EXPLORING TOPICS WITHIN THE MATHEMATICS TEACHER EDUCATION CURRICULUM THROUGH THE USE OF TECHNOLOGY

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Introduction

One of the recommendations of the National Educational Technology Standards for Teachers [1] regarding appropriate coursework for teacher candidates is that it "must consistently model exemplary pedagogy that integrates the use of technology for learning content with methods for working with PK-12 students" (pp. 6-7). This recommendation suggests that inquiry into how technology interacts with curricula carries research significance. The goal of this paper is to introduce to the ICTCM community the notion of hidden mathematics curriculum in K-12 teacher education [2] and how it can be revealed through the use of technology. Motivated by work done with elementary and secondary preservice and in-service teachers in various mathematics education courses, the paper shows how technology tools, such as spreadsheets, computer algebra systems, and dynamic geometry software, enable an informal journey into hidden aspects of the formal content of the mathematics teacher education curriculum. By using topics and problems from standards-based curricula, it demonstrates how big mathematical ideas are implicitly present at all levels of the curriculum and may be viewed as hidden threads.

Hidden Mathematics Curriculum

The notion of hidden curriculum in mathematics teacher education was designed to help prospective teachers see connections among concepts taught across the school curriculum [2]. Appreciating these connections is important for elementary teachers in preparing their students for future learning at higher levels. By the same token, secondary school teachers can use their understanding of these connections to build instruction on their students' previous knowledge. Often, in the school curriculum, mathematical structures that connect different concepts across the curriculum are hidden from learners because of their complexity and sophistication. As an example, consider partition of integers – a topic that underlies simple problems on addition all the way to problems involving generating functions. Technology tools such as spreadsheets and computer algebra systems enable an informal introduction of this topic to preservice teachers without overwhelming them with formal mathematical theory and machinery.

From a theoretical perspective, using technology, prospective teachers are given the opportunity to learn advanced mathematical ideas in the social context of competent guidance provided by a college faculty member. Embedding these advanced ideas in the technology allows for their greater accessibility. This approach supports Freudenthal's

[3] notion of the didactical phenomenology of mathematics as a way of developing informed entries into the mathematical culture of humanity. It is also consistent with the tradition of learning through transaction that creates conditions for expert-novice collaboration within the zone of proximal development [4].

Illustration 1. Partition of integers with technology

Consider the following problem which may be encountered at any grade level: *In how many ways can 20 students be arranged into groups of size three, four, or five?* This problem can be understood conceptually as the partition of integers. Arithmetically, there exist six ways to partition 20 into the summands three, four, and five. Algebraically, the equation $3x+4y+5z=20$ has six integral solutions. Both arithmetic and algebraic approaches to the problem can be enhanced by the use of technology. The question is how that can be done with commonly available tools.

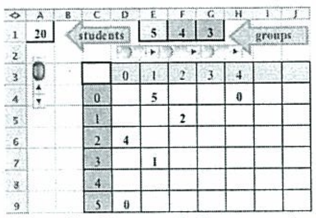


Fig. 1. Spreadsheet solution.

$$x^7 + x^3 + 2x^{11} + x^6 + x^{52} + 6x^{29} + 2x^{47} + x^{51} + x^{55} + 5x^{37} + 4x^{42} + 4x^{41} + 3x^{46} + 3x^{45} + 2x^{50} + 5x^{39} + 3x^{44} + 2x^{49} + x^{54} + 6x^{34} + 4x^{43} + 2x^{48} + x^{53} + x^{58} + 6x^{33} + 6x^{38} + 2x^9 + 3x^{14} + 5x^{19} + 6x^{24} + x^4 + 3x^{13} + 5x^{18} + 7x^{23} + 7x^{28} + 2x^8 + 4x^{17} + 6x^{22} + 7x^{27} + 7x^{32} + 3x^{12} + 5x^{21} + 7x^{26} + 7x^{31} + 6x^{36} + 4x^{16} + 6x^{25} + 7x^{30} + 7x^{35} + 5x^{40} + x^5 + 2x^{10} + 4x^{15} + 6x^{20} + 1$$

Fig. 2 The Graphing Calculator solution.

