

INTEGRATING APPLICATIONS, MODELING, AND TECHNOLOGY INTO A
“COLLEGE ALGEBRA IN CONTEXT” COURSE

Ronald J. Harshbarger
1 University Boulevard
University of South Carolina Beaufort
Bluffton, SC 29909
ronharsh@hargray.com

Lisa S. Yocco
P.O. Box 8093
Georgia Southern University
Statesboro, GA 30460
lisay@georgiasouthern.edu

Most of the students who are non-science majors are intelligent, but are just not interested in mathematics. These people are as likely to be leaders in our communities and country as are people with science and engineering degrees, and so they need the reasoning skills and problem solving skills just as much as science majors do. Non-science majors will likely have careers that require reading for comprehension, problem solving skills, the ability to analyze and interpret, and the ability to work with colleagues. If we give them compelling applications, they will see some reason for mathematics, will not be so bored, and may even come to like mathematics. When solving problems based on real data, giving the source and presenting a table of real data, a scatter plot of the data, and/or the function that models the data is important because it shows that the model being used in an application does not appear as if by magic. It is also useful to have the students analyze the data and develop the function that best models the data. Models are created from real data and problems are then solved using the models. A large number of the problems ask the student to interpret, analyze, and make predictions from the real data models.

Combining Functions

The following table gives the weekly revenue and cost, respectively, for a selected number of units of production and sale of a product by the Quest Manufacturing Company.

Number of Units	Revenue (in dollars)		Number of Units	Cost (in dollars)
100	6800		100	32,900
300	20,400		300	39,300
500	34,000		500	46,500
900	61,200		900	63,300
1400	95,200		1400	88,800
1800	122,400		1800	112,800
2500	170,000		2500	162,500

1. Use technology to determine the equations that model revenue and cost functions for this product, using x as the number of units produced and sold.
2. a. Combine the revenue and cost functions with the correct operation to create the profit function for this product.
b. Use the profit function to complete the following table:

x (number of units)	Profit, $P(x)$ (dollars)
0	
100	
600	
1600	
2000	
2500	

3. Find the number of units of this product that must be produced and sold to break even.
4. Use graphical methods to find the values of x that give positive profit for the product. Producing how many units will give a profit for the product?
5. Graph the revenue and cost functions on the same axes and shade the region where profit occurs. Do the x -values for this region agree with the values of x that gave positive profit using the profit function?
6. Find the maximum possible profit and the number of units that gives the maximum profit.
7. a. Use operations with functions to create the average cost function for the product.
b. Complete the following table.

x (number of units)	Average Cost, $\bar{C}(x)$ (dollars per unit)
1	
100	
300	
1400	
2000	
2500	

- c. Graph this function using the viewing window $[0, 2500]$ by $[0, 400]$.
8. Graph the average cost function using the viewing window $[0, 4000]$ by $[0, 100]$. Determine the number of units that should be produced to minimize the average cost and the minimum average cost.
9. Compare the number of units that produced the minimum average cost with the number of units that produced the maximum profit. Are they the same number of units? Discuss which of these values is more important to the manufacturer and why.

Drug Use

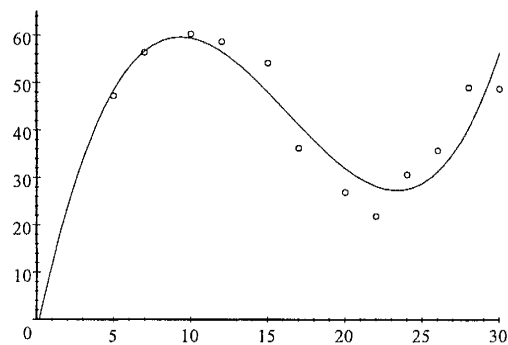
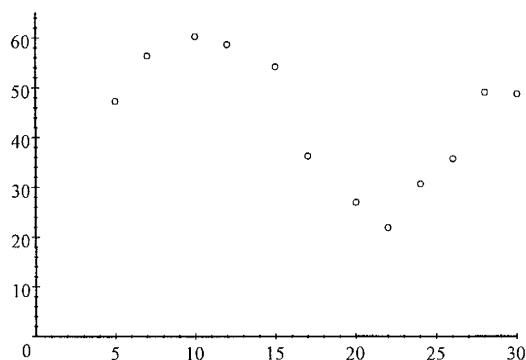
The percentages of high school seniors who have used marijuana are shown in the table for years from 1975 to 1996.

