

THE USE OF MAPLE AND LAPLACE TRANSFORMS IN SOLVING INITIAL VALUE PROBLEMS.

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Abstract:

There is more than one method to solve initial value problems analytically. One of the methods is to use Laplace transforms. We will show some examples of dynamical systems to include spring mass system and a simple pendulum in solving initial value problems with the use of Maple and Laplace transforms

Discussion/Maple Segments:

There is more than one method in solving initial value problems analytically. When solving a spring mass system model we use the method of undetermined coefficients method or variation of parameters to find the particular solution. The other analytical method used to solve a spring mass system or illustrate the motion of a simple pendulum is the use of Laplace transforms. When using Laplace transforms in solving an initial value problem we first take Laplace transform on both side of the differential equation and the then take the inverse Laplace transform to find the final solution of the problem. The Laplace transform of the first and second derivative occurring in the general spring mass system model will contain the initial conditions involved in the problem. By the substitution of the initial conditions in the equation involving the Laplace transform of the function to be solved and then take the inverse Laplace transform of the equation will allow us to solve the complete initial value problem.

We will use Maple with Laplace transforms to solve some examples of initial value problems to include a spring mass system model and the motion of a simple pendulum. Also we will show how to solve an initial value problem that contains a system of differential equations, with the use of Maple and Laplace transforms.

By using integration by parts one can show the following Laplace transforms of the first and second derivatives.

$$L(x''(t)) = s^2 X(s) - x'(0) - s * x(0)$$

$$L(x'(t)) = sX(s) - x(0)$$

Where $X(s) = \int_0^{\infty} x(t)e^{-st} dt$ is the Laplace transform of the function $x(t)$ and L stands for the Laplace -transform.

We will now go through some examples of initial value problems and their solution using Maple and Laplace transforms.

Example1: Solve the initial value problem with the use of Maple and Laplace transforms.

$$\frac{d^2 x(t)}{dt^2} - 4 \frac{dx(t)}{dt} + 3x(t) = e^{3t}, \quad x(0) = 4, x'(0) = 8$$

By taking Laplace-transform on both sides of the equation we will get an equation involving the Laplace transform $\text{laplace}(x(t), t, s)$ of the function $x(t)$ and the two initial conditions $x(0)$ and $x'(0)$. The Maple command that is used to get the Laplace transform of a function is “`inttrans[laplace]()`” followed by the function in parenthesis, where `inttrans` stands for the integral transformation. The Maple segments are as follows.

```
> inttrans[laplace](diff(x(t), t$ 2) - 4*diff(x(t), t)
+ 3*x(t) = exp(3*t), t, s);

s^2 laplace(x(t), t, s) - (D(x))(0) - s x(0)
- 4 s laplace(x(t), t, s) + 4 x(0)
+ 3 laplace(x(t), t, s) = 1/(s - 3)
```

We will substitute the initial conditions in this equation using the “`subs`” command,

```
> subs({x(0) = 4, D(x)(0) = 8}, %)
;

s^2 laplace(x(t), t, s) + 8 - 4 s - 4 s laplace(x(t), t, s)
+ 3 laplace(x(t), t, s) = 1/(s - 3)
```

```
> subs(laplace(x(t), t, s) = X(s), %)
;
```

$$s^2 X(s) + 8 - 4s - 4s X(s) + 3 X(s) = \frac{1}{s - 3}$$

We will now solve for the function $X(s) = \text{laplace}((t), t, s)$ in the above equation using the “`solve`” command.

```
> solve(%, X(s));
```

$$\frac{-20s + 25 + 4s^2}{s^3 - 7s^2 + 15s - 9}$$

Now we will take the inverse laplace transform of the above function using the Maple command "inttrans[invlaplace]()" followed by the above function in parenthesis

> *inttrans[invlaplace](%, s, t);*

$$\frac{9}{4} e^t + \frac{1}{4} e^{(3t)} (2t + 7)$$

We have now solved the initial value problem completely using Maple with Laplace-transforms.

Example2: Solve the following initial value problem that represents the motion of a simple pendulum.

$$x'(t) = y(t), y'(t) = -4 \cdot x(t), x(0) = 0, x'(0) = 8$$

We will first take the Laplace –transform of the set of differential equations one by one and then substitute the initial conditions using the “subs” command as in the previous example.

> *inttrans[laplace](diff(x(t), t) = y(t), t, s);*

$$s \text{ laplace}(x(t), t, s) - x(0) = \text{laplace}(y(t), t, s)$$

> *inttrans[laplace](diff(y(t), t) = -4 \cdot x(t), t, s);*

$$s \text{ laplace}(y(t), t, s) - y(0) = -4 \text{ laplace}(x(t), t, s)$$

> *solve({%,%%}, {laplace(x(t), t, s), laplace(y(t), t, s)});*

$$\left\{ \begin{array}{l} \text{laplace}(y(t), t, s) = \frac{s y(0) - 4 x(0)}{s^2 + 4}, \text{ laplace}(x(t), t, \\ s) = \frac{s x(0) + y(0)}{s^2 + 4} \end{array} \right\}$$

> *subs({x(0) = 0, y(0) = 8}, %*
);

$$\left\{ \text{laplace}(y(t), t, s) = \frac{8s}{s^2 + 4}, \text{laplace}(x(t), t, s) = \frac{8}{s^2 + 4} \right\}$$

> *inttrans[invlaplace](%, s, t);*

$$\{x(t) = 4 \sin(2t), y(t) = 8 \cos(2t)\}$$

Conclusion: The use of Maple and Laplace transforms allow us to solve initial value problems without the knowledge of undetermined coefficients method or the variation of parameters.

References:

1. Soma Velumylyum, "Maple can do Numerical Approximations of Area under Curves. Are you Ready with the Current Technology to Generate some Numbers in front of the Engineering Majors? Contributed paper session, 16th Annual International Conference on Technology in Collegiate Mathematics, Chicago, Illinois, November 2003
2. Soma Velumylyum "Maple can Solve Initial Value Problems. Students do not have to Memorize some of the Mathematical Formulas". Poster session, Maple Summer Workshop- University of Waterloo, Ontario, Canada, July 2002
3. Soma Velumylyum, "Maple does all Mathematical Calculations. What do Calculus Teachers do in Class?" Contributed paper session, ICTCM 14th Annual International Conference on Technology in Collegiate Mathematics, Baltimore, Maryland, November 2001