Figure 1 shows a spreadsheet that allows one to partition integers into up to three unique summands. The software makes it possible to develop a systematic way of solving a three-dimensional problem by reducing it to a two-dimensional problem: fix the number of student groups of size 5 (that varies from zero to four) and for each case partition the numbers (20, 15, 10, 5, 0) into summands three and four. Note that a special technique based on the use of inequalities allows one to use a spreadsheet to partition an integer into four summands [5]. However, mathematical software like Maple allows one to work with even higher number of summands [2].

Alternatively, the problem can be solved through the method of generating functions, a topic that can be revisited within a mathematics education course for secondary teachers. Because summands three, four, and five can enter a partition more than one time, this method enables one to find the number of partitions of 20 as the coefficient of x^{20} in the expansion of the following product of geometric series:

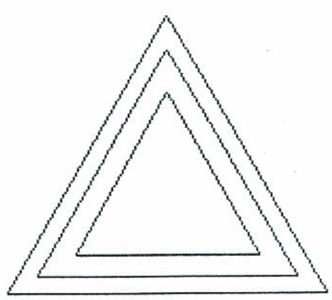
$$(1 + x^3 + x^6 + x^9 + ...)(1 + x^4 + x^8 + x^{12} + ...)(1 + x^5 + x^{10} + x^{15} + ...)$$

The use of a computer algebra system such as the Graphing Calculator (http://www.pacifict.com/) makes it possible to expand the above product to see that two different approaches to the student group problem produce the same answer (Figure 2). In revisiting this method typically taught as a part of an undergraduate course in discrete mathematics, one can address the recommendation of the Conference Board of the Mathematical Sciences [6] regarding the importance of making connections between collegiate and school mathematics in the preparation of teachers.

Illustration 2. From fractions to calculus using technology.

In the context of a class discussion on geometrical representations of fractions, one pre-teacher suggested the sketch pictured in Figure 3 as a representation of the fraction one-third. The teacher claimed that it is possible to construct three self-embedded equilateral triangles such that the area of the smallest is one-third of the area of the largest. Whereas this representation may appear intuitive at the elementary level, its formal demonstration requires the use of trigonometry – a major topic from the secondary mathematics curriculum. Indeed, one can show that the sides of the triangles of areas 1, 2, and 3 square units have the lengths $a_1 = \frac{2}{\sqrt[4]{3}}, a_2 = \frac{2\sqrt{2}}{\sqrt[4]{3}}$, and $a_3 = 2\sqrt[4]{3}$ linear units, respectively. Therefore, the areas of triangles with the sides of length a_1 and a_3 are in the ratio of 1 to 3. In general, one can use this representational idea to represent $1/n$ through n self-embedded triangles with sides $a_n = 2\sqrt{\frac{n}{3}\tan 60^\circ}$.

Geometric construction (Figure 3) can be done with the help of the dynamic geometry software The Geometer's Sketchpad (GSP). To this end, one can construct a triangle of radius $R_1 = \frac{2}{\sqrt[4]{27}}$, then define $R_2 = \sqrt{R_1^2 + \frac{4}{\sqrt{27}}}$ (in general, $R_{i+1} = \sqrt{R_i^2 + \frac{4}{\sqrt{27}}}$), and iterating this process n-1 times, obtain a sequence of n self-embedded triangles. Note that such a sequence of triangles can be dilated with the coefficient k. In this case, $R_1 = \frac{2k}{\sqrt[4]{27}}$, and $R_{i+1} = \sqrt{R_i^2 + \frac{4k^2}{\sqrt{27}}}$. Figure 3 shows the case for k=2, n=3.



n	$\sqrt{t_1^2+\frac{16}{3\cdot\sqrt{3}}}$	Area △B''B'B'''
0	2.48	4.00 cm^2
1	3.04	8.00 cm^2
2	3.51	12.00 cm^2

Figure 3. Triangle representation of one-third through GSP iteration.

