ARE TEXTBOOKS ADDRESSING ALL ACCESSIBLE TOPICS FOUNDATIONAL TO CALCULUS?

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Abstract. In the last decade, the integration of technology[1] in the teaching and learning of mathematics has increased substantially. As a result, many graphical ideas, some numerical ones, and some good modeling problems have found their way into the pre-calculus literature. At the same time, relevant concepts, methods, and approaches, now available via technology, are being ignored. After 15 years of using graphing calculators, it is appropriate to ask: Are we taking full advantage of the main capabilities that this technology offers in order to provide secondary students with the best possible preparation for calculus? In this paper, we list a selection of accessible concepts, approaches and applications, not yet fully present in most pre-calculus books. Then we address this question: how well do current precalculus textbooks in the US incorporate these concepts?

1. Introduction.

The integration of technology in the teaching and learning of mathematics impacts all facets of instruction. More than a quarter of the mathematics content taught before the arrival of the scientific calculator is not taught today [Tooke, 2001]. Indeed, technology's influence on today's classroom instruction is readily observable from changes in content, assessment, and teaching methodologies. The multi-representational capabilities of graphing calculators have blurred grade-level distinctions previously associated with various content tasks, providing more students with access to significant mathematics at younger ages [Usiskin, 2007]. The ability to automate cumbersome calculations via technology allows students at various levels to i) use technology to meaningfully explore concepts and problems previously proposed to the most advanced mathematics students, and ii) to extend the breadth and depth at which these concepts are treated. Technology promotes conceptually-oriented instruction and the inclusion of more relevant applications, exploration and discovery in the mathematics classroom. As a result, teaching has become more student-centered, with inquiry playing a more pronounced role in both the delivery and the content of new curriculum. Not surprisingly, assumptions about mathematics curricula made in a time prior to the integration of technology in the classroom are, in some cases, no longer valid. Topics such as optimization, different

[1] We remark that in this article, unless otherwise specified, by "technology," we mean any technology with the capabilities of modern graphing calculators without symbolic manipulations (i.e. non-CAS calculators).

matrix applications, linear and nonlinear regression, recursion etc. are now accessible to students in secondary and introductory college levels (prior to calculus).

An inspection of modern precalculus texts reveals a curiously uneven approach to technology use. While many texts make use of technology's visualization capabilities - not only in the scope of the content, but also in the way that many concepts are introduced, and in the type of questions that students are asked - the same textbooks typically fail to exploit technology's powerful numeric and multi-representational capabilities. As we informally perused a number of current textbooks, we began to reconsider the role that technology actually plays in today's classrooms. Seventeen years after the introduction of the first Texas Instruments graphing calculators, a comprehensive appraisal of precalculus textbooks with respect to technology inclusion is in order. In this paper we aim to address the following question: Are our textbooks taking full advantage of the main capabilities that technology offers, in order to provide precalculus students with the best possible preparation for calculus?

2. Possible Impact of Technology on the Study of Functions before Calculus

Below, we list possible changes that technology facilitates on the traditional coverage of functions.

I. The study of basic transformations, such as $f(x)+a, f(x+a), -f(x), a\cdot f(x)$, $|f(x)|$, and $f|x|$, facilitates the study of families of functions, each with a root or parent function.

II. With every family of continuous functions the interplay among the analytical, graphical, and numerical representations should be presented. Hence, students can support graphically and/or numerically analytical solutions and vice versa, whenever possible; moreover, they can find some irrational solutions that would not be available analytically [Demana and Waits, 1998]. We refer to this as a technologically balanced approach.

III. In addition to the properties traditionally considered for every family of continuous functions, the integration of technology allows us to include:
   a. finding the range of all functions studied,
   b. determining irrational zeros, hence all the real zeros,
   c. finding local extrema, with intervals where the function is increasing or decreasing,
   d. using sequences to explore the local and end behavior,
   e. comparing relative growth of functions or families of functions,
   f. considering relevant examples of data that can be modeled via regression by the family of functions studied,
   g. optimization problems.

IV. The toolbox that we provide to the students can now include the use of nontraditional tools, such as lists, sequences, and recursion to solve different problems.

