USE OF TECHNOLOGY IN UNDERGRADUATE STUDENT RESEARCH

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Abstract
In this paper, we will demonstrate the use of technology, specifically computer algebra systems such as Maple, in students' research projects. We have presented a sample of projects from the mathematics capstone course and the capstone honors seminar in our institution where the students used technology to complete projects in various areas of undergraduate mathematics and statistics.

Keywords: Maple, Minitab, Geometer's Sketchpad, Symbolic Computation Systems, undergraduate research

Introduction
We have used technology in the mathematics and statistics classes for over fifteen years. We believe that technology, when used appropriately, can enhance a student's understanding of complicated topics. Research in mathematics at the undergraduate level is somewhat challenging. Most mathematics students, in the undergraduate level, do not have the maturity and the background knowledge to be able to complete meaningful and publishable research projects. It has been our experience that technological tools such as Maple may compensate for some of the shortcomings of an inexperienced researcher. These tools are powerful enough to allow a student to conduct various mathematical or statistical experiments to achieve reasonable results. In addition to statistics, there are certain branches of the discipline of mathematics, such as numerical analysis, operations research and optimization and some elements of applied mathematics, which lend themselves well to the use of technology. There are even more theoretical branches of mathematics such as number theory which, as shown in recent publications, can benefit from the powerful features of computer algebra systems. Students at TLU, including all of the mathematics majors, have to complete a capstone research course, which includes an individual research project, as part their requirements for graduation. Honor students are also required to complete a capstone course which involves team projects. We will present some of the recent projects and discuss the benefits, as well as some of the drawbacks in this paper. For related background information on the use of Maple, Geometer's Sketchpad and other related software, we encourage you to consult the authors' earlier papers [1] and [2].

1 A summary of the student projects

Following is a sample of group projects where were completed by teams of students in the capstone honors class. For the sake of brevity we have included only the title of the projects, the software which was used ,and a brief description.

1. Freshman Fifteen- . Students used real data, and Minitab software, to study the so-called "freshman fifteen" myth.
2. Use of Force by SAPD, are minorities targeted [San Antonio Express, April 28th 2002]? Students used statistical methods with Minitab to investigate whether San Antonio Police Department used force disproportionately on minorities.
3. Did Mark Twain write the letters? Letters published in 1861 in the New Orleans Daily Crescent were signed as "Quintus Curtius Snodgrass". Students used statistical analysis with Minitab to examine the question.
4. Polling agencies- Who did better in predicting the outcome of the 2004 election? Students compared the polls from several agencies (Rueters/Zogby, Rasmussen, CNN/USA/Gallup, Mason-Dixon, etc.) with the actual election results.
5. Bleeding Hearts but tight Fists, George Will's article, is he right? This is one of the more controversial Newsweek columns by George Will. Students did their own research to verify/dispute Will's claim.

Following is a sample of projects completed by mathematics students as part of their senior capstone course. In this list, we have included the title, the student's name and the software used.

1. Michael McFarren, "Chebyshev Acceleration Methods For solving Linear Systems ", 1995, C and MAPLE
2. Whitne. Daum, "Computational Pitfalls in MAPLE",1998, Maple
3. Shannon Hanes, " Statistical Analysis of Logical Thinking Patterns among Adults", 1998, EXCEL
4. Jereme Matthews," The Black and Scholes Partial Differntial Equation", 1998, Maple
5. Roger Hooker, " Frank-Wolfe Algorithm", 1998, MS ACCESS
6. Carrie Arlitt, " Convergence rate of Runge-Kutta Methods", 2001, Maple
7. Kent Kraft, "Chi-Square Test for Improving Finite Difference Methods with Extrapolation ", 2002, Maple
8. Crystal Wiedner, "Optimizing ω in the SOR Method", 2005, Maple
9. Robin White, "Analysis of a Revised Newton's method", 2005, Maple
10. Jacob Robbins, "Improving Iterative Methods for Determining Eigenvalues via Aitken Acceleration", 2006, Matlab
11. Jennifer Weaver, "Volumes of Irregular Solids of Revolution", 2007, Maple
12. Dan Smith, "A Newton-SOR method for Solving Nonlinear Systems of Equations", 2007, Maple.

