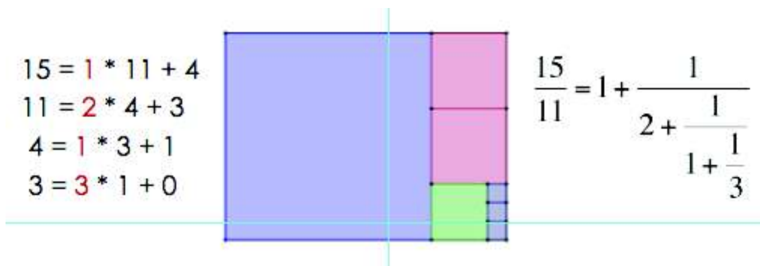

Figure 1: Euclidean Algorithm and Continued Fractions of Rational Numbers

The next natural question is what happens if the numbers are not rational. An investigation in *GeoGebra* with the Golden Rectangle helps teachers discover that some types of irrational numbers have an interesting pattern in their continued fraction expansion and that the expansion is infinite. The Golden Rectangle is one of the best examples to use and the representation of the golden ratio as a continued fraction is as beautiful as the Ratio itself. Convergence of the golden ratio can be explored from the continued fraction expansion and the connection between the golden ratio and the Fibonacci numbers is displayed naturally.

Further investigation in *GeoGebra* helps teachers recognize that only numbers that are solutions of quadratic equations have a periodic continued fraction expansion. The most amazing and crucial discovery is the relationship between similarity in the geometric representation and the periodicity in the continued fraction expansion. Lagrange used continued fractions to find the value of irrational roots and proved that a real root of a quadratic irrational is a periodic continued fraction. The inservice teachers in our institute have made the same discovery thanks to the use of *GeoGebra*.

The topic of continued fractions is a rich mathematics topic with many connections to the school curriculum. Teachers can explore different ways of representing rational and irrational numbers using geometric representations. The abstract Euclidean Algorithm for finding the greatest common divisor can be understood visually. The connection of periodicity and similarity is striking. Research supports the belief that students are better able to develop meaning for symbols and procedures when they can draw and manipulate graphical representations that are meaningful to them. With this topic not only our teachers learn mathematics content but also technology, since they have to construct the geometric representations in *GeoGebra*, as well as pedagogy since the teachers experience a beautiful lesson based on inquiry and discovery while constructing their own understanding mathematically.

Baravelle Spirals

Another example of the use of GeoGebra in the Institute and the middle school math classroom is the exploration of Baravelle Spirals. These simple constructions in GeoGebra can be explored geometrically and its relationship with infinite geometric series and patterns becomes a topic of crucial importance in the middle school math curriculum. Mathematically, the Baravelle Spiral is a geometric illustration of a basic calculus concept, the sum of an infinite geometric series.

Baravelle spirals are defined as nesting spirals formed by marking the center point of the sides of a polygon, then connecting the points of the adjacent sides forming an isosceles triangle inside of the original polygon. If this process is repeated in an iterative way, a series of triangles will form what it seems to be a curve but it is made entirely of straight lines. The spiral effect comes from the way the structure is constructed.

When considering the areas of the nested triangles of one of the spirals inside of the polygon, a geometric infinite sequence can be defined. The investigation at this point is if the sum of those areas approaches some specific number. This exploration can be reinforced with the use of the spreadsheet that is available in GeoGebra.

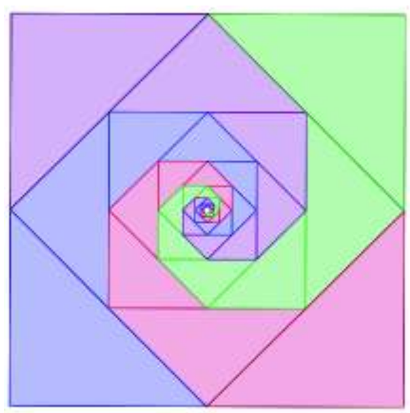

Figure 2: Square Baravelle Spiral

With a simple insertion of a slider in the GeoGebra construction, a nested series of triangles can be made that are not necessarily connected to the midpoint of the sides. This new simple modification brings more investigations on the sum of the areas, a review of trigonometry, the geometric series formed and the limit of the sum of the areas.

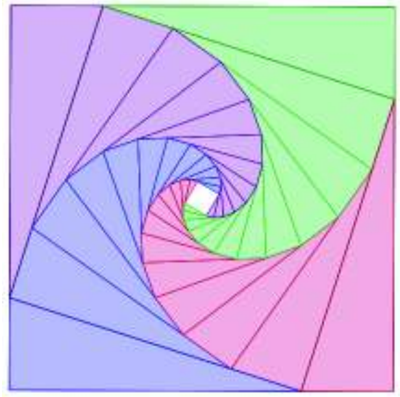

Figure 3: Modified square Baravelle Spiral

When considering polygons of different number of sides and the areas of the triangles that form the Baravelle spirals, a pattern can be discovered and a general formula can be found for the area of the first nested triangle and the ratio that reduces the areas of the consecutive triangles. With these two pieces of information, the limit of the sum of any polygonal Baravelle spiral can be found.

