MATH AND MURDER: NEWTON'S LAW OF COOLING
An Application from *Why Did You Cut Your Couch In Half?*

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As educators we are always looking for ways to get our students excited about mathematics. In this paper, I will outline an activity that does just that, a murder mystery where the students from your class are the suspects. This activity uses Newton's Law of Cooling as the main tool, and can be used to give an application of solving logarithmic/exponential functions in a Trigonometry or Precalculus course or for applications of separable equations in Calculus and Differential Equations courses.

Newton's Law of Cooling states the rate at which the temperature of an object changes is proportional to the difference between the temperature of the object and the surrounding temperature, i.e.

$$\frac{d\theta}{dt} = -k(\theta - r).$$

The solution to this equation is

$$\theta(t) = (\theta_0 - r)e^{-kt} + r$$

where θ_0 is the temperature at time 0, r is the room temperature at time t, $\theta(t)$ is the temperature of the object at time t, and k is the constant of proportionality. The course in which you are using the activity will determine if you start with the differential equation or if you start with the solution.

To begin the activity teachers have two options. They can have students work in groups or they can have the entire class work together to find the time of death of the first victim. If the instructor has students working in groups, they should check the students' answer as to the time of death before the second murder is introduced. A sample first murder would be:

> Tragedy at QLSU! Dr. Smith, the chair of the math department was found dead in his office Tuesday afternoon at 2:30. Apparently his TI-92 was sabotaged, causing him to be electrocuted. When found, his body temperature was 87.6° and his office is kept at 72°. He was moved to the morgue, which is kept at 40°. After 2 hours, his body temperature is 78.3°. When did he die?

To solve we must first find k. We have the body temperature at time 0, namely 87.6°. The body temperature 2 hours later is 78.3°, and the room temperature is 40° (the body is in the morgue). After plugging the values in we have

$$78.3 = (87.6 - 40)e^{-2k} + 40$$

$$\frac{78.3 - 40}{87.6 - 40} = e^{-2k}$$

$$\ln(0.804622) = \ln(e^{-2k}) = -2k$$

$$k = 0.108691.$$

Using this value, the fact that a body's normal temperature is 98.6° (temperature at time of death) and the room temperature at time of death is 72°, we have

$$98.6 = (87.6 - 72)e^{-0.108691} + 72$$

$$\frac{98.6 - 72}{87.6 - 72} = e^{-0.108691}$$

$$\ln(1.70513) = -0.108691t$$

$$t = -4.9097.$$

Therefore the time of death was 4 hours and 55 minutes earlier, at 9:25 that morning.

Creating this activity does take time and teachers will want to plan ahead. (A sample activity is included at the end of the document and teachers are welcome to use this with their students' names if they wish.) When creating your own activity, the first step is to collect the schedules of each student. The schedules are used as the alibi sheets. The killer is a conscientious student and would never dare to skip a class. Therefore, if a murder takes place when a student is in class, they are no longer considered a suspect.

As schedules are collected, the instructor will also need each student to give a hobby or activity they enjoy. The hobbies will be used as methods in which the victims are killed. When creating your murder scenarios, choose ones that are unrealistic and humorous, like being run down by a stationary bike for someone who likes the gym, or infected by a computer virus for one who loves computer games.

The next step is to choose the killer. This student should be someone who has a schedule that is either sparse or unusual compared to the rest of the class. You will also need to determine how many murder victims there will be. In general, 5 or 6 victims are

sufficient to make the activity interesting enough for your students to learn the concepts well. From here, by using the alibi sheet, determine when the victims should die. These times should be chosen to eliminate as many suspects as possible. Note that there are times when it is impossible to remove all the suspects you need to, so it may be necessary to say students have fled the country or even changed classes to avoid being the killer's next target. These methods will allow you to eliminate any students who haven't already been eliminated by the murders.

Now that you have determined the times of the murders, you must create the specifics of each crime scenario. To do this you choose a time that the body is found, the temperature of the body when found, the temperature of the room in which the body is found, the temperature of the morgue (if the body is moved there), and a time passed before the second reading is taken. Working backwards, using Newton's Law of Cooling, you then find the temperature of the body at the second reading. For example, assume you want the person to die at noon. You decide the body is found at 1:30 with temperature 84° in a room kept at 70°. The body is then moved to the morgue that is kept at 40° and the second reading is taken 1 hour later. You need to find the temperature of the body at the time of the second reading. To do this we first need to find k. We use the fact that the body temperature at time of death is 98.6°, the time of death is -1.5, and the room temperature is 70°. Plugging the values into our equation, we have

$$98.6 = (84-70)e^{-k(-1.5)} + 70$$

$$98.6 - 70 = (84-70)e^{-k(-1.5)}$$

$$\frac{98.6-70}{84-70} = e^{-k(1.5)}$$

$$\ln(2.04286) = \ln(e^{-k(-1.5)}) = 1.5k$$

and find

$$k = .476233.$$

Then to find the body temperature after one hour, we plug in the values and simplify

$$(84-40)e^{-.476233(1)} + 40$$

to find the second temperature reading is 67.3°. From here you can create your scenarios and have the students find the time of death for each murder.

