CLASSIFYING STUDENTS' MISTAKES IN HANDHELD DEVICES*

Kevin Anderson, andersk@missouriwestern.edu
Tim Miller, milltim@missouriwestern.edu
Tingxiu Wang, twang1@missouriwestern.edu
Department of Computer Science, Mathematics and Physics
Missouri Western State University
4525 Downs Drive, Saint Joseph, MO 64507

Abstract: Students make mistakes with hand held devices, so do hand held devices. The problem is that students often do not realize that technology can make mistakes, and they simply accept whatever results are given by the technology. This paper classifies students' mistakes in handheld devices, and give some examples of mistakes made with graphing calculators.

There have been many discussions about the mistakes that students make with graphing calculators and computer algebra systems (Benson, Demmel, Howell, Steward). These discussions mainly give examples to caution students that calculators and computers may give wrong or questionable answers. Few classify the mistakes. In this paper, we discuss four categories of mistakes with graphing calculators and computer algebra systems.

1. ORDER OF OPERATION ERRORS

A very typical kind of mistake is related to the order of operation. Order of operation errors can occur in two cases: either due to many students not being as aware of order of operation as they should be; or due to translation errors from the text book to the calculator. There is no simple way to tell if students do not understand the order of operations rules, or if they are having difficulty translating equations from their text book.

1.1. Students Are Not Familiar with the Order of Operations

When students do not understand order of operation, two kinds of operations can cause students' trouble. One involves fractions and the other operations on exponents. The "pretty print" feature on the TI-89 can really help students see the difference between the operations they need to perform and the expressions they entered into the calculator. While the TI-83 and TI-84 will not clearly display these errors, for an expression involving only addition, subtraction and multiplication, hence students may not notice a mistake if they are not familiar with order of operation.

Example 1.1. To evaluate $7-8\times 6^2+12\div 4$, they may enter $7-8*6\wedge 2+12/4$ with TI-83/84/89. Even if they do not pay attention to the order of operation, they do not get an error. As for other scientific calculators, they may get an error with a simple model, but they still can get a correct answer with an advanced one.

* This paper was presented at the 21st International Conference on Technology in Collegiate Mathematics.

Example 1.2. (Fraction) To evaluate $\frac{25-4\times5}{12+6^2}$, some students would just enter $25-4*5/12+6\wedge2$. They can see that the display of TI-89 is different from what they need to perform. Therefore they can realize that there is a mistake. However, the display of TI-83/84 will not bring this error to their attention.

Fraction with TI-89	■ $25-\frac{4\cdot5}{12}+6^2$ $\frac{178}{3}$ 25-4*5/12+6^2 MAIN DEG AUTO FUNC 1/30
Fraction with TI-83	25-4*5/12+6^2 59.33333333

Example 1.3. (Exponents) To evaluate $A=5000\left(1+\frac{.06}{4}\right)^{4\times5}$, some students would enter $5000(1+0.06/4)\wedge4*5$. The TI-89's display can easily bring the error to the student's attention, but TI-83/84 do not.

Exponents with TI-89	■ $5000\cdot\left(1+\frac{.06}{4}\right)^4\cdot5$ 26534.1 5000(1+.06/4)^4*5 MAIN DEG AUTO FUNC 1/30
Exponents with TI-83	5000(1+.06/4)^4* 5 26534.08877

1.2. Students Do Not Enter an Expression Correctly into a Calulator

Even if students know the order of operations, they can make mistakes. Students often do not realize that extra parentheses are needed even if the expression does not have them.

Example 1.3. (Graphing): Graphing equations such as $f(x)=\frac{2+x}{x+5}$ without proper parenthesis (Figure 1), students get the incorrect graph (Figure 2). Students might not realize the mistake with TI83/84, but they should notice a different display on the TI-89.