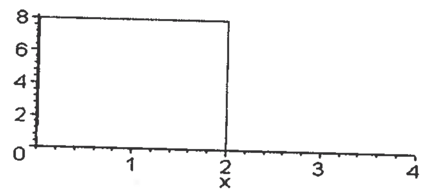
2 A sample project using Maple

The following example shows the use of Maple in determining the coefficients of Fourier series as part of a student project. Note that the series converges to the average of the right and left limits at x = 0 where we have a discontinuity.

```
> f:=x->piecewise(0<x and x<2,8,x>2 and x<4,0);
```

$$f := x \to \text{piecewise}(0 < x \text{ and } x < 2, 8, 2 < x \text{ and } x < 4, 0)$$

```
> plot(f(x),x=0..4);
```




```
> period:=4; L:=period/2;
```

$period := 4 \quad L := 2$

```
> an:=int(f(x)*cos(n*Pi*x/L),x=0..2*L)/L;
```

$$an := \frac{8\sin(n\pi)}{n\pi}$$

```
> bn:=int(f(x)*sin(n*Pi*x/L),x=0..2*L)/L;
```

$$bn := -\frac{8(-1+\cos(n\pi))}{n\pi}$$

```
> simplify(an) assuming n::integer;simplify(bn) assuming n::integer;
```

$$0, \quad -\frac{8(-1+(-1)^n)}{n\pi}$$

```
> A:=int(f(x),x=0..2*L)/(2*L);
```

$A := 4$

```
> Fn:=an*cos(n*Pi*x/L)+bn*sin(n*Pi*x/L)assuming n::integer;
```

$$Fn := -\frac{8(-1+(-1)^n)\sin\left(\frac{n\pi x}{2}\right)}{n\pi}$$

```
> FF:=(k,x)->4-sum(8*(((-1)^n-1)/(n*Pi))*sin(1/2*n*Pi*x),n=1..k);
```

$$FF := (k, x) \to 4 - \left(\sum_{n=1}^{k} \left(\frac{8(-1+(-1)^n)\sin\left(\frac{1}{2}n\pi x\right)}{n\pi}\right)\right)$$

```
> limit(FF(k,0),k=infinity);
```

4

```
> limit(FF(k,1),k=infinity);
```

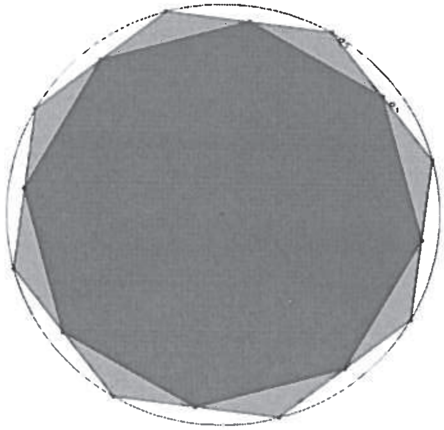
$$\lim_{k \to \infty} 4 - \left(\sum_{n=1}^{k} \left(\frac{8(-1+(-1)^n)\sin\left(\frac{n\pi}{2}\right)}{n\pi}\right)\right)$$

```
> evalf(FF(100,1));evalf(FF(1000,1));f(1);
```

7.974537753, 7.997453522, 8

3 A sample project using Geometer's Sketchpad and Maple

We were early adopters of graphing calculators, Minitab, and Maple. Given that it is somewhat surprising that we have been using geometric exploration software a relatively short period of time. Much motivation for geometric speculation is visual and with Geometer's Sketchpad students can speculate about geometric relations and easily create multiple cases in which they can test their speculations for plausibility. The project presented here arose from a small class demonstration concerning the ratio of the areas of a mid point polygon to its parent polygon. (We started with regular convex polygons and formed the mid-point polygon by connecting the mid points of the edges in adjacent order.)