Although, it takes time to create this activity for each class, I have found it to be worth the effort. Students in my Trigonometry, Precalculus, Calculus, and Differential Equations classes are actually laughing and enjoying mathematics and they see an application to separable equations, integration, logarithms, and exponential functions. I feel, like many of you, that anything that gets the students involved in and excited about learning is definitely worth a little extra work.

Sample Activity:

Alibi Sheet

The Math Murderer is conscientious about his/her studies and does not skip classes. Below is a list of when members in this class are unavailable, i.e. when they have an alibi. Use this information to catch the killer.

Student 1: M, W: 8-10, 11-12
T, R: 8-11, 3:30-5

Student 2: M, W, F: 8-10, 2-3
T, R: 8-9:15

Student 3: M, W: 8-10, 1-3
T: 2-3:15
R: 12:30-3:15
F: 8-10, 1-2

Student 4: M, W. F: 8-12, 2-5
T: 4-5
R: 11-5

Student 5: M, W: 8-12, 4:30-5:45
T, R: 9:30-5
F: 8-12

Student 6: M, W, F: 9-10, 12-1
T, R: 8-9:15, 2-3:15
M-F: 3-5

Student 7: M, W, F: 8-2
T: 9:30-2
R: 9:30-12:15

Student 8: M, W, F: 8-1:30
T, R: 8-11

Student 9: M, W, F: 8-10, 2-3:15
T: 2-3:15
R: 9-12, 2-3:15

Student 10: M, W: 8-10, 11-12, 1-5
T: 11-12:15
R: 11-12:15, 2-5
F: 8-10, 11-12, 1-2

Student 11: M-F: 8-12, 2-5

Student 12: M, W, F: 8-10
T, R: 9-10:15, 12-1:15

Student 13: M: 8-10, 11-12
W: 8-12, 2-5
F: 8-10, 11-12, 2-3

Student 14: M-F: 8-12, 2-3

Student 15: M: 8-12, 1-2
W, F: 8-11, 1-2

Student 16: M, W, F: 8-10, 1-5
T, R: 11-5

Student 17: M, W, F: 8-11
T, R: 8-9:30, 1-5

Student 18: M, F: 8-10, 2-3
T, R: 11-12:15
W: 8-10, 1-3

Student 19: M: 8-11, 1-5
W, F: 8-11

Student 20: M, W, F: 8-11
T: 1-5
R: 2-5

Student 21: M, W, F: 8-11
T, R: 11-1:45

Student 22: M, W, F: 8-2
T: 11-5
R: 11-12:30

Student 23: M, W, F: 8-10, 3-4
T: 2-3:15
R: 9:30-12:20, 2-3:15

Student 24: M, W, F: 8-10, 11-3

Student 25: M, W, F: 8-12, 2-3

Math and Murder: Newton's Law of Cooling

(Note this activity uses the murder of Dr. Smith as part of the mystery, so don't forget to eliminate the necessary suspects! Students 1, 8, 12, 14, and 17)

A second tragedy has struck QLSU! While playing basketball Monday, Student 3 went to go for a slam-dunk and someone had slid a trampoline under the basket. 3 went right through the roof as the Math Murderer had succeeded again. The temperature outside when she went through the roof was 72°. When the police came to investigate at 6:55 pm, they measured her body temperature to be 76.3°. Two hours later, they measured the body temperature again and it had fallen to 74.5°. When did Student 3 die?

Answer: around 12:12

The Class has suffered another loss. Student 20 was found in his room in front of his computer at 12:15 on Tuesday afternoon. The apparent cause of death was an infection from a computer virus. When his body was found, the police report stated his body temperature was 78.3° in a room kept at 70°. After an hour, his body temperature had fallen to 75.6° When did Student 20 die?

Answer: around 9:06

The Math Murderer Strikes Again! On Thursday at 2:10 pm, the body of Student 1 was found at the Mall. She was trampled by a coin operated horse. The temperature outside was a constant 72°. When the body was found the police had recorded the temperature at 83°. The body was immediately moved to the morgue which is kept at 40°. Three and a half hours later, 1's body temperature was recorded to be 55.4°. When did she die?

Answer: around 11:10.

Another victim found at QLSU! On Friday at 6:00 pm, Student 19 was found dead. While working out at gym, he was run over by a stationary bike. The room is kept at 72°. When the police found the body, the temperature was 90.3° 19 was moved to the morgue kept at 38°. Three and a half hours later, the body temperature had fallen to 75.2°. When did Student 19 die?

Answer: around 2:10

Police catch the Math Murderer! Do you know who did it? Answer: Student 15.

Bibliography:

1) Berkey, D. *Calculus*. New York: Saunders College Publishing, 1984.
2) Boyce, W. and DiPrima, R. *Elementary Differential Equations and Boundary Value Problems (6th edition)*. New York: John Wiley and Sons, Inc., 1997.