A MODELING APPROACH TO COLLEGE ALGEBRA

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Two Versions of College Algebra

At the University of Pittsburgh we have two forms of College Algebra and Pre-Calculus: 1) for students headed to Calculus and 2) for students in the non-math/non-science majors. These latter students generally take only one additional quantitative course, usually statistics. Our lowest level course is College Algebra. We do not teach Intermediate Algebra; instead we offer College Algebra Part 1 and College Algebra part 2, which teaches College Algebra over two terms using the same text and syllabus as the one term College Algebra, awarding 1.5 credits for each part. In general the majority of students in a College Algebra course, of either type, choose majors that are non-math/non-science related or are undecided about their major. In the Applied College Algebra course for non-math/non-science majors we take a modeling approach. The reason is that these students tend to be in disciplines where they do a lot of reading, they tend to be more aware of current trends and social issues and *really* want to know "When am I ever going to use this stuff?" They are the students who may be writing about data or table and charts that someone else has given them. Thus motivating students with real world examples and problems tends to increase their interest, understanding and success in the course.

Student Background

In the fall of 2006 a study was done to compare the nature and success of students in the two types of College Algebra courses which this author taught. College of Arts and Science students with a math SAT less than 600 must take a placement test. Meeting with an advisor, they are placed in the appropriate course. A survey given on the first day of class shows the makeup of the students:

Math 0031 College Algebra
(Prepares students for Calculus when followed by math 0032 Trigonometry)

Summary	Number	Percent
Female	101	69.7
Male	44	30.3
Total	145	
Amount enrolled after drop/add	132	
Highest Math Taken		
Algebra I	1	0.7
Geometry	3	2.1
Algebra 2	28	19.6
Trigonometry	34	23.8
Pre-calculus	45	31.5
Calculus	32	22.4
total	143	

Math 0025 Applied College Algebra
(Prepares the non-math/non science majors students for a quantitative course using a modeling approach.)

Summary	Number	Percent
Female	90	62.9
Male	53	37.1
Total	143	
Amount enrolled after drop/add	128	
Highest Math Taken		
Algebra I	0	0
Geometry	9	6.3
Algebra 2	39	27.3
Trigonometry	39	27.3
Pre-calculus	32	22.4
Calculus	17	11.9
total	136	

Calculator Skill 0-5		
Average	2.39	
Median	3	

Majors/interest

non-math/non-science	45	31.0
Undecided	39	26.9
science	61	42.1
total	145	100.0

Calculator Skill 0-5		
Average	2.17	
Median	2.5	

Majors/interest

non-math/non-science	105	73.4
Undecided	25	17.5
science	13	9.1
total	143	100.0

In general the Applied College Algebra students had less high school math. Only 22.4% of the Applied students had taken a Pre-calculus compared to 31.5% in College Algebra. In addition College Algebra had twice as many students who had taken high school calculus. The College Algebra had 31% of students who were non-math/non-science majors. It is not known why these students were signed up for College Algebra as opposed to Applied College Algebra, it could have been scheduling conflict or that the Applied College Algebra section was closed.

Structure of the Courses

The structure of the two courses is similar. Each is 3 credits and meets for three lectures and one recitation per week. In addition students have written homework and three computer homework assignments per week. Both were taught in large lecture format. The problem solving recitations are small sections of 25-30 students lead by undergraduate teaching assistants.

Math 0031 College Algebra

Text: *College Algebra and Trigonometry* 2e. Penna, Beecher, Bittinger. Addison Wesley 2005.

Syllabus:

1. Topics
   - Fairly traditional treatment of topics
   - Functions, function notation, and transformations of functions, composition of functions
   - Linear Functions
   - Quadratic functions
   - Heavy emphasis on polynomial and rational functions
   - Exponential and logarithmic functions
   - Matrices
2. Four Applied problems
3. Mymathlab computer homework, 3 times per week
4. Written graded homework and 5 quizzes

Math 0025 Applied College Algebra

Text: *Explorations in College Algebra* 3e. Kime, Clark, Michael. John Wiley. 2005.

Syllabus:

1. Uses real world data to motivate topics and model of concepts
   - Examples from social sciences
   - Heavy emphasis on rates of Change and linear functions*
   - Heavy emphasis on exponential and logarithmic functions*
   - Power and quadratic functions
   - Light treatment of polynomial and rational functions
   - Transformations and composition of functions
2. 8-10 Applied problems called "Explorations"
3. Wileyplus computer homework, 3 times per week
4. Written graded homework and 5 quizzes

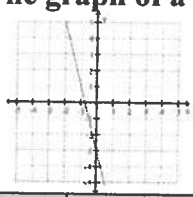
*Authors found when talking with service departments that linear and exponential models were the functions most used.

Analysis

At the end of the course students in both courses took a 33 problem multiple choice final exam, 20 of which were in common. Many instructors who oppose a modeling approach fear that students will not have the appropriate manipulation skills to succeed in the next math course. In this study there were only 7 problems that showed a statistically significant difference on the responses of the two groups. Only three of these problems showed a percentage difference of more than 9 percentage points. These three questions were indicative of the amount of emphasis each group devoted to the topic in the syllabus. The results follow:

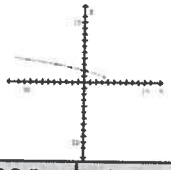
(Note: Math0025=Applied College Algebra and Math0031=College algebra)

7) **The graph of a linear function f is given. Write a formula for f.**



Math0025	92.72	111	two-tailed P value equals 0.0070
Math0031	83.62	116	very statistically significant.

