

# GOOD TEST PROBLEMS FOR CALCULUS COURSES THAT REQUIRE THE TI-89 GRAPHING CALCULATOR

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When the Mathematics Department was faced with replacing the TI-86 graphing calculator with the CAS equipped TI-89 calculator in 2003, there was some concern that providing students with a machine that could do algebra and calculus would reduce the course to a series of lessons on button pushing. With the TI-86 being phased out the only other option was to require the TI-84. However, that calculator was insufficient for use in the calculus sequence, differential equations, and linear algebra courses.

In response to this change we began to put together problems that would adequately assess student understanding while allowing them full use of the technology. Over the past six years we have written and collected exam problems and projects that are challenging, benefit from use of technology yet cannot be solved exclusively by a series of keystrokes. Our primary goal is to assist our students to develop a better conceptual understanding of calculus through use of CAS technology while avoiding a strictly button-pushing approach.

This paper discusses a variety of problems that have proven to be successful assessment tools in Calculus I and Calculus II. In compiling a database of problems for our classes, some problems have been carefully selected from the Stewart Calculus book used for this course sequence. Others have been written by Mathematics faculty members. The following are examples of those written or modified by our faculty. These are currently used on tests and departmental final exams. Applications problems and projects which successfully blend technology use with conceptual instruction constitute a significant portion of course assignments and are covered in a separate paper.

**Function Notation.** The TI-89 has the ability to store user-defined functions with standard notation. With either define or store, function operations can be done on the homescreen.

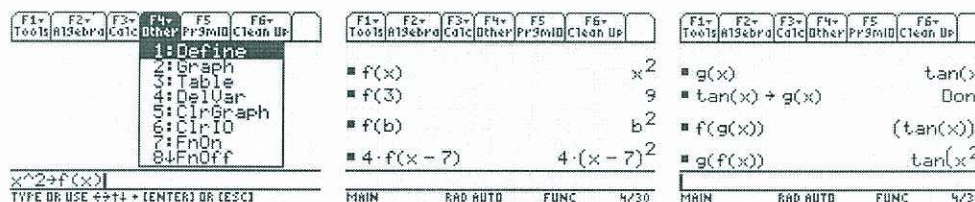


Fig. 1. Function notation on the TI-89

Use of this feature helps to reinforce function notation and function operations, a concept that frequently causes confusion in weaker students. Problems 1, 2, 5, and 6 below make use of stored functions.

**Equation Solving.** The solve( ) feature of the TI-89 reduces the emphasis on algebraic manipulations and facilitates the presentation of complicated and more in-depth problems. While it is essential to demonstrate some algebraic solutions by hand, it is advantageous to allow and train students to make best use of the solve( ) feature. Benefits include helping eliminate student failure due to weak algebra skills and an increased focus on the calculus concepts. Problems 2, 5, and 6 utilize the solve( ) feature.

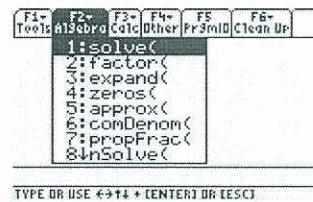


Fig. 2. The solve( ) feature

**Definition of the Derivative.** This problem requires that the student has an understanding of definition of the derivative, the relationship of the secant line to the tangent line, basic graphing, and has a facility with function notation. Parts (a) and (c) ask for answers in a form that elicit an informed response rather than a press of a key. Because the TI automatically simplifies its answers, asking for the unsimplified version results in most students finding the secant line slope by hand. The request for the point-slope form of the tangent line prevents use of GRAPH/A:Tangent.

**Problem 1.** Let  $f(x) = 2x^2 - 5$ . Show all work or explain how you used the TI-89.

- State the unsimplified formula for the slope of the secant line from  $x = 1$  to  $x = 1 + h$ .
- Write down the limit expression for  $f'(1)$  using function notation. Find  $f'(1)$ .
- State the point-slope form of the equation of the tangent line to the graph of  $f'(x)$  at  $x = 1$ .
- Sketch a graph of  $f(x)$  and the tangent line.

Solution:

a. 
$$\frac{(2(1+h)^2 - 5) - (2(1)^2 - 5)}{(1+h) - 1}$$

b.  $f'(1) = 4$ .

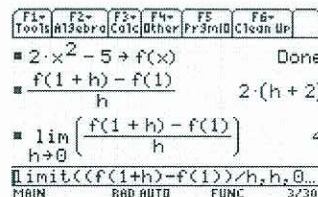


Fig. 3. TI-89 solution to Part (b)



c.  $y - (-3) = 4(x - 1)$ .

d. Let  $Y1 = 2x^2 - 5$  and  $Y2 = 4(x - 1) - 3$ .