<i>Year of Graduation</i>	1975	1977	1980	1982	1985	1987
<i>Percent</i>	47.3	56.4	60.3	58.7	54.2	36.3

<i>Year of Graduation</i>	1990	1992	1994	1996	1998	2000
<i>Percent</i>	27.0	21.9	30.7	35.8	49.1	48.8

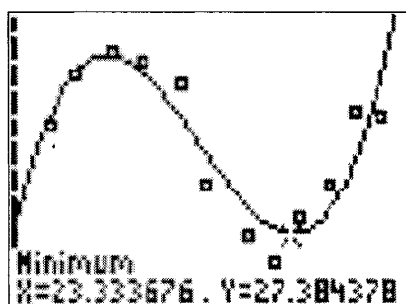
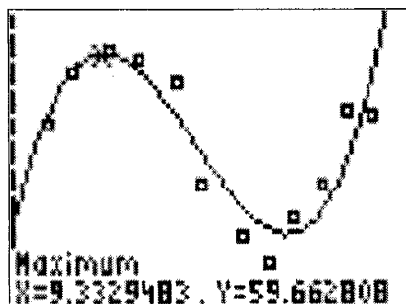
Source: National Institute on Drug Abuse

The scatterplot of the data, with x representing the year (since 1970) is below. The scatterplot was created using Scientific Notebook.



The cubic model is $y = 0.0235x^3 - 1.1526x^2 + 15.3679x - 2.4897$. The graphs of the scatterplot and the model, created with Scientific Notebook, are shown below.

Use the TI-84 Plus emulator (Smart View) to find the maximum and the minimum



The maximum occurred at $x = 9.333$ during 1980 and the minimum occurred at $x = 23.334$, during 1994. We can find the same information using the derivative of the model.

Market Equilibrium

Suppose the demand for a product is given by

$$p = 36 - 3q \text{ dollars}$$

and the supply by

$$p = 4q + 1 \text{ dollars,}$$

where q is the number of hundreds of units demanded and supplied.

Excel can be used to find market equilibrium numerically, graphically, or with matrices.

Market Equilibrium

$$p = 36 - 3q \quad p = 4q + 1$$

Units (hundreds)	demand	supply	demand-supply
q	$\$p$	$\$p$	$\$p$
0	36	1	35
0.5	34.5	3	31.5
1	33	5	28
1.5	31.5	7	24.5
2	30	9	21
2.5	28.5	11	17.5
3	27	13	14
3.5	25.5	15	10.5
4	24	17	7
4.5	22.5	19	3.5
5	21	21	0

$$3q + p = 36$$

$$4q - p = -1$$

Matrix A	q	p	c
	3	1	36
	4	-1	-1

Inverse of A	0.142857	0.0142857
	0.571429	-0.428571

Solution	q (hundred)	5
	p (dollars)	21

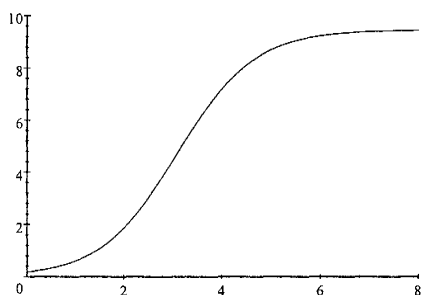
DVD Players

The sales revenues from DVD players from 1999 to 2004 can be modeled by the logistic function

$$y = \frac{9.46}{1 + 53.08e^{-1.28x}}$$

where x is the number of years after 1998 and y is in billions of dollars.

This function can be graphed using Scientific Notebook, and the sales revenue in 2004 can be estimated.



$$f(6) = \frac{9.46}{1 + 53.08e^{-1.28(6)}} = 9.2336$$

Federal Support for Education

Federal on-budget funds for all educational programs (in millions of constant 2000 dollars) between 1965 and 2000 are given by

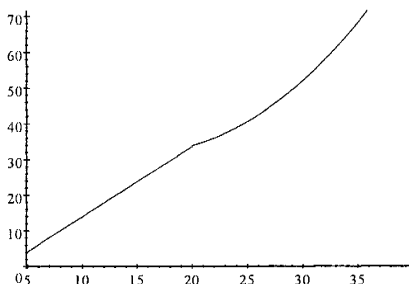
$$P(t) = \begin{cases} 1.965t - 5.65 & \text{when } 5 \leq t \leq 20 \\ 0.095t^2 - 2.925t + 54.429 & \text{when } 20 < t \leq 40 \end{cases}$$

where t is the number of years after 1960.

- Graph $P(t)$ for $5 \leq t \leq 40$ and describe how the funding varies during this period.
- What was the amount of funding in 1980?
- What was allotted in 1998?

To graph this function using Scientific Notebook, it must be entered as a 3 x 2 matrix:

$$f(x) = \begin{cases} 1.965x - 5.65 & \text{if } 5 \leq x \leq 20 \\ 0.095x^2 - 2.925x + 54.429 & \text{if } 20 < x \leq 40 \end{cases}$$



If the function is defined as " $f(x)$ " then it can be graphed, evaluated, differentiated, and integrated.