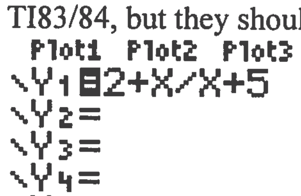


Figure 1 with TI-83

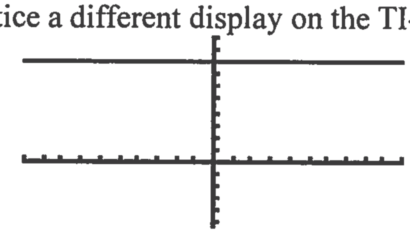


Figure 2 with TI-83

2. SYNTAX ERRORS

When students made a syntax error, a calculator will either not perform the operation, or give students an alternative result. In addition, just like there is no simple way to tell us if students do not understand the order of operation, or if they do not enter an expression into a calculator correctly, there is a fine line between a syntax error and an order of operation error. For example when calculating $\sin\left(\frac{6}{5\pi}\right)$ a common student error is to enter $\sin(6/5*\pi)$ into the calculator. This begs the question. Did the student not understand order of operation, or the syntax of the calculator? One syntax error that can not be confused with an order of operation error is what we call a "close parenthesis" error. It appears on the TI-82/83/45/85/86 series, but not the TI-89's. This is not just a student error, but an error on the part of TI's programming. Consider the function $y=\sin(x+2\pi)$, which is of course equivalent to $y=\sin(x)$. Both of the above functions are distinctly different from $y=\sin(x)+2\pi$. Unfortunately if the student forgets to "close" the parenthesis the TI-83 automatically "closes" the parenthesis at the end of the expression. If the student tries to graph $y=\sin(x)+2\pi$ but forgets to close the parenthesis (Figure 3), the calculator will then return the standard sin graph (Figure 4)

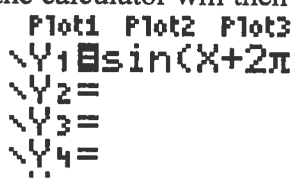


Figure 3 with TI-83

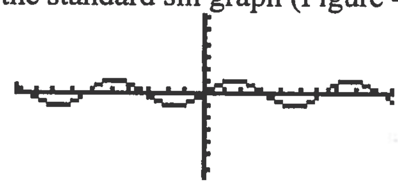


Figure 4 with TI-83

Students not only need to be reminded of proper order of operation/syntax, they also need to be aware of the eccentricities of the HHC they are using. Students using the TI-82, 83 must take care to make sure that they properly "close" their parenthesis to avoid this error. Other syntax errors include entering $(\sin x)^k$, and $(\log x)^k$ into a calculator.

3. IMPROPER SETUP OF HHC

Many authors have given examples that some mistakes are caused by improper setups. For example, Bogley and Robson (1996) mentioned two common mistakes caused by a wrong or improper setup of the calculator:

- Using an improper window when graphing;
- Using degrees instead of radians, or vice-versa.

These kinds of mistakes often occur when students graph functions and calculate trigonometric values. These mistakes are well known and correctable by changing the setup of the device. So we will not give examples. However, for graphing calculators that have symbolic functions, such as the TI-89, the setup can include numerous different modes: Auto Mode, Exact Mode, Approximate Mode, Function Mode, Parametric Mode, Polar Mode, Sequence Mode, 3D Mode, and Differential Equations Mode. Students can make mistakes under a combination of all these modes. For example, students might get an incorrect result with Degree Auto Mode, while they could get a correct one with Radian Auto Mode. Therefore, the proper modes in graphing calculators become even more critical when using the TI-89 in upper level mathematics classes. When using the TI-89 to perform any calculus operation (limits, differentiation, integration, etc.) that involve trigonometric or inverse trigonometric functions Radian Mode is required to get what we expect, not Degree or Gradian Mode. However, selecting the proper mode for a calculation can be tricky. In the following examples, screenshots were made using the TI-89 Titanium with the most current operating system (v. 3.10). Errors can also be made when variables are used that have been assigned specific values in earlier work.

3.1 Radian-Auto Mode vs Degree-Auto Mode, or Approximate-Auto Mode

Example 3.1. Find the derivative of $f(x) = \sin x$. The following screens give different results under different modes. The result from the Radian-Auto Mode is what we expect. The result from the Degree-Auto mode is not what we would normally want. Even worse, if the calculator is in Degree-Approximate mode.