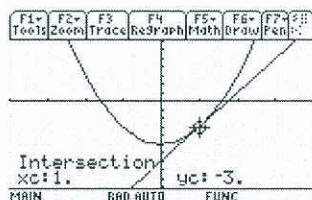
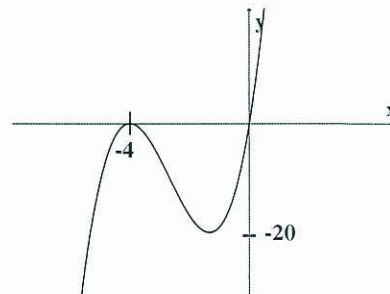


Fig. 4. TI-89 solution to Part (d)

**Critical Points.** A standard calculus problem typically asks for either a numerical answer or an answer in terms of a parameter. In this problem, enough information is provided in the graph to find a numerical answer while the equation includes a parameter. The calculator is used for computation but the student must understand calculus in order to determine what computations are necessary. A more challenging version of Problem 2 would state only that the graph shown is a cubic function with roots at  $x = 0$  and  $x = -4$  and would not give the equation form. Since only calculus techniques will provide an exact answer, numerical approximations are not permitted in order to prevent the student from using the Graph/Math features to solve the problem.

**Problem 2.** For the graph shown, the equation is of the form  $f(x) = a \cdot x(x+4)^2$ . Use calculus to answer the following.



- Find the location ( $x$ -value) of the minimum. State the value exactly.
- Determine the exact value of  $a$  that creates a minimum of  $-20$  as shown on the graph.

Solution:

- The student begins by storing the parameterized function and finding the first and second derivatives. Proofreading the stored functions is always recommended before solving for the critical points.

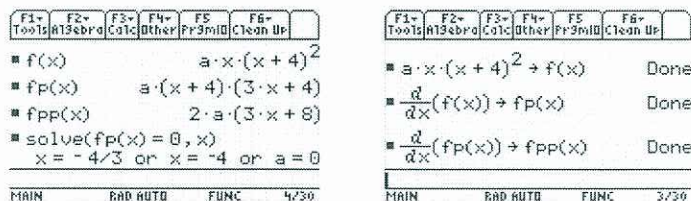


Fig. 5. TI-89 solution to Part (a)

- If the student understands that the function evaluated at the critical point which lies between the roots should equal  $-20$ , then they will be able to set up a very simple equation to solve on the TI. Graphing the function and using Graph/Math/Minimum to check the accuracy of their result is advised.

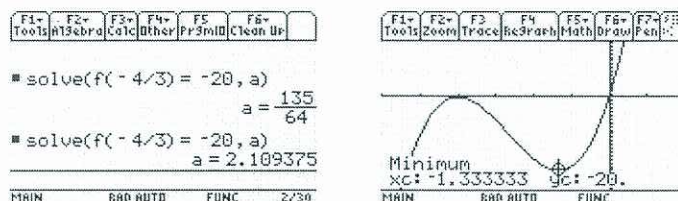


Fig. 6. TI-89 solution to Part (b)

**Derivative Rules.** Rather than take away the calculator during part of an exam, it is more practical to request a derivative for which the TI solution differs significantly from the unsimplified solution found using derivative rules. This provides a way to test on derivatives done by hand while allowing students to have their calculator through the entire test. The following are samples of functions that work particularly well for this problem type.

**Problem 3.** Find the derivatives of the following functions. Do these by hand showing all steps. Do NOT simplify. Output from a TI-89 calculator is not acceptable and will receive no credit.

a.  $y = \frac{1}{\sqrt{x^2 + 2}}$       b.  $y = \frac{e^{-x}}{1 + x^3}$       c.  $y = \sin(x^3) \tan(x)$

Solution:

a.  $y' = -\frac{1}{2} (x^2 + 2)^{-3/2} \cdot 2x$

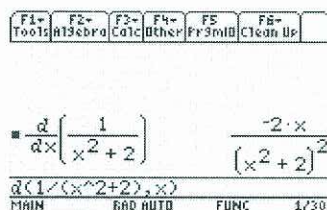


Fig. 7. TI-89 solution to Part (a)

b.  $y' = \frac{-e^{-x}(1 + x^3) - 3x^2(e^{-x})}{(1 + x^3)^2}$

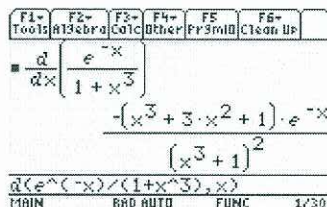


Fig 8. TI-89 solution to Part (b)

c.  $y' = 3x^2 \cdot \cos(x^3) \cdot \tan(x) + \sec^2 x \cdot \sin(x^3)$

$$\frac{d}{dx}(\sin(x^3) \cdot \tan(x))$$

$$\frac{3 \cdot x^2 \cdot \sin(x) \cdot \cos(x) \cdot \cos(x^3)}{(\cos(x))^2}$$

$$\frac{d(\sin(x^3) \cdot \tan(x), x)}{}$$

Fig. 9. TI-89 solution to Part (c)

**The Derivative and Rate of Change.** Assessing conceptual knowledge through essays or writing is usually impractical for large enrollment classes on an in-class test or final exam. We have found that this matching format works well because it addresses most of the basic formulas and requires some extra thought on the part of the student since the baseline function is a velocity function.