12) **Find the domain and the range for the function**



Math0025	82.88	111	two-tailed P value equals 0.0427
Math 0031	92.24	116	statistically significant

13) **The given graph represents a translation of the graph of $y = x^2$. Write the equation of the graph.**

Math0025	91.89	111	two-tailed P value equals 0.0310
Math 0031	98.28	116	statistically significant

(Note: both groups did very well on this question)

14) **A polynomial of degree three which has zeros at 3, -3, and 2 and a leading coefficient of 1 is:**

Math0025	54.05	111	two-tailed P value is less than 0.0001
Math 0031	84.48	116	extremely statistically significant

(Note: math 0025 Applied College Algebra de-emphasized polynomials; however the 4th edition of the text has an expanded section on polynomials.)

16) **Sketch a graph of the function**

$$g(x) = \begin{cases} x^2 - 9, \text{ if } x < -1 \\ 1, \text{ if } -1 \le x \le 1 \\ x^2 + 9, \text{ if } x > 1 \end{cases}$$

Math0025	79.28	111	two-tailed P value equals 0.0258
Math 0031	87.93	116	statistically significant

(Note: The 4th edition of the text used in the Applied course now expands piecewise functions.)

17) **The population of a town is 60,000people. The population is expected to decrease at a rate of 5% every 10 years. Determine the exponential model P(t) and decide how many years will it take for the population to reach 45,000**

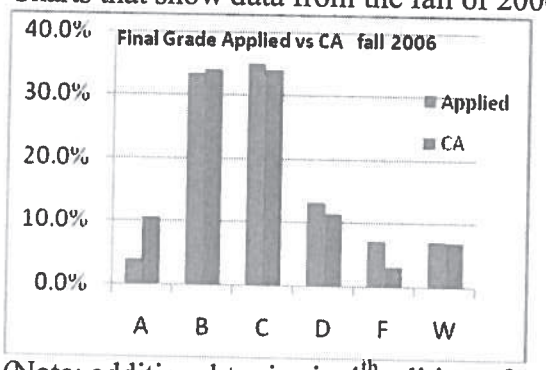
Math0025	**70.27**	**111**	**two-tailed P value is less than 0.0001**
Math 0031	31.90	116	extremely statistically significant

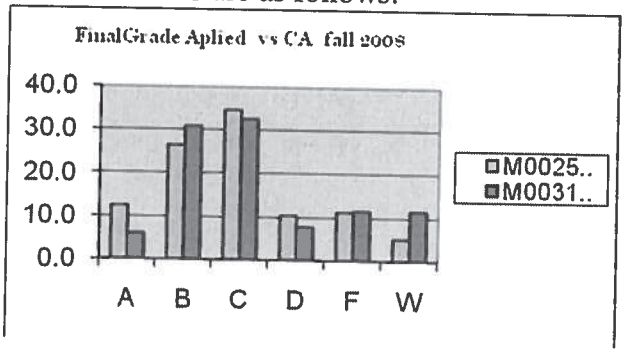
19) **The graph of $y = log_2 x$ is equivalent to the graph of (note: must use the change of base formula)**

Math0025	18.02	111	two-tailed P value is less than 0.0001
Math 0031	43.10	116	Extremely statistically significant

Final Grades

The final exam grades for each section show that students perform almost equally as well. Charts that show data from the fall of 2006 and fall of 2008 are as follows:





(Note: additional topics in 4th edition of text used in the Applied course may have increased students understand of topics for fall or 2008.)

Summary[i]:

Students who are in College Algebra and who will not be taking calculus deserve a math course that will be useful in other qualitative courses. Taking a modeling approach allows students to see the connection with other courses they are currently taking or will take. Using real world data enhances their understanding of qualitative ideas and does not decrease their understanding of algebraic ideas and topics. This study showed that overall the Applied Algebra for non-math/non-science students, which used a modeling approach, had completed less math classes in

high school (completed up to trig) while the College Algebra students mostly took up to pre-calculus. Coming into college with more completed math classes in high school has a big effect on the outcome on certain topics in each course. It also showed that the traditional College Algebra students answered more questions correctly: 14.66/20 = 73.3% compared to the Applied students 68.3%. This difference was not significantly significant however, and students performed at the 75% or higher level on 13 additional questions related to material covered in each course. When compared to students in a traditional College Algebra course, the Applied students performed as well as their counter parts in a traditional College Algebra course on 15 of 20 common final exam questions, and statistically better on 2 of the 15. Of the five questions where the traditional College Algebra students had scores that had a statistically significant, the difference on common final exam questions can be traced back to the emphasis paid in the course syllabus. Departments that are considering replacing a traditional College Algebra with a modeling approach should know that this study showed that students in the modeling course also learn basic algebra techniques and perform nearly as well as their traditional counterparts. This is in spite of the fact that students in the modeling course had less math high school math courses than their traditional counterparts.

[i] Special thanks to undergraduate research assistant Joana Ruderman of the University of Pittsburgh who helped with the analysis of this data.