Radian-Auto Mode	■ d(sin(x),x) cos(x) d(sin(x),x) MAIN RAD AUTO FUNC 1/30
Radian-Approximate Mode	The same as the above
Degree-Auto Mode	■ $\frac{d}{dx}$(sin(x)) $\frac{\pi\cdot\cos(x)}{180}$ d(sin(x),x) MAIN DEG AUTO FUNC 2/30
Degree-Approximate Mode	■ $\frac{d}{dx}$(sin(x)) .01745329252·cos(x) d(sin(x),x) MAIN DEG APPROX FUNC 3/30

Similar results occur when students evaluate limits, indefinite integrals, and differential equations. In general, the Radian-Auto Mode gives the results we expect. Students will get different results from what we expect (consequently we consider them wrong) if they forget to change the setup when finding limits, derivatives and integrals and solving a differential equation in the Degree-Auto Mode, Degree-Approximate Mode, or Degree-Exact Mode. Readers are encouraged to evaluate the following examples with Degree-Auto Mode, Degree-Approximate Mode, Degree-Exact Mode, Radian-Auto Mode, Radian-Approximate Mode, and Radian -Exact Mode: $\int \sin^{-1} 2x\,dx$, $\lim_{x\to\infty} \tan^{-1} x$.

When solving second order differential equations, all modes yield the same answer provided that the characteristic equations do not have a complex root. However, if the characteristic equation of a differential equation has a complex root, students will get a different result from what we expect.

Example 3.2. Solve $y'' - 2y' + 10y = 0$. Its characteristic equation has non-real roots $m = 1 \pm 3i$.

Radian-Auto Mode	■ deSolve(y'' − 2·y' + 10·y ▸ y = @11·e^x·cos(3·x) + @12·▸ ...lve(y''-2*y'+10*y=0,x,y) MAIN RAD AUTO FUNC 1/1
Radian-Approximate Mode	■ deSolve(y'' − 2·y' + 10·y ▸ y = @5·$(2.718)^x$·cos(3.·x) ▸ deSolve(y''-2y'+10y=0,x,y... MAIN RAD APPROX FUNC 1/30

The moral of all these examples is this: For calculations related to calculus, students should set the calculator in Radian and Auto (or Exact) mode; otherwise they can get some extremely ugly results, which are different from what we expect.

3.2. Auto Mode vs Approximate Mode or Exact Mode

Just now we discussed that results in the Degree Mode can be very different from what we expect. However, Approximate Mode can give students a completely wrong answer.

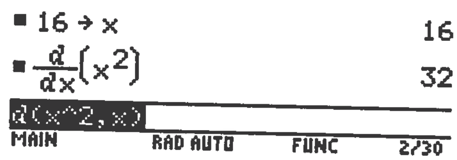
Example 3.3. Find $\lim_{x\to\infty}\left(1+\frac{0.3}{x}\right)^x$ (Li & Wang, 2007).

Radian-Auto Mode	■ $\lim_{x\to\infty}\left(\left(1+\frac{.3}{x}\right)^x\right)$ 0. limit((1+.3/x)^x,x,∞) MAIN RAD AUTO FUNC 1/30
Radian-Approximate Mode	■ $\lim_{x\to\infty}\left(\left(1+\frac{.3}{x}\right)^x\right)$ 0 limit((1+.3/x)^x,x,∞) MAIN RAD APPROX FUNC 1/31
Radian-Exact Mode	■ $\lim_{x\to\infty}\left(\left(1+\frac{.3}{x}\right)^x\right)$ $e^{3/10}$ limit((1+.3/x)^x,x,∞) MAIN RAD EXACT FUNC 1/30

Although students can get a wrong answer with TI-89 for $\lim_{x\to\infty}\left(1+\frac{0.3}{x}\right)^x$, they can get $\lim_{x\to\infty}\left(1+\frac{0.4}{x}\right)^x$ correctly in any mode. In fact, for a general constant k, they still get a correct answer e^k for $\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x$.

3.3 Internal Memory

Sometimes, students would assign a special value to x in their graphing calculators. If they forget to clear the memory, they will just get a numerical value whenever they do a symbolic calculation. The following screenshot shows that $x = 16$ is stored in the memory, and when calculate $\frac{d(x^2)}{dx}$, students can only get 32, instead of $2x$. They will not get what they want.