Area P_1 = 127.34 cm^2

Area P_2 = 149.19 cm^2

P1 is MPP,
P2 is Polygon

$$\frac{(\text{Area } P_1)}{(\text{Area } P_2)} = 0.85$$

The class tried regular octagons of different sized (in GSP just stretch or shrink the original figure) and discovered that this ratio appears to be constant. And these discoveries lead to questions about regular polygons with different numbers of sides. With GSP a large number of students were willing to try cases and to state hypotheses. This became a teaching moment and an illustration of the difference of inductive and deductive logic. Finally I led them through a derivation of a formula for the ratio that included their findings as special cases. That result is that the ratio of the areas of a regular MPP to its Polygon of n sides is given by $\sin\left(\frac{(90*(n-2))}{n}\right)^2$

And now we were ready to experiment with non-regular convex polygons. After much experimentation and referencing the literature we discovered that **the area of a convex midpoint polygon is half the area of the original polygon plus one forth the area of the star shaped polygon** formed by connecting every other vertex of the original polygon and continuing until returning to the starting point. The area of the star shaped polygon must be computed algebraically and the students appreciated finding that one could use Maple to easily calculate these areas.

Consider an irregular pentagon with vertices in Cartesian Coordinates at (0,0), (4,0), (5,4), (3,6), and (1,4)
Define some matrices using the endpoints of segments making up the perimeter.

```
> with(linalg); A:=matrix(2,2,[4,0,5,4]);
B:=matrix(2,2,[5,4,3,6]);C:=matrix(2,2,[3,6,1,4]);
```

$$A := \begin{bmatrix} 4 & 0 \\ 5 & 4 \end{bmatrix} \quad B := \begin{bmatrix} 5 & 4 \\ 3 & 6 \end{bmatrix} \quad C := \begin{bmatrix} 3 & 6 \\ 1 & 4 \end{bmatrix}$$

Using linear algebra we can calculate the area of the original pentagon, APENT as:

```
> APENT:=1/2*(det(A)+det(B)+det(C));
```

$$APENT := 20$$

The coordinates of vertices of the sample pentagon were selected so that the mid-points of edges could be found by inspection. Thus, we proceed to finding the area of the mid point pentagon.

```
> E:=matrix(2,2,[9/2-2,2,2,5]);
F:=matrix(2,2,[2,5,0,5]);G:=matrix(2,2,[0,5,1/2-2,2]);
```

$$E := \begin{bmatrix} \frac{5}{2} & 2 \\ 2 & 5 \end{bmatrix} \quad F := \begin{bmatrix} 2 & 5 \\ 0 & 5 \end{bmatrix} \quad G := \begin{bmatrix} 0 & 5 \\ \frac{-3}{2} & 2 \end{bmatrix}$$

And the area of the mid point pentagon is AMPPENT

```
> AMPPENT:=1/2*(det(E)+det(F)+det(G));
```

$$AMPPENT := 13$$

If we now find the area of the associated Star Shaped polygon we can verify the formula that we found in the literature. Let the area of the Star Shaped polygon be ASSP

```
> H:=matrix(2,2,[5,4,1,4]);
J:=matrix(2,2,[1,4,4,0]);K:=matrix(2,2,[4,0,3,6]);
```

$$H := \begin{bmatrix} 5 & 4 \\ 1 & 4 \end{bmatrix} \quad J := \begin{bmatrix} 1 & 4 \\ 4 & 0 \end{bmatrix} \quad K := \begin{bmatrix} 4 & 0 \\ 3 & 6 \end{bmatrix}$$

```
> ASSP:=1/2*(det(H)+det(J)+det(K));
```

$$ASSP := 12$$

VERIFICATION of formula:

```
> 1/2*APENT+1/4*ASSP;    13
> AMPPENT;    13
```

None of this work is original, but it was all new to our students. Working with technology takes a lot of the fear and tedious work out of learning mathematics by examining cases and speculating on relationships that may later be proven deductively.

Acknowledgements: We would like to thank the Texas Lutheran University Research and Development Fund, which partially supported this research.

REFERENCES

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