THE ART AND SCIENCE OF DESIGNING HIGHER LEVEL CLICKER QUESTIONS

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Introduction

Posing and answering questions is at the heart of learning of mathematics. When studying mathematics we try to answer questions by making and proving conjectures, by using heuristics to solve problems, or by organizing information into appropriate representations. We look for specific patterns, apply concepts, find counter-examples, or develop proofs. Mathematics is an active study of ideas to find solutions, to determine the validity of statements, and the limitations of methods.

When technology became available to use in the mathematics classroom that was based on the use of multiple-choice questions, we were initially skeptical but very intrigued. Other disciplines particularly the sciences were finding that students were learning concepts as well if not better than before the use of personal response systems (Mazur) [1]. Students were engaged and actively involved in the discussion of new material and in the review or application of ideas. Students were seeing immediately if they misunderstood a concept through timely feedback during class time and well before a summative assessment. Peer discussion was integrated into the learning of the discipline.

At first glance the use of multiple-choice questions seemed unusual perhaps because these items would be in text as part of the materials for the course. In our experience, questions that were posed in our quizzes and examinations were not multiple-choice. Students would show their work as they evaluated, determined, solved, and represented mathematics. As instructors we would see how they approached problems in written form and learn about misconceptions in these end of unit/chapter assessments.
Even our labs, projects, and homework assignments did not use multiple-choice items.

Yet, when you think about the nature of teaching mathematics and of the verbal communication that takes place in the classroom, rather than just what is in text or print, we often use questions that have a more narrow focus. Is this statement true? Followed by how do you know this? Is this closed? Over what domain? Which of these representations is the most appropriate? Why? Usually the narrow questions are the starting point into the solving or representation of a problem. The follow-up questions that go deeper into the process allow us to better gauge how much our students understand. But if these questions are verbal only and only a few students respond or even if many nod their head what does this mean?

As instructors we have seen common misconceptions that now we could address directly in the form of multiple-choice questions and learn immediately how many of our students were struggling with these ideas. So the evolution of our pedagogy incorporating pre-written key multiple-choice items for class began. Fortunately, during this time many projects were taking place to develop collections of "clicker items", particularly for the study of calculus. Some projects began to focus on levels of questions and items were developed that were quick fact checks, concept items, or application items. In the Good Questions Project at Cornell, see http://www.math.cornell.edu/~GoodQuestions/, calculus items were developed and identified as Quick Check, Probing, and Deep (Terrell) [2]. Meaningful multiple-choice items were also being developed as part of the Project Math Quest (Zullo et.al.) [3] and for CRS in Statistics Courses (Murphy) [4].

At the time of our initial work, these projects did not have published items for the teaching of college algebra. Yet, their work provided ideas for such development. Over the past year several textbook authors/writing teams were developing collections of such items to correspond with specific textbooks. Now Project Math Quest also has pre-calculus items available at their site. These sources include both the questions and the answers. Sometimes annotations are given, but usually a cognitive level is not identified with the items as in the Good Questions Project for calculus.

As we developed new items, we found it helpful to use a framework for levels of questioning. We chose to use the new version of Bloom's Taxonomy developed by Lorin Anderson, et. al. with the six levels of remembering, understanding, applying, analyzing, evaluating, and creating (Anderson & Krathwohl) [5]. These levels correspond to increasing cognitive demand from the beginning recall of knowledge in the remembering level, to the high level of creation where students synthesize ideas to create a new "product". The benefit of using this framework is that the taxonomy is well recognized and defined and has been used in either the original or new version for over fifty years.

Sample Item Design Discussion

Let us first consider several variations of the same problem. Figure 1 shows the original form of our question focusing on an understanding of the relationship between the number of unique factors of a polynomial equation and the number of its solutions over the domain of the real numbers.

How many solutions does the equation
$(x+4)(2x+5)=0$ have?

a) 0
b) 1
c) 2

Figure 1 1st Version of Algebra Item

Depending on the student's prior learning experiences, the question posed in Figure 1 may be a basic recall of knowledge question, but the fact that they must consider carefully the factors themselves in the equation also requires that the student have conceptual knowledge about the nature of the solutions and how these two linear factors provide solutions to the equation. When problems of this type were discussed in our algebra classes, about a third of the students initially voted for b) 1. At this point we found it crucial to ask the students to explain their reasoning rather than to first solve for the solutions to the equation. The questioning and discussion that followed were very enlightening to understand what students were thinking about numbers and solutions. Many of the students felt that only the factor of (x+4) would provide a solution since it would provide a "whole" number result and the factor of (2x+5) would provide a fraction so could not be considered a solution to be counted. Does this belief arise from solving many practice problems that have integer rather than rational non-integral solutions?