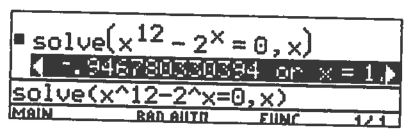


4. HHC DO NOT PERFORM SOME CALCULATIONS PROPERLY

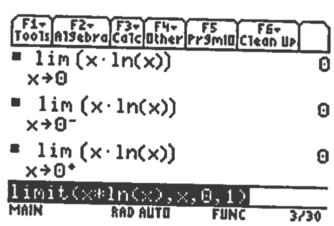
Errors caused by improper setup are often correctable. However, for the following examples, students will get wrong answers no matter what setup they use. This is very bad because students cannot prevent them. So they must beware of these kinds of mistakes, and have some idea about the capabilities of the HHC. All of the following calculations are done in Radian-Auto mode. Among the following examples, some are new, and some have been published. For completeness and easy reference, we include them together.

4.1 Graphing Calculators Return a Wrong Answer

Example 5.1(Li & Wang, 2007). How many real solutions does $x^{12}-2^{x}=0$ have? Although the equation has three real solutions, graphing calculators only give two. Here's what TI-89 produces.



Example 4.2. Find $\lim_{x\to 0} x\ln x$*, $\lim_{x\to 0^-} x\ln x$*, and $\lim_{x\to 0^+} x\ln x$. These are other examples of wrong answers with TI-89. The asterisk indicates an incorrect calculation by the calculator. It seems that TI-89 does not always make a distinction between left-, right-, and two-sided limits for functions that are internally defined such as $\ln x$, $\sqrt{x}$, $\sin^{-1}x$, etc.

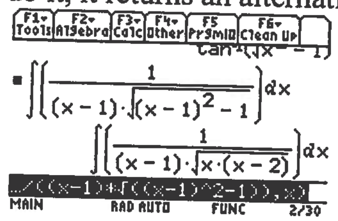


Example 4.3 (Friedberg, Insel and Spence, 2005). Solve $\begin{cases} 4935937x+4935936y=1 \\ 4935938x+4935937=2 \end{cases}$. By algebraic methods, the solutions of the equation are $x=-4935935, y=4935936$. However, with TI-83/84/85, students will get a wrong augmented

matrix, $\begin{pmatrix} 1 & 0.999999797404 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, which means no solution. But the TI-98 does it correctly.

4.2. Calculators Return the same expression without evaluation, or take too long to get an Answer.

Example 4.4. $\int \frac{1}{(x-1)\sqrt{(x-1)^2-1}}dx$. This integral can be calculated by hands by substitution. But the TI-89 cannot do it, it returns an alternative expression of the integral.



Problems like this also occur when evaluating $\int \frac{1}{(x-k)\sqrt{(x-k)^2-1}}dx$ for constant k.

Similarly, for $\int (x-k)\cdot e^{-(x-k)^2}dx$, students will be disappointed. The calculation of Laplace transforms starts out well: $\int_0^\infty t\, e^{-st}dt$ (for $s>0$). However, the calculation of $\int_0^\infty t^4\, e^{-st}dt$ takes nearly ten minutes.

REFERENCES:

1. Benson, S. (a) Lies my calculator told me: Creating learning opportunities through calculator "mistakes," http://www2.edc.org/cme/showcase/KY/calcliesarticle.pdf
2. Benson, S. (b) More Lies My Calculator Told Me: Creating learning opportunities through calculator "mistakes," http://www2.edc.org/cme/showcase/KY/MoreLies.pdf
3. Demmel, J., Can You Trust Your Calculator or Computer, UC Berkeley Math 16a.
4. http://www.cs.berkeley.edu/~demmel/Ma16A_Fall08/roundoff.html
5. Friedberg, Insel and Spence (2005), *Elementary Linear Algebra*, 3rd edition, Prentice Hall, p27.
6. Howell, M. *Lies My Calculator Told Me*, AP Central
7. http://apcentral.collegeboard.com/apc/members/courses/teachers_corner/11703.html
8. Li, X. & Wang, T. *Bilingual Teaching and Its Assessment on Mathematical Thought and Appreciation,* Journal of Guizhou Normal University (Natural Science Edition in Chinese), May (2007).
9. Steward, J. *Lies My Calculator and Computer Told Me*, James Steward Center for Mathematics, http://www.stewartcalculus.com/data/default/upfiles/LiesCalcAndCompTold.pdf