INSPIRATION FOR PROBLEM CREATION

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How might preservice teachers solve a problem like the following (Figure 1)?

Determine the values of the three different digits *d*, *a*, and *n* given in the following:

$$\begin{array}{r} dna \\ \underline{+dan} \\ and \end{array}$$

Figure 1: Three Variables Problem

When presented with this task, preservice teachers typically choose to use problem-solving techniques including "guess-check-revise" and using some basic number sense. But when they were encouraged to look for ways in which this and other problems might be tackled using technology (specifically, the TI-nSpire CAS handhelds), preservice teachers explored the capabilities of the technology and developed new strategies and skills. I also asked them to find ways in which the original problems could then be altered to make them more technology-proof—that is, less likely to be solved using technology. This exercise helped preservice teachers develop their technological pedagogical content knowledge (TPACK) (Mishra & Koehler, 2006) as they considered how they might encourage their future students to appreciate problem solving in classrooms where technology is also valued and encouraged.

Problem Solving in Technology Classrooms

Ever since technology began to make its way into classrooms, teachers have had to find ways to capitalize on its potential while recognizing the possibility of its misuse. For example, when the National Council of Teachers of Mathematics (NCTM) acknowledged the role of calculators in the classroom in its *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), its statements were sometimes misinterpreted as advocacy for allowing students to use calculators at all times in lieu of learning basic facts and algorithms. NCTM clarified its stance in its 2000 *Principles and Standards for School Mathematics*, stating, "Technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions" (NCTM, 2000, pg. 25). Since then, mathematics teachers have looked to curricula and support materials for assistance in finding an appropriate balance between technology and skills in their classrooms.

But technology development never stops, and teachers can find themselves having to catch up with advances in the field without a clear idea how to handle them. The internet has posed a particular set of problems as it has morphed over the past two decades from a mode of communication between researchers to an information source for the general public. Mathematics teachers have found that problem-solving tasks that they had always used may no longer be appropriate in their classrooms since their students may be able to locate these problems on the internet along with answers, solution strategies, and even variations on these tasks (Wanko, 2007).

And as computer algebra systems (CAS) have become available online and in handheld technologies, teachers find that their curricula may need to adapt to recognize that students have access to this technology. It should be noted that these changes are not necessarily viewed negatively—in fact, many teachers embrace the potential for having their students explore mathematical ideas in a richer, more meaningful way, just as NCTM advocates (NCTM, 2000). However, the problem comes when educators do not recognize these fundamental shifts in how technology can be used in their classrooms and their TPACK falls out of sync with their students' access to the technology.

In a methods course for preservice secondary mathematics teachers, I decided to address this need for TPACK while helping them explore functions of the TI-nSpire CAS. Preservice teachers were given sets of "Calendar Problems" (28-31 problem-solving tasks that appear monthly in issues of the *Mathematics Teacher*) and were asked to identify problems that could be solved using the technology. Teams were instructed to explore different ways in which the technology could be used to solve various problems and to prepare presentations that were to be given to their peers on using the TI-nSpire CAS.

In addition, preservice teachers were asked to modify the original problems—maintaining the integrity of the underlying mathematics while making them less likely to be solved using the technology. This aspect of the assignment proved to be particularly difficult for some preservice teachers, as they had to understand the technology, the underlying mathematics, and the original task, and they had to apply some creative thinking to develop a new version of the task.

The overall assignment yielded some interesting results, some of which are described below. Following these, I discuss some of the implications for this type of assignment and the future of using technology in preparing mathematics teachers.

Explorations by Preservice Teachers

Each team of preservice teachers made frequent use of the Solve function on the TI-nSpire CAS handhelds, solving equations with one variable or systems of equations with multiple variables and substituting expressions. For example, one team solved the Two Unknowns Problem (Figure 2) first by writing two equations, solving one for one variable by hand and substituting it into the other equation, solving it using the technology (Figure 3, first line). This approach was very common and in cases like this one, the two unknown numbers were given by the handheld, which they could input to get the final answer to the problem (Figure 3, second line).

Two numbers have a product of 19,551 and a sum of 280. Find their difference.

Figure 2: Two Unknowns Problem

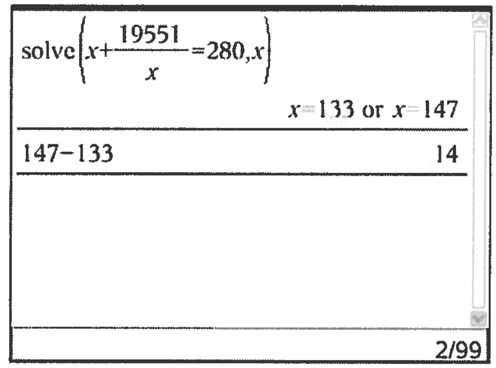

Figure 3: Technology Solution for Two Unknowns Problem

In revising the Two Unknowns Problem, preservice teachers recognized the importance of exploring different expansions of polynomial expressions and they decided to look at another situation involving only two variables, but one that required more additional information. They recognized that the technology could do binomial expansions, but that the solver would need to know what binomial he or she would want to expand—making their new task one that could still benefit from using technology, but requires human thought to know which expression to use (Figures 4 and 5). This revision also demonstrates another common approach used by preservice teachers to make a problem more technology-proof: requiring a different kind of solution or justification to be provided. In this case, the problem doesn't simply ask for a numerical solution, but instead it asks for supporting work with the prompt "show how to find…" and it limits the solver's possible approaches with the prompt "without finding the values of…".

Without finding the values of x and y, show how to find $x + y$ knowing that $x^3 + y^3 = 120{,}744$, $x^2y = 54{,}432$, and $xy^2 = 63{,}504$.

