

A DYNAMICAL SYSTEM MODEL OF INFORMATION OPERATION EFFECTS

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Irregular warfare or low intensity conflict is not a new form of warfare. For centuries, this form of warfare has been the preferred method of weaker forces to attack a more powerful adversary, and important to this type of warfare is the ability to control or influence the population. Conventional military models have consistently proven that they are not equipped to handle the focus on the human element, as well as the importance of winning popular support during this conflict. To counter this deficiency, the Department of Defense Analysis (DA) at the Naval Post Graduate School (NPS) has developed a multidisciplinary program of instruction to better equip student research efforts to gain insights and key relationships between an insurgency and the population.

Students enrolled in the Department of Defense Analysis program cover a wide spectrum of experience, capabilities, and academic backgrounds. Each class cohort is comprised of members from all services, and approximately 30 percent of the cohort is international officers. The typical student is a Special Operations officer with seven to ten years of operational experience. Most students in the program do not have a math or engineering background, and college algebra is typically the last math course the students have taken. The multidisciplinary program of instruction is constructed to enhance the student's ability to think clearly, creatively, and analytically. Among the course work for those working toward a master's degree in Defense Analysis is a three course sequence in mathematics that includes dynamical systems, probability, statistics, and game theory.

Mathematical Modeling is the first course in the sequence of instruction. During this course, students are exposed to modeling with dynamical systems, covering topics such as exponential growth and decay, logistics growth, predator-prey, competing species, the S-I-R model of infectious disease spread, Lanchester's equations, empirical modeling, and linear programming. The second course in the sequence is Modeling for Military

Decision Making. This course builds on the Mathematical Modeling course and introduces probabilistic models, statistical concepts, and simulation.

Since Special Operations officers can engage in winning the “hearts and minds” of a population, constructing a model involving the effects of influencing a population is pertinent. What follows is a two-part project that spans both the Mathematical Modeling and Modeling for Military Decision Making courses and leads students through the development of a discrete model of state change that incorporates distributions.

Assume that a particular system has eleven states, S_0, S_1, \dots, S_{10} that might represent the distribution of public opinion regarding a particular issue, its acceptance of a new idea, its sympathy toward rebellion (rejection of authority), etc. We construct the model so that state S_0 reflects that portion of the population that completely rejects (or is completely hostile to) the issue, authority, new idea, etc. On the other hand, S_{10} reflects that portion in complete agreement with the issue (acceptance of authority, the new idea, etc.). State S_5 reflects ambivalence, no opinion, or perhaps ignorance of the matter.

Assume that:

- a sufficiently large, random sample of the population has a distribution that can be apportioned to the S_j , $j = 0, 1, \dots, 10$, so that the system’s states reflect the larger population’s rejection or acceptance of the matter being modeled. It follows, that $\sum_{j=0}^{10} S_j = 1$ at all times.
- there is no state lower than S_0 and none higher than S_{10} .
- state distributions can change with time, presumably due to influences such as the media, key persons, studies, observations, reflection, etc.
- time intervals are taken to be sufficiently short so that transitions occur only between adjacent states. That is, during the period n to $n+1$, whatever change occurs takes place from S_j to S_{j+1} or S_j to S_{j-1} .
- at any time n , a portion of S_j might transition from S_j to S_{j+1} at rate λ_j and from S_j to S_{j-1} at rate μ_j .
- $0 \leq \lambda_j \leq 1$ and $0 \leq \mu_j \leq 1$.
- the transition rates, λ_j and μ_j , depend on the state, j , and they might also depend on time, n .
- an initial distribution $S_0(0), S_1(0), \dots, S_{10}(0)$ is given, known, or assumed.

Figure 1 is a depiction of the various states and transmission coefficients described in the assumptions.

