EMPLOYING THE VOYAGE 200 TO EXPLORE TWO OPEN PROBLEMS IN ELEMENTARY NUMBER THEORY

JAY L. SCHIFFMAN
ROWAN UNIVERSITY
201 MULLICA HILL ROAD
GLASSBORO, NJ 08028-1701
e-mail: schiffman@rowan.edu

ABSTRACT: Open problems in elementary number theory have fascinated both professional and amateur mathematicians throughout the rich history of mathematics. In this paper, we will demonstrate how the VOYAGE 200 enables students to better understand the intricacies of primes in general, and in particular, the two open problems known as *Goldbach's Conjecture* and *The Twin Prime Conjecture*. In a letter to Leonhard Euler in 1742, Christian Goldbach asserted that every even integer > 2 seemingly was expressible as the sum of two primes. Euler was unable to prove or disprove Goldbach's Conjecture and the problem remains open to this day. A related problem involves the notion of twin primes. A twin prime pair consists of a pair of odd primes that differ by two. The first such pair is $\{3,5\}$. It likewise remains open as to the infinitude of the number of such twin prime pairs. While these problems are easy to state and many mathematicians believe they will eventually be proven in the affirmative, the problems are rather challenging. The great mathematical minds throughout the ages have failed to conclusively resolve them in either the affirmative or the negative. Our goal is to employ our technological marvel to demonstrate the conjectures for a small range and shed new light on these problems while simultaneously discovering some neat mathematics.

We will assume some familiarity with the Voyage 200 learning tool in the sense that the reader can navigate with the basic functions and keystrokes on the calculator. Our initial goal is to demonstrate Goldbach's Conjecture for the even integer 50 in the sense that we seek all the ways one can express 50 as the sum of two primes. Utilizing the VOYAGE 200, define three functions in the Y= EDITOR. Here the is Prime command is illustrated in **FIGURE 1** with our function inputs in **FIGURE 2** below:

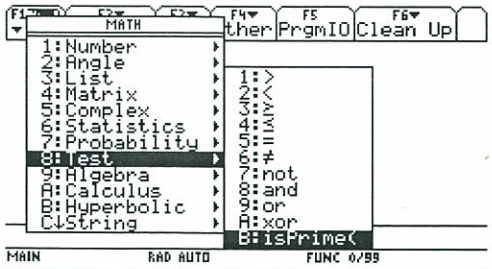

FIGURE 1: The Is Prime Command.

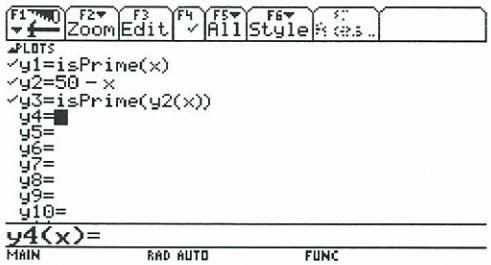

FIGURE 2: The Function Inputs.

In **FIGURE 3,** create a table. Begin the table with the initial odd prime 3 and proceed in increments of 2. The table is produced in **FIGURES 4-5** below:

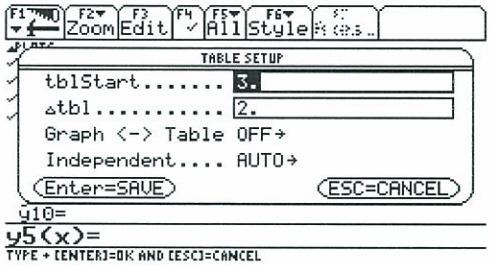

FIGURE 3: The Table Setup.

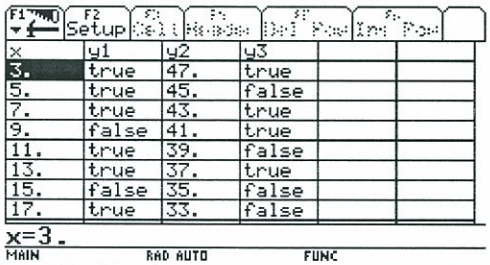

FIGURE 4: Goldbach's Conjecture Revealed For The Even Integer 50.

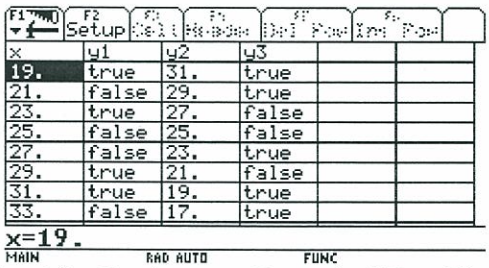

FIGURE 5: Goldbach's Conjecture Revealed For The Even Integer 50.