One can be asked to explore the behavior of the sequence of areas and the sequence of perimeters, as n grows larger. This leads to the calculation of limiting values for the sequences $x_n = \frac{n}{n+1}$ and $y_n = \sqrt{n+1} - \sqrt{n}$ (see [7, p. 49]). One can use tools of Calculus to show that $\lim_{n\to\infty} x_n = 1$ and $\lim_{n\to\infty} y_n = 0$.

Illustration 3. Using inequalities in demonstrating properties of functions through graphing

Prospective teachers can learn mathematics by illuminating specific properties of functions such as monotonic behavior, asymptotic behavior, and their mutual relationship using the Graphing Calculator. The program allows one to graph relations in addition to functions. This, in turn, makes it possible not only to plot graphs traditionally, but also to exhibit properties not explicitly displayed by the graphing the function itself. For example, consider the function $f(x)=\sqrt{x+1}-\sqrt{x}$ - a continuous version of the sequence y_n discussed above. One can use inequalities to support a visual demonstration that $f(x)$ decreases monotonically and approaches zero as x increases. The ability of the Graphing Calculator to graph a relation from any two-variable equation or inequality can be utilized in graphing the space between two graphs in the neighborhood of any given value x_0. In comparing the values of $y=f(x)$ for different values of x, it is helpful to show the change of y through its value expressed as a bar (Figure 4).

Graphically, this bar represents a set of points (x, y) which satisfy the inequality $\sqrt{y}\sqrt{\sqrt{x+1}-\sqrt{x}-y}\sqrt{0.01-|x-x_0|}>0$. Indeed, the far-left radical constrains consideration to positive y-values. The middle radical constrains consideration to y values under $f(x)$ only. Finally, the far-right radical squeezes horizontally to a given thickness (0.01 in the case of Figure 4) the space under $f(x)$ in the neighborhood of x_0. In general, the space of thickness 2ε between $f(x)$ and the x-axis in the neighborhood of x_0 is defined by the inequality $\sqrt{y}\sqrt{f(x)-y}\sqrt{\varepsilon-|x-x_0|}>0$ when $f(x)>0$ and $\sqrt{-y}\sqrt{y-f(x)}\sqrt{\varepsilon-|x-x_0|}>0$ when $f(x)<0$. Such use of the Graphing Calculator illustrates "the way in which software can embody a mathematical definition" [6, p.132] as it involves the development of skills in using inequalities as tools in rather sophisticated computing applications.

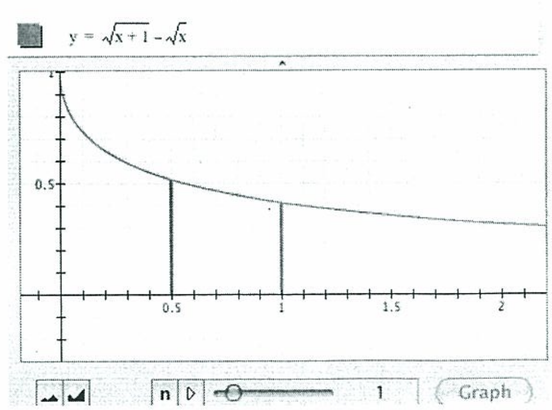


Figure 4. Using inequalities in constructing bars.

Conclusion

Each of the illustrations given in this paper demonstrates a certain aspect of the interplay between technology and curriculum. The illustrations demonstrate multiple ways in which mathematical definitions can be integrated with the tools of technology. When prospective teachers learn to interact with these tools, they also have the opportunity to learn mathematics. Furthermore, using the notion of hidden mathematics curriculum benefits teacher education programs by enabling faculty to introduce computational tools without teaching special courses on technology. Overall, the important role that technology plays in opening windows into hidden mathematics curriculum for prospective teachers though the simultaneous study of mathematics content and pedagogy has been emphasized. More information on this approach can be found in other works of the authors [8, 9].

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