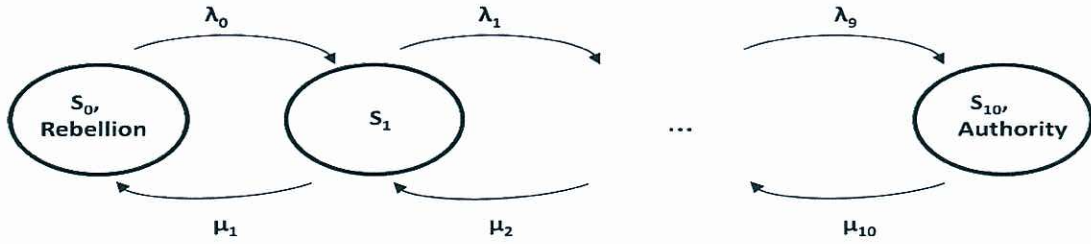


Figure 1 Project state change schematic.

Therefore, at time $n+1$, a portion of the population is in state S_j provided:

- at time n , a portion of the system that was in state S_j remained in S_j .
- at time n , a portion of the system transitioned (upward) from state S_{j-1} to S_j .
- at time n , a portion of the system transitioned (downward) from S_{j+1} to S_j .

Then the time evolution of state S_0 , can be described as

$$S_0(n+1) = S_0(n) - \lambda_0 S_0(n) + \mu_1 S_1(n),$$

and the time evolution of state S_{10} as

$$S_{10}(n+1) = S_{10}(n) - \mu_{10} S_{10}(n) + \lambda_9 S_9(n).$$

Note that in the above equations, we have taken λ_j and μ_j to be time independent.

Students are then tasked to:

1. Develop equations describing the time evolution of the other nine states.
2. Assume that the transmission rates are $\lambda_0 = \lambda_1 = \dots = \lambda_{10} = 0$ and that $\mu_0 = \mu_1 = \dots = \mu_{10} = 1/11$. Also, assume that $S_0(0) = S_1(0) = \dots = S_{10}(0) = 1/11$. Find and plot the distribution of the system states $S_j(n)$, $j = 0, 1, \dots, 10$, at time $n = 0, 25, 50, 100, 200$, and 500 . Describe this scenario and its evolution.
3. Assume that the transmission rates are $\lambda_0 = \lambda_1 = \dots = \lambda_{10} = 1/11$ and that $\mu_0 = \mu_1 = \dots = \mu_{10} = 0$. Also, assume that $S_0(0) = S_1(0) = \dots = S_{10}(0) = 1/11$. Find and plot the distribution of the system states $S_j(n)$, $j = 0, 1, \dots, 10$, at time $n = 0, 25, 50, 100, 200$, and 500 . Describe this scenario and its evolution.
4. Assume that the transmission rates are $\lambda_0 = \lambda_1 = \dots = \lambda_{10} = 1/11$ and that $\mu_0 = \mu_1 = \dots = \mu_{10} = 1/11$. Also, assume that $S_j(0) = 0$, $j \neq 5$, and $S_5(0) = 1$. Find and

plot the distribution of the system states $S_j(n)$, $j = 0, 1, \dots, 10$, at time $n = 0, 25, 50, 100, 200, 500$, and 1000 . Describe this scenario and its evolution.

5. Repeat requirement 4, but modified as follows: assume that $S_j(0) = 0$, for all $j \neq i$, and $S_i(0) = 1$ for some $i \neq 5$. What changes did you note?

6. Experiment with initial distributions and transmission coefficients, and then discuss results.

Students implement their model using Microsoft Excel, and they are allowed to work in groups of no more than three. Typically, there is a sufficient number of foreign students in each class, so each group is charged to contain at least one person from another country. Students are given one week to complete this portion of the project. Group submissions are required to describe the modeling process, to convey their understanding of the problem, and how they solved it, to include figure captions, and to submit a product that could “stand alone.” Figure 2 and Figure 3 provide an example of student work for requirements 4 and 6, respectively.