**Problem 4.** Let  $v(t)$  represent the instantaneous velocity of an object in ft/sec. For each item in column one, list the matching equivalent item(s) from column two. A number may have more than one letter associated with it.

- |   |   |
|---|---|
| 1. $v(4)$   | A. Slope of the secant line to $v(t)$ from $t = 2$ to $t = 4$ |
| 2. $v'(4)$  | B. Slope of the tangent line to $v(t)$ at $t = 4$             |
| 3. $\frac{v(4) - v(2)}{4 - 2}$                        | C. Instantaneous acceleration at $t = 4$                      |
| 4. $\lim_{h \rightarrow 0} \frac{v(4 + h) - v(4)}{h}$ | D. Net distance traveled between $t = 2$ and $t = 4$          |
| 5. $\int_2^4 v(t) dt$                                 | E. Instantaneous velocity at $t = 4$                          |
|   | F. Average acceleration from $t = 2$ to $t = 4$               |

Solution:

1 – E      2 – C, B      3 – F, A      4 – C, B      5 – D

**Tangent lines and Concavity.** The calculator can easily find derivative functions and evaluate them at a given point. Therefore, determining tangent line equations and linear approximations can be done with the GRAPH/A:Tangent key. The presentation and wording of Problem 5 requires that students relate several concepts including tangent line slope, derivative at a point, concavity, and the graph of a tangent line to a function.

**Problem 5.** Let  $f(x) = \ln(x^2)$  for  $x > 0$ .

- Find the equation of the tangent line to  $f(x)$  that has slope 2 and state the equation in point-slope form.
- Does the tangent line lie above or below the function near that point? How do you know?
- Sketch a clear graph of the function and the tangent line.



Solution:

- a. Tangent line through (1,0):  $y - 0 = 2(x - 1)$ .

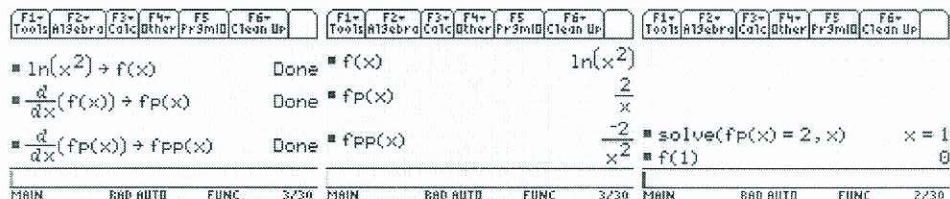


Fig. 10. TI-89 solution to Part (a)

- b.  $f''(x) = -\frac{2}{x^2} < 0$  for all  $x$ . Therefore the graph of  $f(x)$  is concave down everywhere and the tangent line must lie above the function at any point.
- c. The graph of  $f(x)$  confirms the results in part (b).

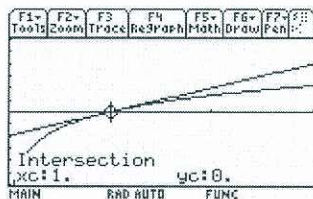


Fig. 11. TI-89 solution to Part (c)

**Integration.** An introductory version of an initial value problem tests on several key integration ideas. This problem can even be introduced before integration by parts is taught. Giving a problem that requires the calculator for integration focuses attention on the importance of the integration constant. Appropriate choice of function names will enhance learning, e.g. storing  $f'(x)$  as  $fp(x)$  for “fprime” and  $\int fp(x) dx$  as  $f(x)$ . The final solve statement reinforces the idea that the initial condition is necessary to finding the integration constant.

**Problem 6.** Let  $f'(x) = x^2 \sin(x)$ . If  $f(0) = 5$ , find  $f(x)$ .



Fig. 12. TI-89 solution to Problem 6

So  $f(x) = (2 - x^2) \cdot \cos x + 2x \cdot \sin x + 3$ .

Our department continues to develop and refine calculus and precalculus problems that thoughtfully employ the powerful technology of the TI-89 CAS for teaching and assessing. When homework and lecture problems are carefully designed, we have found that the graphing calculator enhances student understanding of calculus concepts.