Now consider the two similar items in Figures 2 and 3. Figure 2 is a true/false item and Figure 3 is a multiple-choice item with an additional response selection. (Both items may be considered multiple-choice items in terms of the voting and the use of personal response systems, since both provide a few select responses for student input.)

The equation $(x+4)(2x+5) = 0$ has 2 solutions.

a) True
b) False

Figure 2 True/False Version of Item

How many solutions does the equation $(x+4)(2x+5) = 0$ have?

a) 0
b) 1
c) 2
d) 3

Figure 3 Extra Option Version of Item

What is gained or lost in rewriting our original item? In the true/false form we focus only on the choice of 2 solutions. Is this done to focus on their attention on 2 linear factors, 2 solutions? Is this a better way to focus this attention? In the third variation an extra selection is added for discussion. Can a quadratic equation or a polynomial equation with two linear factors have 3 solutions? Do we need to address this directly?

Now consider the variations of this problem in Figures 4 and 5.

How many solutions does the equation $(x+4)(2x+8) = 0$ have?

a) 0
b) 1
c) 2
d) 3

Figure 4 Double Root Version of Item

How many solutions does the equation $(x+4)(2x^2-8) = 0$ have?

a) 0
b) 1
c) 2
d) 3

Figure 5 Version with 3 Solutions

As you can see, the questions in Figures 4 and 5 are addressing unique solutions and the importance of completely factoring each of the factors of the equation. Sometimes in student work the expression for a factor may yet contain a common factor or may need to be factored further as two binomials. Cognitive dissonance may occur when these questions follow our first question. What is meant by unique solutions? Why did we have factors that look very similar to our original problem and yet arrive at a different result? Now in our discussion we learn that the number of solutions has reduced. But we still have 2 factors. Similarly, why does changing the second factor from (2x+8) to ($2x^2$-8) result in 3 unique solutions instead of 1 or 2 solutions? What if the second factor was ($2x^2$+8) instead of ($2x^2$-8)? What if the domain changed from the set of real numbers to the set of complex numbers? What if the domain did not change?

The depth of knowledge and understanding required to determine the number of unique solutions given the factors, albeit sometimes incompletely factored expressions, is increasing over the set of similar questions. It is not only the level of items but also the increasing depth or scope of the items developed for class or adapted in class at the time the misconceptions occur, that is important.

Let us consider two graphical items depicted in Figures 6 and 7 below. A graph of any polynomial function of degree three or less may be displayed as part of the item for Figure 6 and any of the four functions given may be displayed for item 7. Alternatively, all four curves may be graphed and labeled and students could be asked to identify which of the curves would represent a specific function.

The function f(x) is displayed in this graph. How many zeroes (roots) does it have?

a) 0
b) 1
c) 2
d) 3

Figure 6 General Item

What function is displayed in this graph?

a) f(x) = (x-4)(2x-5)
b) g(x) = (x-4)(2x-5)
c) j(x) = (x+4)(2x+8)
d) k(x) = (x+4)($2x^2$-8)

Figure 7 Specific Function Item

Again these new questions consider the zeroes or roots of polynomial functions, but now students must analyze graphs and apply their knowledge of both this representation form and their knowledge of zeroes of a function. In the question in Figure 7, the students must also determine what the roots are to determine the answer to the question whereas in Figure 6 students may be asked to determine the zeroes after answering this question.

Application of understanding of zeroes of polynomial equations may also be accomplished by real world application items. The model of the flight of a projectile with height as a function of time is an appropriate physics application that could be examined as it relates to a quadratic relationship. The time elapsed until the object lands after flight (so has a height of zero) would be one of the zeroes of the polynomial function. Multiple-choice items could be developed and discussed that focus on a realistic domain for this application, the maximum height achieved by the projectile, as well as items concerning the zeroes of the model.

Another application where understanding the forms of a quadratic equation is exemplified in the following function along with voting items in Figures 8, 9, and 10. This profit-price model and associated items were developed by adapting the model from a problem of McCallum et.al. (McCallum)[6].

