GEOMETRY OF THE EARTH AND UNIVERSE COMPUTER LABS

Sarah J. Greenwald
Appalachian State University
Department of Mathematics
Boone, NC 28608
greenwaldsj@appstate.edu

Introduction

While geometry means measuring the earth, too often it is presented in an axiomatic way, divorced from reality and experiences. A segment on the geometry of the earth and universe stretches the imagination, connects students to current research, and highlights real-world applications of geometry and its connections to art, philosophy, physics, astronomy and geography. This segment can be aimed at students with no math background, at graduate students in geometry and topology, or at any level student in between.

We will explore a series of labs on the geometry of the earth and universe that have been used in a liberal arts mathematics class and in courses for teachers. The labs are available on-line [3]. Each one develops visualization skills while taking advantage of various technologies and manipulatives. Explorations of web-based movies, interactive web-based games, *The Geometer's Sketchpad*, *Microsoft Excel*, and manipulatives such as slinkies, globes, and *Zome* educational construction toys are used to engage the students with the material. Each lab includes such explorations as well as reflections by the students on the various activities. These reflections include answering a series of questions, preparing a presentation, or writing a report.

Lab 1: Representing Spaces

In the first lab, students explore the theme of representing spaces by examining perspective drawing in *Microsoft Excel*, whether *The Simpsons* are 2-D or 3-D, and how Escher represents spaces in his art [3]. This lab challenges their notion of what mathematics is and it enables the students to explore paintings and cartoons in a new mathematical way that is fun and creative.

Lab 2: Research Problems

Next, each group of students selects a different problem about the geometry of the earth or universe [3]. The problems connect with Book 1 of Euclid's *Elements* to prepare for related proofs but a less technical version of this lab is aimed at students in a course for non-majors. Students turn in a report and present their web and book research to the rest of the class and then we go over the answers, many of which only require a globe and string or other manipulatives but are also reinforced with dynamic geometry software.

For example, students are asked whether whether the Pythagorean Theorem holds on the earth between approximately Umanak, Greenland; Goiania, Brazil; and Harare, Zimbabwe. After their report to the class, we use a long piece of string to create and mark the lengths of the bases, *a* and *b*, of this spherical right triangle on a globe. Next, we move the marked string from the globe to the table in order to create a flat right triangle with the same bases. We complete the Euclidean triangle to create a Euclidean hypotenuse. Finally we place the Euclidean hypotenuse back onto the sphere, between Zimbabwe and Greenland, to see that the Euclidean hypotenuse is longer than the spherical hypotenuse, *c*. Since the length of the Euclidean hypotenuse is the square root of $a^2 + b^2$ then we have demonstrated that of $a^2 + b^2 > c^2$ on the sphere. We also use dynamic geometry software [6] in order to create a spherical right triangle and measure $a^2 + b^2 - c^2$.

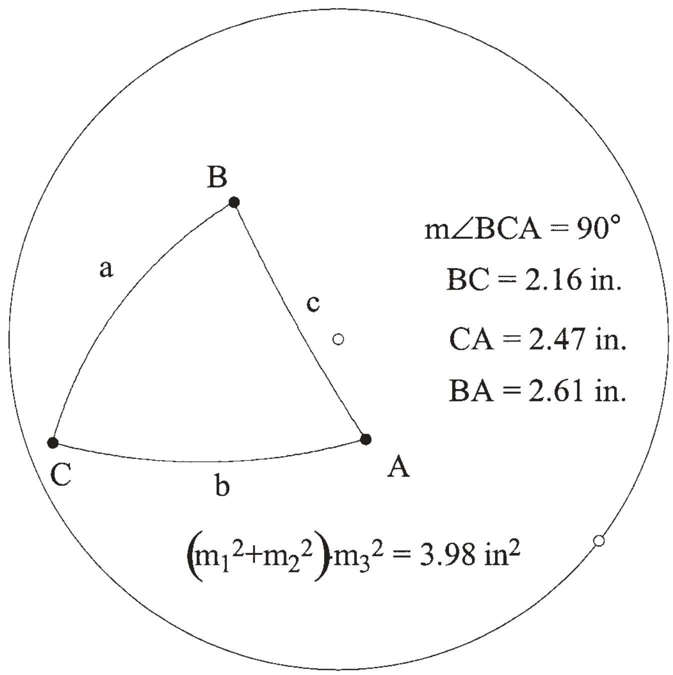

Figure 1. Pythagorean Theorem on a Sphere in Dynamic Geometry Software

One can then drag the vertices of the triangle to explore the Pythagorean Theorem for different right spherical triangles.