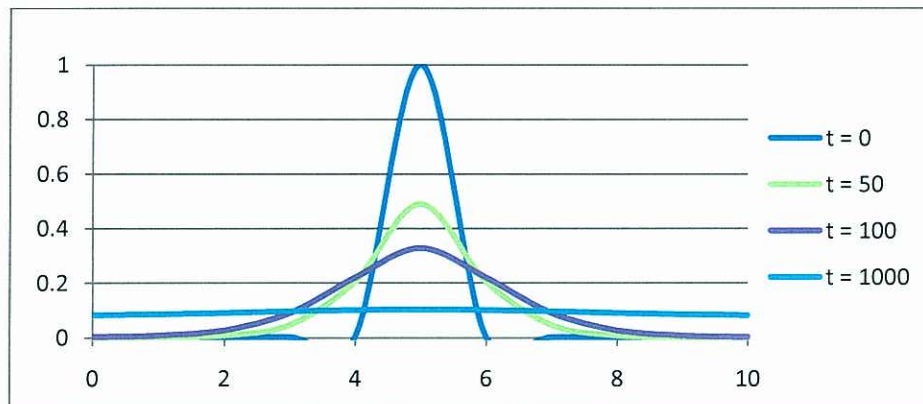


Figure 2 State evolution of an initially perfectly ambivalent sample, $S_5(0) = 1$, subjected to equal leftward and rightward transmission (requirement 4).

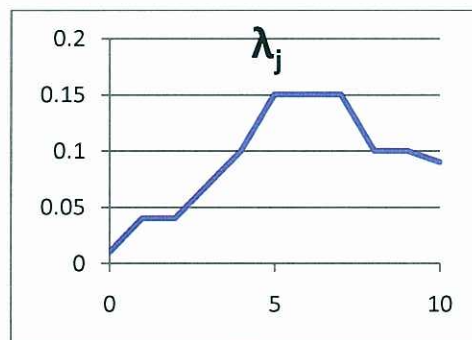


Figure 3 Student-created, time independent rightward transmission distribution as a function of state (requirement 6).

One part of the mathematical modeling process is to identify shortcomings in order to improve the model. Among the shortcomings of the model:

- in the presence of only leftward (or rightward) influence, the entire population moves to the left-most state (or right-most). It is unlikely that this actually happens.
- transition rates are not persistent (rather than constant); however, some events are more persistent than others.
- movement between states is not necessarily due exclusively to media or other influences; events themselves can affect perceptions and thus can lead to changes in state distributions.

To rectify those shortcomings, students are then tasked to:

7. Develop a model that incorporates diminishing transition rates.
8. Demonstrate the effect of diminished transition rates on an initial, uniformly distributed population, and comment on the results.
9. Subject an initial, uniformly distributed population to a sequence of randomly generated transitions that diminish with time; that is, subject the population to both leftward and rightward influencing events. (The sequence of randomly generated transitions should be introduced at random times.) Discuss your implementation and results. Figure 4 depicts an example of student work for requirement 9.

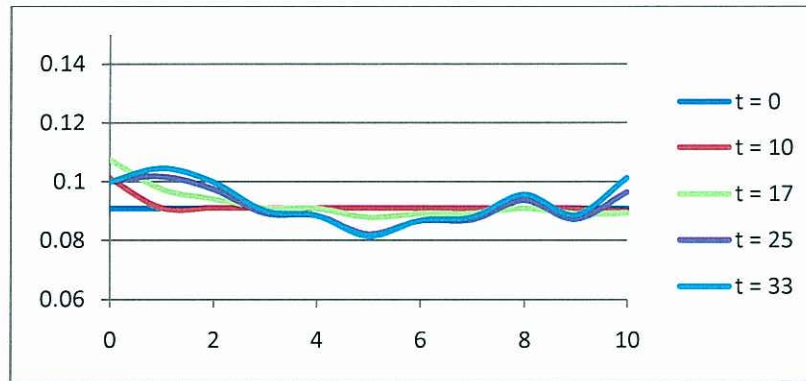


Figure 4 Student submission depicting the effect of diminishing transmission rates on an initially uniformly distributed population. Application times are shown in the legend. Transmission rates are those depicted in Figure 3.

As part of winning “hearts and minds”, one might consider the distribution of $\lambda_j(n)$ and $\mu_j(n)$ to be influence strategies for the authority and rebellion, respectively. We impose the requirement that the influence effort of the respective side satisfies $\sum_{j=0}^{10} \lambda_j(n) \leq 1$

and $\sum_{j=0}^{10} \mu_j(n) \leq 1$. That is, the time-dependent transmission rates can be distributed among the various states (j), so that the total influence effort sums to at most 1.

