CREATING PICTURES IN EXCEL TO MOTIVATE THE LEARNING OF FUNCTIONS

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The concept of function and understanding their multiple representations are important in helping students develop what some researchers refer to as function sense (Eisenberg and Dreyfus 1994; National Research Council 1989). However, making connections between the visual and symbolic representations of functions is difficult for students (Vinner 1989; Eisenberg and Dreyfus 1994). Dick (1992) and Wilson and Krapfl (1994) suggested that the use of multiple representations, interpretation from one representation to another, and analysis which allows students to relate the graphic, numeric, and symbolic information are critical areas that students should be exposed to in order to develop a better understanding of functions. Although each of these representations is available in a graphing calculator environment, software programs, such as Microsoft© Excel, also provide an environment for students to explore multiple representations of functions. In this paper, an activity is described which allows students to work with the different representations of functions and at the same time create fun and interesting pictures in Excel by graphing functions or other equations over a specified domain.

Why Use Excel?

As already noted, Excel, as with graphing calculator technology, provides an environment in which multiple representations of a function may be investigated. Excel allows a function to be defined symbolically, by creating a table of data through the use of the spreadsheet, and also graphing the function using a scatter plot of the data. Another reason Excel is used is because some graphing calculators have a limit on the number of functions that may be input; however, it is possible to graph line segments and circles using a graphing calculator's drawing tools. Excel also allows different functions to be graphed using a different color and this can aid students in more clearly identifying the different parts of a piecewise-defined function. It is also easy to draw vertical line segments in Excel and, if an error is made, it is easy to correct it whereas on the graphing calculator, it may mean having to clear the picture and start all over again. Figure 1 shows a picture of a house created using Excel and a TI-83+ graphing calculator.

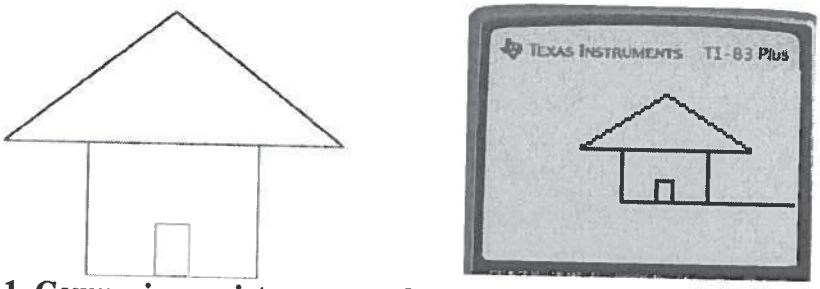


Figure 1. Comparing a picture created using Excel and the TI-83+ graphing calculator.

Creating the Pictures

To create pictures, a restricted domain of the function is used so students have to determine the interval of domain for each part of the picture. Once a part of the picture is graphed, points may need to be added to the interval of domain in order to create a smooth curve. So, for instance, if the upper and lower parts of a circle are graphed, it may be necessary to have increments of one-tenth over the interval of domain in order to make the curve appear smooth. Figure 2 shows a series of graphs of the upper and lower parts of the circle $x^2 + y^2 = 4$. The first image was graphed using three points in the domain, the second image was graphed using nine points in the domain, and the third image was graphed with increments of one-tenth over the interval of domain. Notice how the graph becomes smoother as more points are added to the interval of domain. Although some instructors might consider students having to understand how many points to use in an interval to be a disadvantage to using Excel, it can also add to the value of learning about graphing functions because students learn to distinguish between an estimate to the graph of a curve and the actual graph of the curve.