Figure 4: Revision of Two Unknowns Problem

$$
\begin{aligned}
(x+y)^3 &= x^3 + 3x^2y + 3xy^2 + y^3 \\
&= (x^3 + y^3) + 3(x^2y) + 3(xy^2) \\
&= 120{,}744 + 3(54{,}432) + 3(63{,}504) \\
&= 474{,}552 \\
\text{So} \quad x + y &= \sqrt[3]{474{,}552} = 78
\end{aligned}
$$

Figure 5: Solution to Revision of Two Unknowns Problem

In solving the Prime Numbers Problem (Figure 6) using technology, preservice teachers used the Factor function (Figure 7) to bypass what was probably the original problem author's intent to use formulas for factoring the sum and difference of perfect cubes. This is a perfect example of goals of classic problem-solving tasks being altered by today's technology.

Prove that neither 999,019 nor 1,000,343 is prime.

Figure 6: Prime Numbers Problem

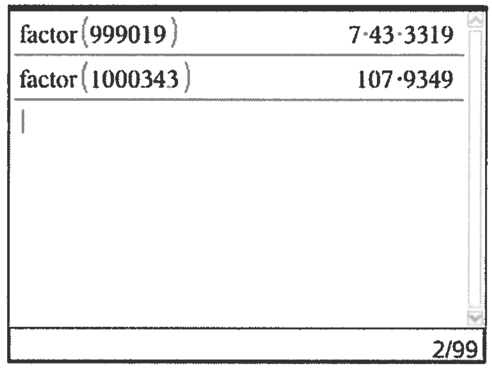

Figure 7: Technology Solution for Prime Numbers Problem

In the revision of the Prime Numbers Problem, preservice teachers decided that they wanted to create a version of the task where students would still have to use the factoring formulas for the sum of perfect cubes. Their revised task (Figure 8) describes a hypothetical person who is discussing his solving strategy and the student is asked to make sense of his approach and to apply it to the general case (Figure 9).

Jacob stated that 1,000,343 is not a prime number because it could be written as 1,000,000 + 343. Is his statement correct? If so, can his strategy be used to prove that other numbers are composite? Explain.

Figure 6: Revision of Prime Numbers Problem

We know that we can rewrite 1,000,000 + 343 as the sum of two cubes: $100^3 + 7^3$ which can factor as $(100 + 7)(100^2 - 70 + 7^2)$ because $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$. So Jacob is right because the sum of two cubes can be factored as two numbers, neither of which is 1, so the sum of two cubes is not prime.

Figure 7: Solution to Revision of Prime Numbers Problem

Returning to the Three Variables Problem (Figure 1), it is interesting to note how the team of preservice teachers approached solving this problem using technology. But before the technology application, they had to do some algebraic simplification and use some reasoning about linear Diophantine equations. They recognized that each of the three variables *d*, *a*, and *n* needs to represent a non-negative single digit integer. The three numbers in the addition problem could be represented using decimal expansion as:

$$\begin{array}{r} 100d + 10n + a \\ \underline{+100d + 10a + n} \\ 100a + 10n + d \end{array} \qquad \text{or} \qquad \begin{array}{r} 200d + 11n + 11a = 100a + 10n + d \\ 199d + n = 89a \\ n = 89a - 199d \end{array}$$

Using the linear Diophantine equation $n = 89a - 199d$, they set up a spreadsheet in the TI-nSpire CAS, showing the possible values for *d* of 0 through 9 in row 1 and the possible values for *a* of 0 through 9 in column A (Figure 8). The other 100 cells in the spreadsheet show all of the calculations for the various combinations of *a* and *d*. A value of 5 appears in cell F11, indicating that when $d = 4$ and $a = 9$, $n = 5$, the solution to the problem (one other potential solution appears in cell B2 when $d = a = n = 0$, the trivial case, which can be excluded because *d*, *a*, and *n* are not all different). Preservice teachers gave several examples of cryptogram puzzles that involve more than three variables as ways to revise the Three Variables Problem.

	A	B	C	D	E	F	G	H	I	J	K
◆											
1		0	1	2	3	4	5	6	7	8	9
2	0	0	-199	-398	-597	-796	-995	-1194	-1393	-1592	-1791
3	1	89	-110	-309	-508	-707	-906	-1105	-1304	-1503	-1702
4	2	178	-21	-220	-419	-618	-817	-1016	-1215	-1414	-1613
5	3	267	68	-131	-330	-529	-728	-927	-1126	-1325	-1524
6	4	356	157	-42	-241	-440	-639	-838	-1037	-1236	-1435
7	5	445	246	47	-152	-351	-550	-749	-948	-1147	-1346
8	6	534	335	136	-63	-262	-461	-660	-859	-1058	-1257
9	7	623	424	225	26	-173	-372	-571	-770	-969	-1168
10	8	712	513	314	115	-84	-283	-482	-681	-880	-1079
11	9	801	602	403	204	5	-194	-393	-592	-791	-990
12											
13											
14											

B2 =89·a2-199·b$1

Figure 8: Technology Solution for Three Variables Problem

Conclusion

Preservice teachers devised many different ways to use the TI-nSpire to find solutions to problem-solving tasks, many of which shifted the need away from traditional problem solving strategies to other solving approaches which required problem solving using technology. They recognized the need for teachers to understand the technology to which their students have access and appreciated the challenge in revising the tasks they may use to accommodate the technology that is available.

In devising revisions of the tasks, preservice teachers came to recognize the value in the kinds of questions that are posed—that the technology may make calculations and other mathematical functions easier, but that their students can still be challenged to explore the underlying mathematics when the tasks require them to do so.

References

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