Students must then choose a side (rebellion is state $j = 0$, and the authority is state $j = 10$), and then address the following:

10. Assume and describe an allocation strategy that your opposing side might implement. Counter your adversary's allocation strategy with one of your own. Assume an initial, uniformly distributed population, and then implement the two strategies (assumed adversary's and your own) through (i) persistent application of the two opposing strategies, and (ii) randomly introduced, non-persistent influence events shaped to reflect your strategy as well as that of your opponent. Discuss your implementation and its results. (Examples of student submissions for this requirement are shown in Figures 5 and 6.)

11. What practical means could one side implement to counter the adversary's influence strategy?

12. What are the most important aspects that would make one susceptible to influential media? What are the most effective forms of media influence? How could they be incorporated into this model? What affects the persistency of influential events or influential media? Discuss.

13. Are the most effective aspects and forms of influence culturally independent? Discuss.

14. Humans are susceptible to "herd" effects, meaning that others' collective opinions and beliefs can influence acceptance of ideas, beliefs, perceptions, etc., regardless of the sanity, veracity, or efficacy of the general, accepted perception. Could this be incorporated into the influence model? How? What could be done to counter the "herd" effect influence? Discuss.

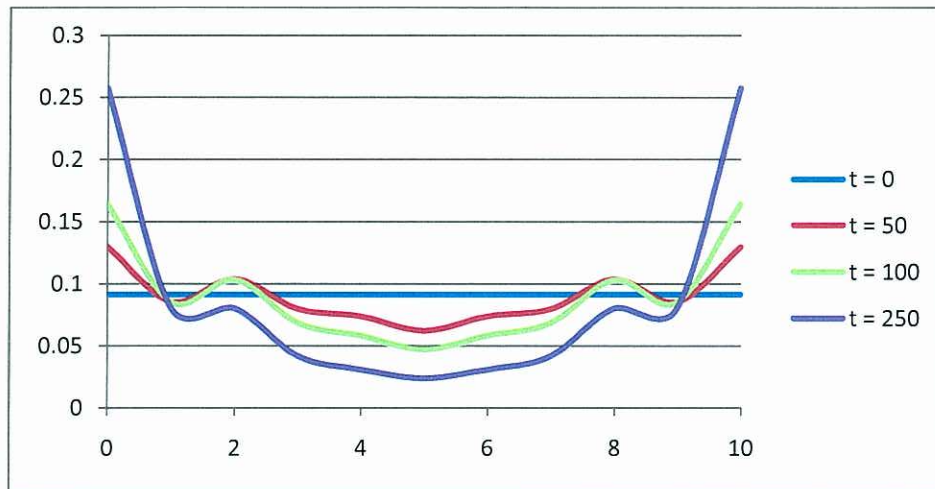


Figure 5 Time evolution of state distribution based on influence strategy depicted in Figure 3 on a uniform initial opinion distribution. Leftward influences mirror rightward influence.

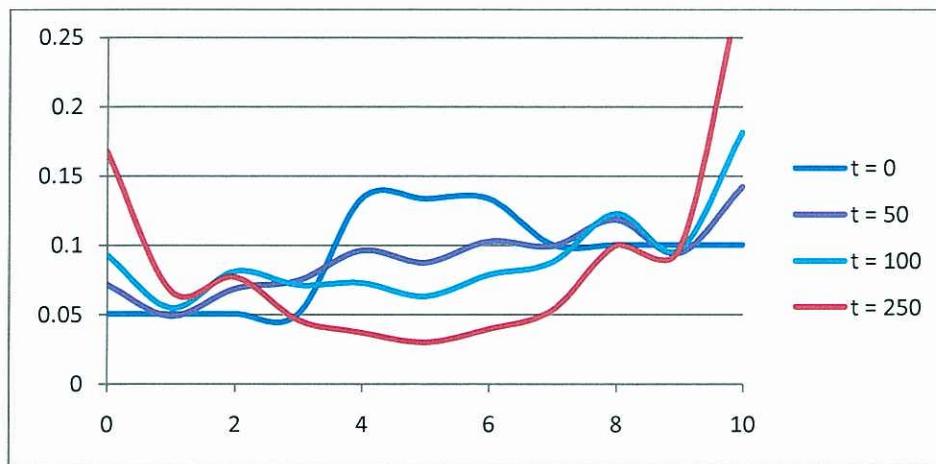


Figure 6 Time evolution of state distribution based on influence strategies depicted in Figure 3 on a non-uniform initial opinion distribution. Note that final opinion distributions differ from that in Figure 5.