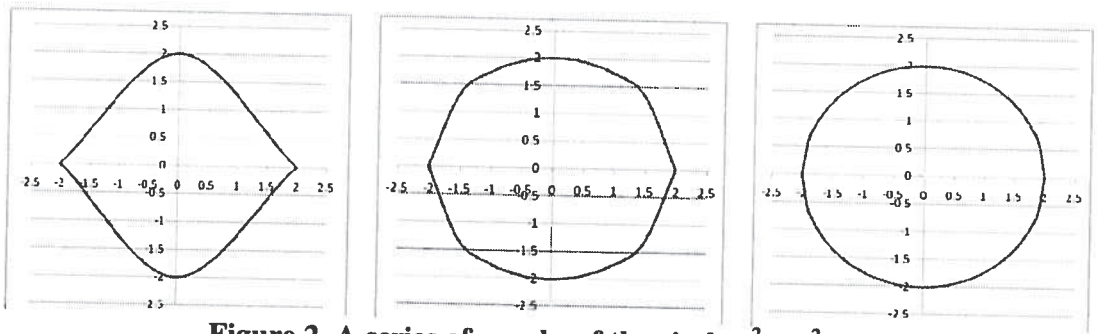
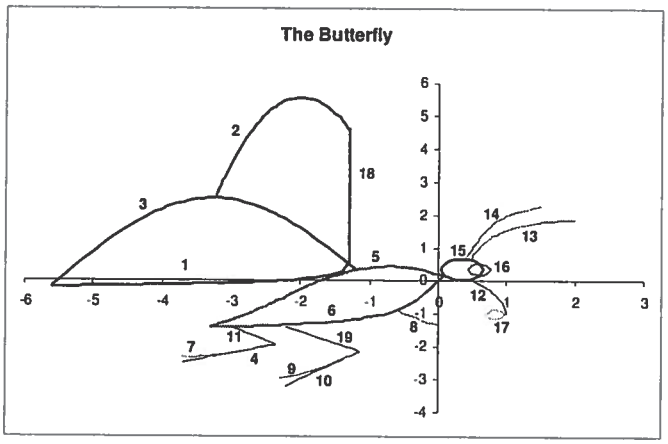


Figure 2. A series of graphs of the circle $x^2 + y^2 = 4$.

Student Activities

There are many student activities that could be done using pictures such as those created in Excel. One possible activity is to provide students with a picture along with the functions or equations used to generate each part of the picture. For the assignment, students would be asked to calculate the domain of each function or equation represented in the picture. This is a valuable activity for students because it emphasizes the calculation of the domain and requires that exact values be used, especially in the case of graphing a square root function. It should be noted that if the endpoints of the interval of

domain are estimated, the value may lie outside of the actual domain of the function, in which case an error is given in Excel. Once the domain is calculated for each part of the picture, then students could use Excel to enter a symbolic representation for each function, input values in the domain based on their calculations and allow the software program to calculate the corresponding range values. They would then use the *Scatter with Smooth Lines* feature in Excel to graph each function and create the picture. Figure 3 shows an example of a butterfly with the function or equation corresponding to each label in the picture.



1) $y = -0.3\log(-x-1.3)$
2) $y = -2(x+2)^2 + 5.55$
3) $y = 2.5\sin(.7x-15)$
4) $y = .4x-1$
5) $y = \sin(x-4)-.55$
6) $y = \arctan(2x)$
7) $y = \sin(x+2)-1.33$
8) $y = -\sin(x+1)-0.5$
9) $y = \sin(x+1)-2$
10) $y = x-1$
11) $y = -\frac{5}{6}x-3.9$
12) $y = \arccos(x)-1$
13) $y = \arctan(3x-1.25)+0.5$
14) $y = \arctan(3x-1.5)+1$
15) $(x-.35)^2+(y-.35)^2 = 0.1$
16) $(x-.6)^2+(y-.35)^2 = 0.03$
17) $(x-.84)^2+(y+1)^2 = 0.02$
18) Line segment connecting (-1.32, 0.5) and (-1.33, 4.7)
19) Line segment connecting (-2.2, -1.4) and (-1.2, -2.1)

Figure 3. The butterfly and corresponding functions or equations.

It is also useful for students to create their own pictures. When students create their own pictures, they are learning about the characteristics of functions because they have to understand what type of function could best be used to generate a portion of the desired curve for creating the picture. An example of a picture that was created by a student is provided in figure 4. This picture was created using line segments and portions of quadratic functions, except for one part along the back of the neck of the dog, which required an exponential function.

