THINKING OUTSIDE THE DIFFEQ BOX

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Introduction

Instructors of Differential Equations often find ourselves dismayed at the lack of enthusiasm with which our students approach our subject. We believe that the study of differential equations is both fascinating as it relates to theory and highly useful as it relates to practice, but our students do not necessarily share our devotion. In an attempt to discover the reason for the discrepancy, we did the obvious: we asked the students and we listened to what they had to say.

The consensus seemed to be that the subject was "boring." Probing more deeply, it seems that the students are likely to see the work they are doing for the course more as exercise working than as problem solving. They see busywork with little room for creativity.

If students are bored, they are unlikely to spend more than the bare minimum amount of time required to complete an assignment. Further, they are unlikely to engage deeper levels of processing, and thus unlikely to progress to higher levels of understanding. Neither of these is conducive to optimal student learning.

Because students often equate "learning" with "grade received," they also often request the opportunity to earn extra credit to improve their grade. Assigning them more problems of the same type they have already been assigned may give them more points but it will not improve their understanding: doing a lot of C work is still doing C work. However, the request for extra credit can be used to elicit deeper, and different, processing than they have already done, and if the assignment is also perceived as allowing for creativity, it can result in significantly more time and effort as well as thought.

Any students can benefit from working such assignments. Students whose skills are less than strong will have the opportunity to strengthen them; students whose skills are strong will have the opportunity to stretch themselves. Students who are not interested in the subject may find something to spark some interest; students who are interested may find new ways of thinking about the subject. Students who are tech-savvy will have the chance to show off; students who are less technologically literate will have the chance to play with something new and different. If the assignments are sufficiently interesting, it may even happen that students will recruit their non-mathematical friends to take a hand in the doing of the project and in the reporting of it. The payoff for the instructor is in seeing our students do more than they perhaps thought they could, and certainly more than they initially thought they would.

Many of us now teaching collegiate mathematics were students at a time when current technologies did not exist. Many of us were students when the technologies were significantly less sophisticated than they are now, and we ourselves were less sophisticated as well. We point out, however, that these assignments need not require a high level of technical expertise on the part of the instructor. We often say that the job of the teacher is to empower our students to surpass us, and this is one area in which that is not only possible but perhaps even probable. The students either already had, or in the course of working the assignment developed, their own expertise in video technology, including editing and sound work. The ones who used video techniques used them either for reporting results or for documenting the work, or both. A bench log is not necessarily pencil on paper.

In terms of implementation, we note that setting a due date well before the end of the semester would be helpful in two ways. One, it could allow for extensive feedback to the students, including discussion of methodology, measurements, presentation issues and so on. Two, it would allow students to present their own work to their classmates, which both presenters and audience seem to enjoy.

Two Sample Assignments

We present two sample assignments, precisely as they were given to the students in a first Differential Equations course. One required students to invent their own units and subunits and solve the differential equations to test whether the familiar simple harmonic motion formulas still held when not working in the metric system. The second required them to work a non-standard problem using Newton's Law of Cooling. Both assignments presented here included a somewhat humorous context, and both assignments encouraged students to work seriously, to present their results seriously, but not to take themselves seriously. For the most part, students entered into the spirit of the assignments, as will be seen.

Assignment One: "It's Spring!" Unit-independent Springs and Gravity

While working spring problems and seeing computer simulations of bouncing springs, I realized what was missing in all this theory was any check that our results had any basis in reality. Perhaps all this business of harmonic motion was just part of the international metric conspiracy, and was intended to boost the popularity of their measuring units and increase sales (there is also a rumor that they will be incorporated into the next version of Windows). How do we know if those equations are correct, or that they don't depend on the units being used? Calculus formulas for trig functions only work in radians, after all, and not in degrees. What should an inquisitive student do?

This is what an inquisitive student should do. Design a method to check this out with your own unadulterated measuring units. For mass you could use some objects that have consistent size, say coins (pennies if you are frugal, dimes or quarters for the affluent) or washers or nails or whatever is convenient. Also define your own unit of length, and sub units. Time - can you trust seconds and minutes? You should define your own time unit. Galileo, while observing the lamps in church swaying like a pendulum, used his pulse to time the swings (he should have been paying attention to the service, probably would have saved no end of trouble for him later on). One hopes he remained calm, for if he got excited all his timings would be off. You do have the option to make up your own names for your units - here's a chance to immortalize your name, or that of a friend, or you could just pick something weird (but keep it clean).