V. Some concepts foundational to calculus can be presented using different representations in the way they were developed and are better understood, i.e., via approximations.

VI. Since technology enables students to revisit problems from different perspectives based upon the depth of their mathematical knowledge, it is possible to use a spiral approach to some of these concepts, like optimization, both through different courses preceding calculus [Quesada & Edwards, 2005], as well as using the different functions studied.

For a selection of examples addressing many of these ideas, the interested reader may want to look at [Quesada, 2007].

3. Textbook analysis

Twelve current and popular precalculus textbooks sold in USA were investigated to determine how well they incorporated the concepts discussed in the previous section of this document. To do this, a scoring rubric was established. Every concept area, except equation solving and local and global behavior was scored in two parts. In the first part, the integration of the topic throughout the text was scored. That is, we looked not only for the introduction of a given concept, say for instance range, when studying linear or quadratic functions, but also if this concept was addressed for all families of functions. To score this first part, a Likert scale of 1-5 was employed, with 5 being the optimal score. The second part of the score measured the technological balance of the instructional approach. A Likert scale of 0-2 was used, with 2 being the optimal score.

The scale described above was not used for equation solving and local and global behavior. For the equation solving approach, the integration of graphical and numerical approaches was measured; therefore, two Likert scales from 0-2 were used, with 2 again representing the high score. For local and global behavior, there was no apparent progression within each concept. For this reason, a Likert scale was not applicable. Thus, a check box system was utilized to determine whether or not local and global behavior concepts were integrated into the text. The results of this analysis are shown in Table 1.

Table 1: Mathematical Concepts and Their Treatment in School Texts

Concepts	Scale	Average
Domain & Range	(1-5)	2.92
Balanced Approach	(0-2)	0.75
Extrema	(1-5)	3.5
Balanced Approach	(0-2)	1.17
Relative Growth	(1-5)	2.33
Balanced Approach	(0-2)	0.5
Transformations	(1-5)	3.58
Balanced Approach	(0-2)	0.83
Solving Inequalities	(1-5)	3.25
Balanced Approach	(0-2)	0.75
Eq. Solving Approach		
Graphically	(0-2)	1.25
Numerically	(0-2)	0.92

Concepts	Scale	Average
Local Behavior		
Graphically	(0-1)	0.92
Numerically	(0-1)	0.42
Rational Functions	(0-1)	1
Piecewise	(0-1)	0.17
Local Discontinuities	(0-1)	0.25
Global Behavior		
Graphically	(0-1)	1
Numerically	(0-1)	0.50
Complete Graph	(0-1)	0.33
Rational Functions	(0-1)	1
Piecewise	(0-1)	0.58

As Table 1 indicates, for domain and range, the average concept score was slightly less than 3. A score of 3 implies that the textbooks included traditional domain and range examples for most families of continuous functions. However, the consistent use of graphical and numerical approaches to address these concepts, particularly the concept of range was lacking.

The average score for relative growth was a 2.33. A Likert score of 2 was given to textbooks that included an example discussing the relative growth of two functions graphically. In general, textbooks tend to ignore or omit discussions of relative growth that focus on numerical aspects of functions despite the fact that such explorations, via tables, is straightforward, efficient, and conveys the numerical sense of which functions grow faster or slower. In our experience, many students find it easier to use tables than graphs to answer relative growth questions. Problems requiring students to find intersections of graphs lying outside the standard calculator viewing window were noticeably absent, despite the fact that such problems require knowledge of relative growth. For instance, when asked to find all intersections of the functions $f(x)=x^2$ and $g(x)=0.01x^3$, students who rely exclusively on the standard viewing window (Figure 1.a) typically fail to identify the intersection (100, 10000) shown in Figure 1.b. As seen in Figure 1.c, using the table to evaluate both functions for increasing values of x, allow the students to easily determine not only which function grows faster for arbitrarily large values of x, but also to find a reasonable interval where to look for the hidden intersection. Students need to recognize when the algebraic approach is easier to use, and how the theory of polynomial equations helps to determine the number and parity of possible solutions.