We seek the outputs that are true in the columns headed by $y1$ *and* $y3$ in a given row. These will generate the proper representation of the even integer as the sum of two odd primes. Also since $\frac{50}{2} = 25,$ we do not need to consider any $x > 25.$ Otherwise one will obtain duplicates of previous outcomes only in the reverse order. The four representations

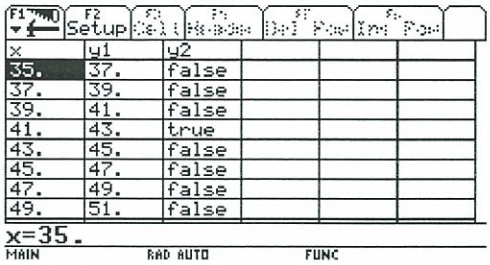

FIGURE 11: Table Revealed.

Here are the twin prime pairs $\leq 50: (3,5), (5,7), (11,13), (17,19), (29,31), (41,43)$. We have a total of six twin prime pairs. View the true outputs in the column headed by y_2. For example, in **FIGURE 9,** when $x = 17$ *and* $y_1 = 19$, we see that y_2 is ***true*** asserting that 17 and 19 indeed constitutes a twin prime pair.

On Page 435 of the TI-89 manual, a program for the Next Prime is given. We reproduce this in **FIGURE 12:**

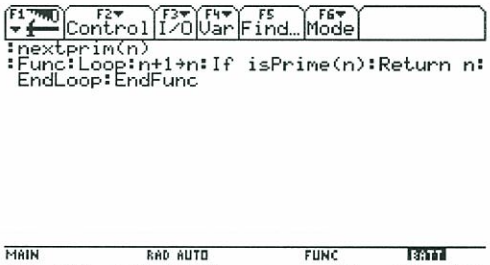

FIGURE 12: The TI-89 Program For The Next Prime.

To cite a simple example, 11 is the prime successor to the prime 7 as we view in **FIGURE 13:**

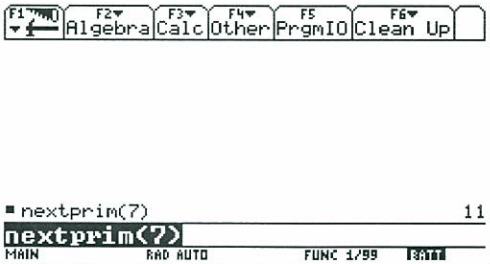

FIGURE 13: 11 Is The Prime Successor To 7.

(i). We secure the distance between 23 and the next prime. In order to achieve our goal, we need to input the following functions as displayed in **FIGURE 14:**

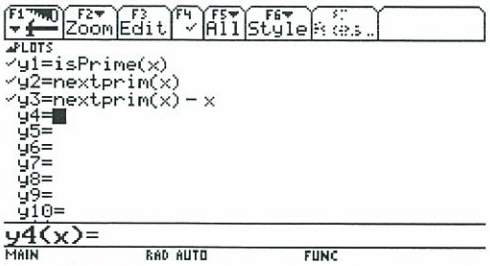

FIGURE 14: Functions Needed To Obtain The Prime Successor.

Our TABLE SETUP is shown in **FIGURE 15** with the start being the prime 23 and the incremental value 1.

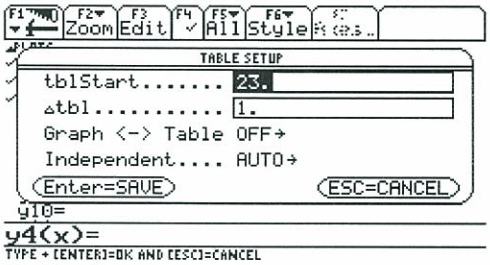

FIGURE 15: The Table Setup To Illustrate The Prime Successor Of 23.

The table is generated in **FIGURE 16** below with the column headed by $y1$ verifying that 23 is indeed prime with a distance of 6 from the next prime 29.

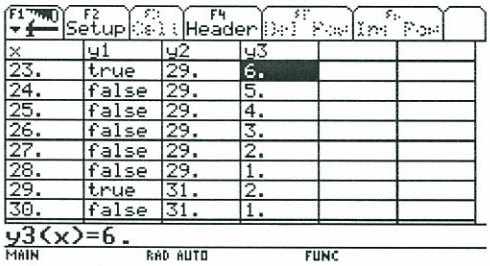

FIGURE 16: 29 Is The Prime Successor To 23, The Initial Time Two Primes Are At Distance 6.

In the identical fashion, one can demonstrate that the greatest distance between two primes in the range of positive integers from one to one thousand is twenty (between the primes 887 and 907) while the greatest distance between primes in the range of positive integers from one to ten thousand is thirty-six (between the primes 9551 and 9587) while between one and one hundred thousand it is 72 (between the primes 31397 and 31469). A concluding activity to challenge the reader might be to find the next larger pair of twin primes after 140737488353699 and 140737488353701, the largest known twin prime pair in 1975. The Next Prime Program on The Voyage 200 enables us to achieve our goal. After some work, the next twin prime pair is revealed: 140737488356207 and 140737488356209. One can discover an exciting world of prime number excursions with judicious use of the Voyage 200. Happy trails!