Describing and predicting the behavior of any complex dynamical system with a pure analytical approach is a difficult prospect. This effort is further compounded when trying to capture the complexities of human interactions or the influence of a message on an individual or group. In recent years, agent based models (ABM) have captured the attention of social scientists in their efforts to better understand the macro impact of micro-level human interactions and interdependences. ABM allows analysts to create and experiment with an artificial world populated by interacting agents. These agents act, interact, and react to their changing environment according to a set of micro-level behavioral rules. The objective of these models is to search for the emergence of phenomena from the micro level of systems to the more complex macro level. For an

introduction to agent-based modeling, see Epstein and Axtel's *Growing Artificial Societies*¹.

The work here presents a simple influence model, designed to reinforce DA students classroom experience. The model's objective is to build upon and extend the student's analytical experience gained from the influence model developed in their Mathematical Modeling course. The dynamics of influence and its impact on a population and its attitude toward an established institution is the focus of this effort. The model involves two categories of actors. "Agents" are members of the general population and may be influenced in their attitude, say, toward an institution. This institution could represent anything from the government, financial organizations, or groups of people. "Influencers" represent persistent messages that either support or oppose the institution. These influencers could be media organizations, activist groups, or a government spokesman. Let's first tackle the description of the agents.

In any grievance model, there must be some representation of grievance. We treat grievance at a given time (t) in this model as simply comprised of two components, attitude (A_t) and opinions (O_t). Attitude represents an agent's current support for a given institution and identifies an agent's leaning as being either pro or con. An agent's attitude can vary over an arbitrarily defined range of 0 to 10; initially, all agents will start in an ambivalent state, neither pro nor con toward the institution. The model contains the ability for a heterogeneous mix of attitudes, but to reinforce the recent analytical modeling of influence it is not employed in this experiment. As an agent interacts with other agents, its attitude could be altered due to others' opinions, here taken to be the average attitude of all individuals in a neighborhood of an agent at any given time.

An agent's attitude toward the institution in the next time period is assumed to be based on these two variables. There are many potential relationships but this experiment assumed:

$$A_{t+1} = A_t - \frac{A_t - O_t}{2}.$$

The logic behind this function is simply that an agent's current attitude is affected by the opinions (the average attitude) of the agents around him. The function suggests that an agent is willing to change its current attitude to meet the group half-way toward their collective opinion.

The influencers are much simpler in design. There exist two types of potential influencers at any given time, a pro and con influencer. These persistent influencers interact with other agents attempting to change their attitudes to match that of the influencer and their goal is to spread their message to all agents. Influencers never

change their attitude and start with either a completely pro attitude, $A_0 = 10$, or a completely con attitude, $A_0 = 0$.

These agents interact on an artificial world or torus lattice that is 33 by 33 patches in size. Agents and influencers randomly move around this space and interact. The movement rule is the same for both agents and influencers. Each turn all agents and influencers move to a random lattice position within their vision range. For this experiment, the vision range was set to one lattice position for both agents and influencers. This implies that each entity randomly chooses one of eight surrounding squares to move into during its turn. Although we have fixed the range of movement and limited the amount of information an agent may receive, the number of agents and influencers an agent may encounter is heterogeneous because of the random movement.

We are interested in the dynamics of persistent message effects, as well as attitude dynamics toward an institution when confronted with conflicting, persistent messages. To track the level support, defined as the number of agents supporting a given institution, the experiment groups each agent in one of 10 states, $S_1 - S_{10}$ depending on their level of grievance towards the institution.