A company sells a specific item at the price, d, in dollars. The profit P(d) earned from the sale of this item, in thousands of dollars, is a function of the price of the item.

P(d) can be expressed in standard, factored, or vertex form as given here.

Standard form: $P(d) = -3d^2 + 42d - 120$
Factored form: $P(d) = -3(d - 4)(d - 10)$
Vertex form: $P(d) = -3(d - 7)^2 + 27$

Which form is most useful to find the prices that make a profit of $0?

a) Standard form
b) Factored form
c) Vertex form

Figure 8 Prices for Profit of $0

Which form of the model is most useful to find the profit if the price is $0?

a) Standard form
b) Factored form
c) Vertex form

Figure 9 Profit when Price is $0

Which form is most useful to find the price that makes the maximum profit?

a) Standard form
b) Factored form
c) Vertex form

Figure 10 Price for Maximum Profit

Each of these questions requires students to analyze the forms of the models carefully and apply knowledge of factored, vertex, and standard forms of a quadratic equation. Students are also translating within the problem between an application in written form and three symbolic forms of the model. Students are using a meaning in context for the y-intercept, x-intercepts (zeroes), and the vertex of the equation. This set of items focuses students' attention on the utility of all three forms of the quadratic in a realistic situation. Students are evaluating the form that is most appropriate to accomplish the goal in each of the multiple-choice items. For these items, most students will utilize five of the six levels of the updated Bloom's Taxonomy, the actions of remembering, understanding, applying, analyzing, and evaluating.

You will notice that none of these multiple-choice items exemplifies the highest level of creating directly in the evaluation or deliberation of the item. However, the follow-up activity that is accomplished may lead to a new discovery as in our early examples in this paper of the creation of a conjecture relating the number of factors in a "completely" factored polynomial equation and the maximum number of unique solutions over the domain of real numbers. Such a conjecture may be developed by students working in small groups, collecting data on polynomial equations and solutions in an exploratory lab.

Conclusion

The development of multiple-choice questions for formative assessment, reflection, and for guiding learning has proved to be an interesting process for both the instructors and our students. The connections between the items used in a class period and the deepening level of the questions are important aspects in this pedagogy in the teaching and learning of mathematics. Our students are providing us with more information about their understanding of mathematics through our follow-up questions and discussion of the mathematics. The multiple-choice items are really a launch into these explorations. It is truly the careful follow-up and by listening to our students' explanations that we can understand their beliefs about mathematics. This process then opens the opportunity for us to support their learning.

References

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Solutions to Clicker Items

Assume the domain for these items is the set of real numbers.

Item 1 in Figure 1: Answer is c) 2 unique solutions. Solutions are x= -4 and x= -5/2.
Item 2 in Figure 2: Answer is a) True there are 2 unique solutions. (-4 and -5/2)
Item 3 in Figure 3: Answer is c) 2 unique solutions. (-4 and -5/2)

Item 4 in Figure 4: Answer is b) 1 unique solutions since the factor 2x+8 would be factored as 2(x+4) so now there is a double root, a solution of only x = -4.
Item 5 in Figure 5: Answer is d) 3 unique solutions since the factor $2x^2$-8 would be factored as 2(x-2)(x+2) along with the other original factor of x+4, giving 3 solutions of 2, -2, and -4.
Item 6 in Figure 6: Depends on the function graphed. If the general function, f(x) =(x-a)(x-b) was displayed on the graph, labeling a and b as the x-intercepts on the graph, a≠b, the result would be c) 2 zeroes, located at x= a and x = b.
Item 7 in Figure 7: Depends on the function graphed. If the graph of the function(x) = $(x+4)(2x^2 – 8)$ = 2(x+4)(x-2)(x+2) is displayed on the graph then the result is d) k(x)= $(x+4)(2x^2 – 8)$. Students would note the values of the x-intercepts or zeroes of the function to help determine this result and would also discuss the factorization of $2x^2$-8.
Item 8 in Figure 8: If P(d) = 0, the most appropriate form to use to solve for d directly is the factored form, answer b. Since this provides two solutions for d immediately.
Item 9 in Figure 9: When d = 0, the most appropriate form to use to solve for P(d) is answer a) standard form. When 0 is substituted for d, the value -120 is arrived at directly. Students would discuss what this negative value signifies.
Item 10 in Figure 10: It is helpful if students recognize that the graph of this function is concave down and the vertex form will provide the maximum height, answer c.