Once you have defined your units you need to find a spring and determine its spring constant. Then put a mass on the spring, and find a way to take measurements so you can plot the experimental motion of the spring. Then calculate the theoretical behavior and compare the two.

Oh, one little detail: how do you find a value to use for g, the acceleration due to gravity in your units?

N.B. You should determine all constants on your own using your own units, not table values.

Level I Project: Write a careful description of all the steps involved in defining units, finding g, measuring spring movement, etc. It should be sufficiently detailed that a person following your instructions could actually do the project.

Level II Project: Take your careful description of what to do and do it. Write up a report of your results.

Assignment Two: "CSI: Mathopolis" Newton's Law of Cooling

You are a mathematical forensic specialist in Mathopolis, the city where all story problems are true: massless ropes and frictionless pulleys are available in every hardware store; skydivers know nothing of wind resistance; even feathers breeze past terminal velocity and drop like a rock, and Newton's Law of Cooling holds exactly for every size and shape of object, animate, inanimate or formerly animate.

The body of of Rene Debarrow has been found in the library of a country house, struck from behind with an antique vase, a far too frequent occurrence. You arrive at the crime scene at 8 p.m. exactly, instantly noting the ambient temperature is 68^{o}, Rene is at 89^{o}. He, like everyone in Mathopolis, had a normal temperature of exactly 98.6^{o}.

You now examine the crime scene in great detail, seeking clues. When you return to take a second temperature reading of the body, you find that your over zealous assistant Igor has sent the body to the morgue. A phone call determines that the body was placed in a locker at 8:30 exactly, but no temperature was taken. The locker is at 34^{o}.

Dashing to the morgue, you take a temperature at 8:45, finding it to be 76^{o}. At 9:15 you take a second reading and find it is 60^{o}.

Level I Assignment: Assume all the times and temperatures are exact. Use Newton's Law of Cooling to find the time of death. Set up the I.V.P.s and solve them, do not use formulas.

Level II Assignment: Assume the temperatures are $\pm\frac{1}{2}$ degree. Determine a range of time in which the death could have occurred.

<u>The Students' Work</u>

In working the spring problem (Assignment One), some students used no technology whatever apart from a calculator; some used video to record what they did, and freeze-frame to take measurements. Some reports were hardcopy, some were hardcopy with visuals, some used PowerPoint or similar presentation software, and one was a movie. None simply did calculations and handed in an "answer." All entered into the spirit of the problem, including wondering whether there really was a vast "metric conspiracy" and inventing some amusing units.

A similar range of technological use showed up in the CSI: Mathopolis problem (Assignment Two). Several students reported their work in hand drawn graphic novel form; a number of them framed their results as reports to a client, sometimes the district attorney and sometimes a suspect; most used at least visuals in their reports, some used presentation software and, again, one produced a short film. As in the first assignment, none simply did calculations and handed in an "answer," but all entered into the spirit of the assignment, most with apparent relish.

It should be noted that not all the resulting work was accurate, a situation perhaps to be expected. Nonetheless, we were more than pleased with the students' efforts to do something they perceived as in some sense significant. A fair proportion of them sought extra consultation as they worked through the assignments, and not a few mentioned that they had actually enjoyed solving the problems. In spite of the humor, they seemed to consider the problems "realistic." This is an improvement over the "boring" judgment they had given to more standard problems.

Conclusions and Follow-up

Not all students chose to take on the extra credit projects, but we found no relation between the students' grades without the project and their decision to undertake it. It was not the case that it was attempted mostly by students who needed as much extra credit as they could get. Nor did we find that it was mostly undertaken by the strong students who were already willing to do as much extra work as they could get. The students who did the projects seemed to enjoy them, judging by the humor displayed in their work and by informal comments received. We know the authors enjoyed the results of the students' work!

Clearly this is an empirical project. It is informed by general findings in the literature (such as the value of time on task) but not by any particular experimental study. No statistical analysis of the results was made because such an analysis would have been full of caveats regarding the non-experimental nature of the project and any results would have had to have been considered essentially meaningless. The design of even a quasi-experimental study would be fraught with difficulties, ranging from the use of historical controls, through self-selection of subjects, to the ethical question of providing some, but not all, students the opportunity to learn more and better. We have therefore opted not to do such a study, but would be interested in reading about one should one of you care to attempt it.