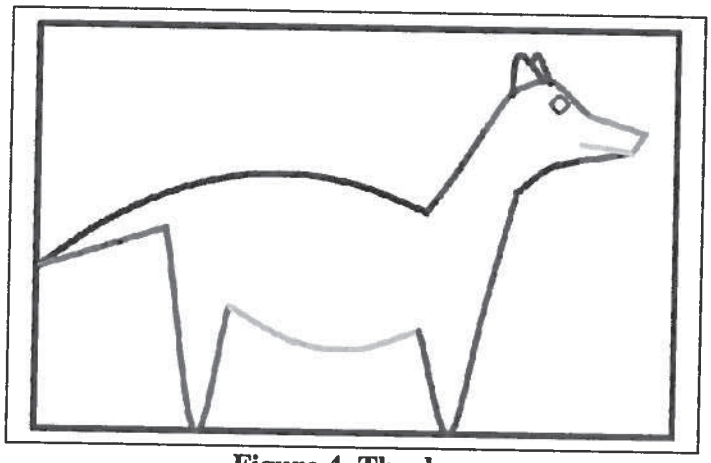


Figure 4. The dog.

It should be noted that when students created their own picture many of the students found it was best to sketch their picture by hand on graph paper before attempting to graph it using technology; although technology can allow functions to be graphed much faster and this is useful when it is hard to determine which function might best represent a part of the picture. This was the case for creating the portion of the back of the neck of the dog. The student thought that a quadratic function could be used to complete that portion of the back of the neck, but eventually discovered that an exponential function was a much better fit. By using a hand-drawn picture, students can plot specific points and then use regression to determine the equations that will model each part of the picture. If the picture is created using only linear and quadratic functions, having students go through the process of solving a system of equations to determine a quadratic function or finding the equation of the line for a linear function is a useful exercise.

Another example of an activity for how the pictures can be used is to give students a picture and have students determine the equation of for each part of the picture. Figure 5 shows a picture of a house that was created using line segments, except for the door knob.

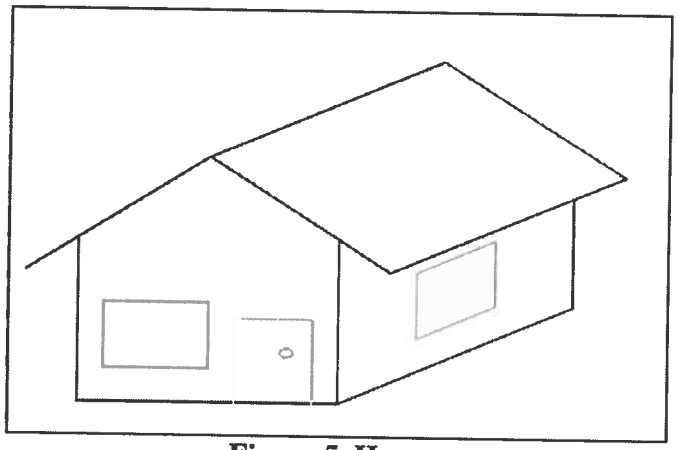


Figure 5. House.

Students can calculate the equation for each line segment within the picture or just determine the slope of each line segment as an introductory lesson on slope. The house picture is good to use in an introductory lesson on slope because it contains line segments with positive slope, negative slope, zero slope, and no slope.

It may also be useful for students to experiment creating pictures using several different technologies so that they can compare each one and determine for themselves the advantages and disadvantages of the different technologies. Although Excel was required to be used in creating the final picture, some students, who had not worked with Excel in the past, used more familiar technologies, such as the graphing calculator, to manipulate the functions they were using in their picture. The graphing calculator also proved to be useful when trying to find an equation using regression. Thus, having access to a variety of technologies can be beneficial to the learning and understanding of functions and their characteristics.

Final Remarks

Creating fun and interesting pictures in Excel by graphing functions can promote students' understanding of function characteristics and can help them get motivated to learn more about various functions. When creating the pictures, students learn to: (1) determine an equation that best models a part of the picture they have created, (2) determine the interval of domain for each function or equation used in the picture, (3) distinguish between an estimate to the graph of a curve and the actual graph of the curve, and (4) distinguish between the different representations of functions. This list represents just some of the ways that students will improve their understanding of functions by creating pictures in Excel.

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