Each case study consists of a fixed population of 99 agents and various numbers of influencers. The goal is to analyze the effects of subjecting the population to various degrees of pro or con messages (interactions). The first case study looked at the impact on the average state (level of support) of the population subjected to equal leftward and rightward influencing messages. Figure 7 depicts the case study at $t = 0$, with 99 agents (white) and the persistent con (red) and pro (blue) influencers. Figure 8 captures the

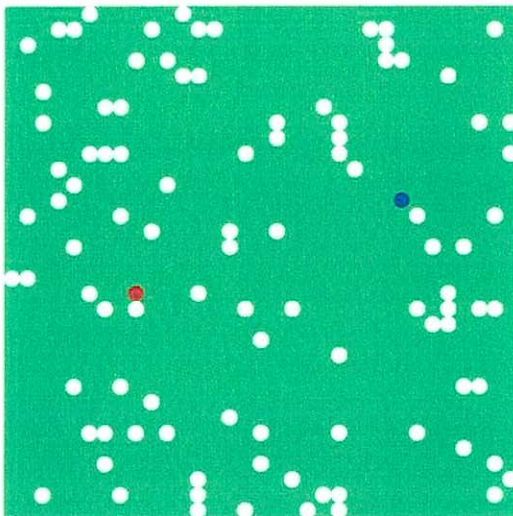


Figure 7 A representative initial condition of 99 ambivalent agents (white) with a persistent agent of each type; pro-issue (blue) and con-issue (red).

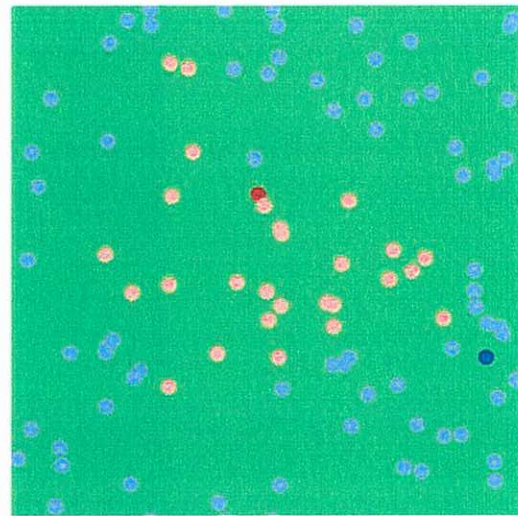


Figure 8 After 500 time steps, the distribution of agent colors reflects the influence of the persistent agents on their random walk. Light blue and light red indicate that the initially ambivalent agent has been influenced in the blue or red direction by a persistent agent or its neighboring agents.

agents' state distribution at $t = 500$. The image indicates that there are now 27 agents (light red) who are leaning toward not supporting the institution and 72 agents who still have some degree of support for the institution. Table 1 provides a snapshot of the average state of all agents from $t = 0$ to 10,000. The table clearly indicates, as expected, that the average state of the population hovers somewhere around 5.0 (ambivalence toward the institution).

Time	State Evolution (Equal Trnasmission Rates)										Mean
	1	2	3	4	5	6	7	8	9	10	
$t = 0$	0	0	0	0	99	0	0	0	0	0	5.00
$t = 50$	1	0	3	7	54	25	4	3	0	2	5.15
$t = 100$	0	6	2	7	28	41	11	3	1	0	5.10
$t = 1000$	0	0	1	7	48	33	5	3	1	1	5.03
$t = 10000$	0	0	1	4	60	27	6	1	0	0	4.84

Table 1 Average state distribution of agents for various times. Mean value of state distribution is found in right column.

Other cases studies studied might include the impact of unequal persistent messages. For instance, what effects might one expect when the con persistent message is twice as prevalent as the pro-leaning message? What if there was not a pro-leaning message?

Among the goals of this project is to have students consider the effect of force presence and actions on contested populations. Later, exposure to more sophisticated agent based programs might help students to better inform commanders on how to influence populations through understanding the host population's beliefs, values, and interests. In addition, students are charged with a bigger question: are there culturally transcendent features of influence structures and mechanisms?

ⁱ Epstein, J., and Axtell, A. (1996) Growing Artificial Societies: Social Science from the Bottom Up (MIT Press, Cambridge, MA).