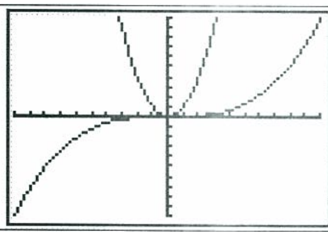

Figure 1.a
[-10, 10]x[-10,10]

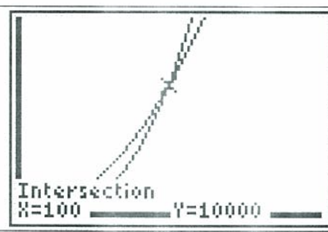

Figure 1.b
[0, 200]x[0,15000].

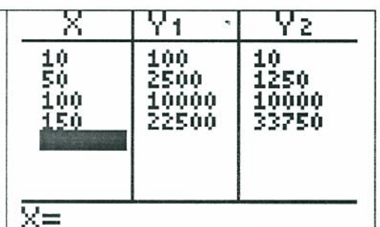

Figure 1.c

Most books do not address these ideas in a recurrent way with algebraic and transcendental equations. Moreover, the interplay between the intersections of the graphs of two functions and the zeros of the difference function is typically not mentioned; yet, finding the zeros numerically or graphically tend to be easier that finding the points of intersection.

Fewer than half of the textbooks addressed the concept of local behavior numerically, and only half addressed the concept of global behavior numerically. Indeed, only one third of the textbooks discussed the idea of the complete graph of a function. This was particularly surprising given the graphical orientation of most texts and the fact that an awareness of when a window displays all the important features of the graph of a function is essential for approaching problems graphically.

Moreover, very little is done with the use of accessible tools such as recursion or the *seq* command. Sequence definition tools, in particular, provide students with powerful multi-representational approaches for exploring limit informally. Figure 2 suggests an exploration of $e = (1+1/x)^x$ with the tools.

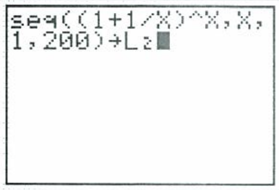	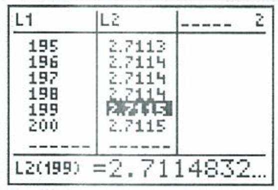	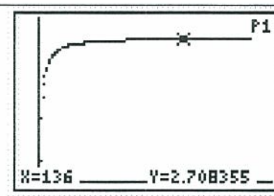
Figure 2.a Sequence definition	Figure 2.b Sequence as table entries	Figure 2.c Sequence as scatterplot

Lastly, most books do not include inequalities involving transcendental functions. Yet, independently of how the equation is solved, asking inequalities entices the learner to think graphically.

4. Summary

As one surveys today's textbooks, it is clear that the recommendations of learned societies (such as MAA and NCTM) and changes in technology have precipitated shifts in precalculus instruction. However, many remnants of the pre-technology curriculum remain. The result is a "continuous extension of the previous edition" approach; hence new, relevant mathematical ideas are juxtaposed with an assortment of topics of dubious worth. Clearly, as evidenced from our inventory of school texts, a need to re-evaluate the essential components of precalculus in a technological world currently exists. We need to ask ourselves the following question as we begin to modernize the precalculus curriculum: Do the topics we teach respond to a conscious decision based on their relevance and accessibility, or are we still teaching topics because traditionally they have been taught at this level? Symbolic calculators have been in the market for ten years, and they bring their own set of curricular and pedagogical questions: Should we not agree on most of the questions raised here before moving forward?

REFERENCES.

1. F. Demana and B. Waits. The role of graphing calculators in mathematics reform. ERIC: ED458108 (1998).
2. "Mathematics topics foundational to calculus at the secondary level," Proc. of the eighteenth Ann. Int. Conf. on Tech. in Colleg. Math., Addison Wesley Longman, Reading, Ma, 2007.
3. *ciation of America, MathDL, Journal of Online Mathematics Applications (JOMA)*, June 2005.
4. J. Tooke. Mathematics, the Computer and the Impact on Mathematics Education. *Computers in the Schools*, Vol. 17, 1-7. The Haworth Press Inc., Nevada (2001).
5. Z. Usiskin. What should not be in the algebra and geometry curricula of average college bound students? A retrospective after a quarter century. *Mathematics Teacher*. National Council of Teachers of Mathematics, Jan. 2007.