

USING CLASSROOM VOTING IN MATHEMATICS COURSES

Kelly Cline, Holly Zullo, and Mark Parker

Carroll College, Helena, MT 59625

kcline@carroll.edu; hzullo@carroll.edu; mparker@carroll.edu

Electronic personal response systems have the potential to dramatically transform the mathematics classroom. With this technology, the instructor can require every single student to actively participate during class by registering a vote on a multiple-choice or true-false question with a hand-held clicker. The instructor receives real-time feedback as to the state of the students' understanding, while the students must engage in the class, preventing them from being passive observers of the mathematics lesson. However, this technology can be used in a wide variety of ways. There are many different types of questions that can be asked, and the instructor must choose from a variety of possible pedagogies which may all have very diverse effects on student learning. For example, one technique is to use classroom voting to give short quizzes. The technology allows this to be done quickly and graded efficiently, so that a quiz question or two can be given at the beginning of every class, thus motivating the students to keep up with the homework and the reading. Another very different option is to use classroom voting as a mechanism to motivate students to discuss mathematics in small groups during class, a technique sometimes called peer instruction (Mazur 1997). A new study at Cornell has demonstrated that this particular method of using classroom voting can have significant effects on student learning in mathematics (Miller, Santana-Vega, & Terrell 2006). In this study, 14 instructors, teaching 330 students, in 17 sections of differential calculus, were each given the option of using classroom voting, and if so, allowed to decide exactly how they wanted to use it. The students were given common examinations, and when the different sections were compared, they found that significantly better results came from instructors who used voting to make the students discuss these questions in small groups before voting.

We have been using classroom voting to motivate student discussion with very positive results for the past three years here at Carroll College, a small liberal arts institution in Montana. At the beginning of the semester, we have the students purchase a numbered list of multiple-choice questions from the bookstore. Several times during each class period, the instructor will call out a question number and give the students a few minutes to read the question and discuss it with their peers in small groups before registering a vote. After the vote, a graph of the results is displayed, and the instructor can go around the class, asking different students to explain which option they voted for and to discuss their thinking. Voting allows students to detect and correct misconceptions early, during the initial class period, rather than after a homework correction cycle, and it usually makes for a much more fun and lively class period for both instructor and students.

Making Classroom Voting Work

In order to make classroom voting work, we have found that on the first day of class, it is important to explain to the students exactly what is going on, and especially to emphasize to the students that the discussions are the real engine of the voting process. The votes are not scored, and do not count for or against anyone's grade. After the vote, when the instructor calls on individual students, it doesn't matter whether they give the right or wrong answer, as long as they have something to offer, some thought to contribute to the discussion. The only unacceptable response is to have nothing to say. Typically the students adapt to this pedagogy very quickly. Then throughout the semester, we integrate several questions throughout each class period, interspersing them between lecture segments whenever possible. One of our colleagues opted for a different approach, only using them at the end of class, as review questions; however, his results were not as good. Instead, the voting seems to be most effective when the questions provoke new issues, to get the students thinking about each new topic, so that they are ready for the instructor to explain the new concept. In the post-vote discussions, it's also very important for the instructor to be quiet, and to listen, as different students explain their views on each problem. Some of the best discussions result when the instructor doesn't confirm or deny the accuracy of each statement, but instead simply asks others what they think. Usually, the students can logically figure out most of the questions themselves, which is far more beneficial than simply being told the correct answer.

When we first considered this pedagogy, we were concerned that it might be hard to get students to participate and buy into this process. However we quickly found that this was not really an issue. Instead the students loved the voting from day one. They liked the process of clicking in with their votes, seeing the class period as being like a fun "game show." Attendance has been outstanding, and student focus groups indicate that they learn more from voting than from traditional lecture. When asked what they would choose if they had to decide between a section of a class with voting versus one without voting, the students are almost unanimous in saying that they would choose the section with voting.

Another concern regarded whether the act of calling on individual students by name after a vote might be too intimidating and create the wrong kind of classroom atmosphere. However this seems to be important, to make sure that as many students as possible participate in the post-vote discussions. In a class of 20 or 25 students, it is often possible to call on every single person by name, during each class period, over the course of about five votes. Further, the practice of calling on students by name after a vote is not nearly as intimidating as it might be in a typical math class, because in this context, the students have already had a chance to discuss the problem with a group of their peers and form a consensus. Thus the instructor is not so much calling on an individual person, as asking them to report on the ideas from their group. After the first few class periods, this act of calling on individual students becomes just a part of the routine.

Classroom voting can be very time consuming, with the votes and discussion often occupying more than half of a class period. However, for the past several years we have

covered exactly the same material, teaching the same syllabi, and giving the same types of exams that we did before introducing classroom voting to our calculus, multivariable calculus, differential equations and linear algebra classes. Nothing has been cut out of our courses to make room for voting. Instead, we use the voting to bring up the same issues that in the past we covered through traditional lectures. We do fewer examples on the board, instead using the voting questions to get the students to work example problems out themselves. This allows the course to proceed at the same pace, teaching the same topics in a different and more student-centered way.