The construction of Baravelle spirals in GeoGebra is a fun activity that brings a numerous series of investigations and explorations in pattern recognition, geometric sequences, sum of infinite series, and even some trigonometry. From there students can go on to the study and exploration of other spirals.

Evidence of the Impact of *GeoGebra* on the MSP Program

Over the course of the last four years, repeated annual studies have demonstrated that the teacher/participants have increased their TPCK. Data was gathered using several instruments that included surveys, classroom observations, and interviews given to all participants in the cohorts.

GeoGebra was described as raising the enthusiasm for the effective and wise application of technology to the teaching/learning enterprise (Fahlberg-Stojanovska and Stojanovski, 2009;Hewson, 2009). Observations of participants in schools and during the summer workshops are also cited as evidence. *GeoGebra* was also credited with changing teacher habits. TWO features were specifically referenced as causing this change: 1) that it is an award winning software system, and therefore has admirable features, and 2) that it provides an effective pedagogical model for teachers.

The aspects of *GeoGebra* that make it award winning, insofar as those aspects are related to changing teacher habits, were described as supporting teacher demonstration, allowing students real-time exploration opportunities, and that even if the *GeoGebra* software is used and eliminated from instruction that it will have raised teachers expectations and standards for future technology-use. They also noted that these are benefits, at least in part, unique to *GeoGebra*.

GeoGebra was also described as providing an effective pedagogical model for teachers. It does so, by modeling the *Standards Mapped for Graduate Education and Mentoring Initiative*. Demonstrations of *GeoGebra* were explained as contributing to the modeling of effective pedagogy for cohort teachers. References to best practices being conveyed to teachers and those teachers using them were noted in the longitudinal observations of summer sessions and semester classes. Participants cited “enormous growth” in their ability to use technology with instruction. Evidence of this growth was generated from classroom observations and work with various classes and the carry-over use from one math course to another and even math education courses at Florida Atlantic University.

Further evidence of the program’s effectiveness to integrate curriculum, instruction, and technology was statistical. Over 14 teachers and 5 cohorts, based on the observation protocols of two researchers, the tendency of teachers to incorporate technology in their classroom jumped from 27.8 percent to 64.3 percent. Moreover the, difference between these two proportions was statistically significant at the ∀=2% level on a one tailed test.

An analysis performed over a large statewide data set suggested that there was a connection between the mathematical skills and activities the project promoted and student achievement. Evidence was provided of the samples involved in the study having higher-than-norm performance.

Conclusion

As described by Mishra and Koehler (2006) Technological Pedagogical Content Knowledge (TPCK) is the basis of good teaching with technology and requires not only content knowledge or pedagogical knowledge but an understanding of the representation of concepts using technologies, how to teach these math concepts using technology, knowledge on the challenges their students will face when presented with this new pedagogy, and how technology can be used to build on existing knowledge and develop new knowledge.

With the availability of dynamic mathematics software, like *GeoGebra*, teachers are able to make graphical representations of math concepts. As the concepts are introduced with pictorial representations, teachers and their students are able to make the connections between the pictures, the math concepts, and the symbolic representation. When presented with a new concept, students need to think, visualize and explore relationships and patterns. This is consistent with the CRA (Concrete, Representational, and Abstract) Model for teaching mathematics currently in better reaching students as they learn and understand mathematical concepts. Technology makes all of this possible for them in a short amount of time.

As a result of their participation in the activities of the MSP project and the use of *GeoGebra*, inservice math teachers gained experience not only on how to use the different tools of the software, but on how to use *GeoGebra* as a tool to enhance their own practice. This experience positively influenced teachers’ perspectives about the use of technology in the teaching and learning of mathematics and increased their own confidence in their ability to use technology in their practice. Moreover, their increased TCPK has impacted their students to the extent of prompting higher scores on annual Florida achievement tests. Many of our teacher/participants have been empowered for district-wide and state-wide influence, and to some degree a number are beginning to exert a national presence.

In the MSP project, effective teaching practices are modeled and a collaborative learning environment are created in which teachers are encouraged to develop strategies and ideas about teaching and learning mathematics while using the technology. The math content has explicitly utilized and intentionally emphasized foundational middle school themes, but this in a way that is new, often discovered through technology based experiments in *GeoGebra* and regularly connected to modern research and applications. The pervasive inclusion of cutting edge software like GeoGebra into our program activities raised teacher perception of the significance of their work, and this sense of importance worked to elevate their enthusiasm for investing time, work, and energy into their own growth. See Appendices A for additional *GeoGebra* resources.

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Appendix A: *GeoGebra* Related Websites

GeoGebra Related Websites

GeoGebra South Florida Institute: http://www.geogebra.org/en/wiki/index.php/GeoGebra_Institute_of_South_Florida

GeoGebra: GeoGebra.org

GeoGebra Wiki Forum: http://www.GeoGebra.org/en/wiki/index.php/Main_Page

GeoGebra Data Files: http://matharoundus.com/GeoGebra