Choosing the Right Questions

During most class periods, we have a dozen or more questions available in the printed list, and there may be only time to ask about four or five. So how should the instructor decide which ones to ask? The best discussions result when the questions are not easy, when there are many plausible answers that different students can select and defend. However, we have found it to be extremely difficult to predict the results of a given question. Some questions which we might expect to be very challenging end up with 95% of the students voting correctly. Other questions which we expect to be elementary, reveal all sorts of misconceptions that we did not anticipate at all, creating great discussions and bringing up very important issues to be clarified. To sort this out, we have found it very useful to keep track of the voting statistics from previous classes. These statistics, when included with the teacher's edition of the question list, can be an excellent guide to which questions will be most valuable.

Misconception Magnets

One group of useful questions that are revealed by past voting statistics are the ones that provoke common misconceptions, often getting a large majority of students to vote for the same incorrect answer. For example, in differential equations, when first introducing the concept of equilibria, we asked the following question:

The differential equation $\frac{dy}{dt} = (t - 3)(y - 2)$ has equilibrium values of

- a) $y = 2$ only
- b) $t = 3$ only
- c) $y = 2$ and $t = 3$
- d) No equilibrium values

In a recent class, 89% of the students voted for answer c), and not a single one voted for a), which is the correct answer. The students understood that the derivative would be zero at an equilibrium value, but only after discussing this question did they get the idea that an equilibrium value is a value of the function, so that if we choose it as an initial condition, then the function will remain at this value permanently. Questions like this often create little discussion before the vote, but afterwards it becomes much more interesting when the class is told that the majority is not correct. Sometimes it is useful to tell them nothing else, and then to simply have them reconsider and revote on the same question, to see if they can figure out what they are missing.

When first introducing the idea of doing matrix algebra with inverse matrices, we posed the question:

True or False Suppose that A , B , and C are square matrices, we know that $CA = B$, and A is invertible. This means that $C = A^{-1}B$.

The students had been introduced to the noncommutative nature of matrix multiplication; however this is a common mistake and they still voted 86% true and 14% false. This question made the students confront this issue right in class, so they were less likely to make the same error on homework.

Discussion Provokers

Perhaps the most useful discussions come from questions where no answer receives a majority of the votes, and instead three or even four different options are selected and defended by substantial percentages of the class. This was the result when we asked the following question, during our introduction to second order differential equations:

The following functions represent solutions of $y'' + ay = 0$ for different values of a . Which function solves this equation for the largest value of a ?

- a) $y(t) = 100 \sin 2\pi t$
- b) $y(t) = 25 \cos 6\pi t$
- c) $y(t) = 0.1 \sin 50t$
- d) $y(t) = 3 \sin 2t + 8 \cos 2t$

In class, the results were a) 5%, b) 29%, c) 37%, and d) 29%, and this question provoked an excellent discussion because three of the answers appeared reasonable to different groups of students. In the post-vote discussion the students were able to sort out the relationship between the differential equation and the different numbers in these functions, with very little instructor guidance, ultimately concluding that the coefficient of t was the square root of a , and thus that the largest coefficient corresponded to the largest value of a .

Another question that provoked this type of diverse voting pattern involved using Euler's method to numerically solve a differential equation:

We know that $f(2) = -3$, and we use Euler's method to estimate that $f(2.5) \approx -3.6$, when in reality $f(2.5) = -3.3$. This means that

- a) $f(x) > 0$
- b) $f'(x) > 0$
- c) $f''(x) > 0$
- d) $f'''(x) > 0$

The results for this question were a) 0%, b) 41%, c) 24%, d) 6%, and e) 29%, indicating substantial confusion over the issue. However, in the post-vote discussion, one student had some particularly good insights, explaining that Euler's method assumes that the function is linear, and in this case it underestimates the true value, so this means that the real function must be concave up, and thus c) is the correct answer. At this point, you could hear students saying "Ah!" around the room as they recognized his logic.

Project MathQUEST: Math Questions to Engage Students

The National Science Foundation has recently awarded us a grant to develop and test a library of classroom voting question for use in linear algebra and differential equations courses (NSF DUE-0536077). Our web site <http://mathquest.carroll.edu> has student editions of these libraries of questions freely available, as well as the libraries that we regularly use in differential, integral, and multivariable calculus. Teacher's editions of these libraries, including past voting statistics and other comments, are available upon e-mail request.

Classroom voting takes time, and some organization, but this teaching technique is well worth the extra effort. It gives the instructor immediate feedback as to the students' level of understanding and it gets the students engaged in the material, by involving them in discussions about important mathematical concepts, which makes classroom voting a powerful new tool for teaching.

References

Mazur, Eric. "Peer Instruction – A User's Manual." Prentice Hall, Englewood Cliffs NJ. 1997.

Miller, Robyn, E. Santana-Vega, and M. Terrell. "Can Good Questions and Peer Discussion Improve Calculus Instruction?" PRIMUS Volume XVI #3, September 2006.