During this lab, students experience a process that is similar to mathematical research. As a result, they develop an ownership of the material as they are exposed to a variety of viewpoints and definitions.

Lab 3: Hyperbolic Geometry

Students are then introduced to hyperbolic geometry through a Java Sketchpad worksheet of the Poincaré disk model that connects with Escher's work from the first lab and the research problems from the second lab [3].

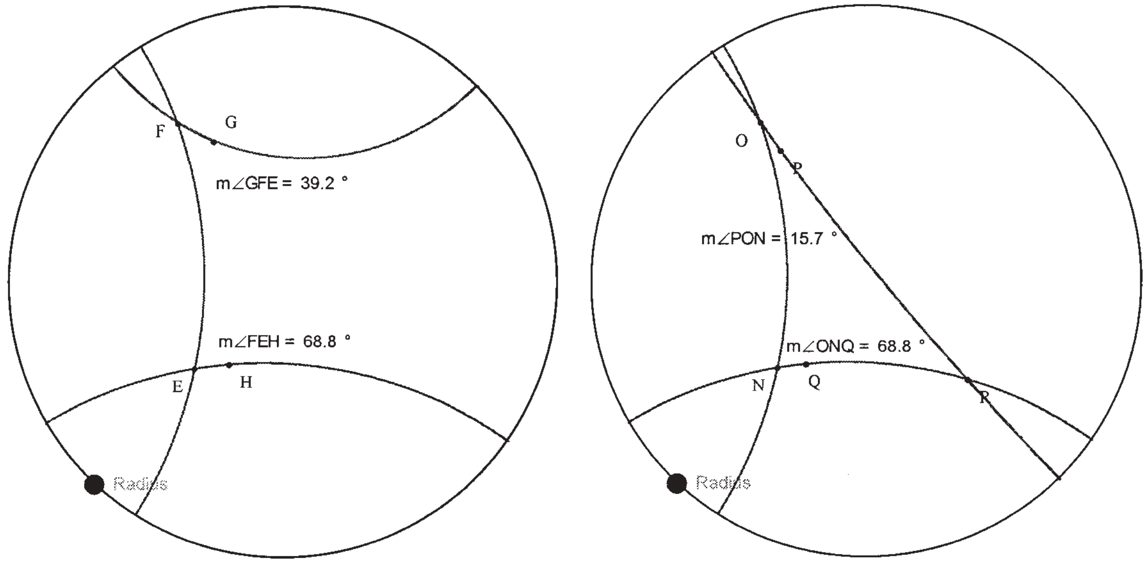

Figure 2. Euclid's 5th Postulate Does Not Always Hold in Hyperbolic Geometry

The Sketchpad explorations are reinforced with manipulatives from Henderson and Taimina's hyperbolic models [5].

Lab 4: 2-D Universes

The themes from the first lab are revisited with web readings about 2-D creatures, Davide Cervone's web movies, and questions about the life of 2-D Marge Simpson [3]. After using slinkies to form 2-D torus and Klein bottle universes,

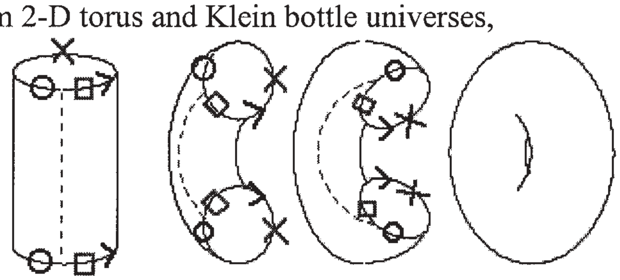

Figure 3. 2-D Torus Universe

students experience what it is like to live on these spaces by playing torus and Klein bottle tic-tac-toe [7]. In the torus game, the top left square is really next to the top right square. They can "scroll" the board in order to help develop their intuition (once a square has been labeled X or O, one can click on it, hold it down, and move the board around to see the identifications). Two-dimensional spherical and hyperbolic universes are also discussed.

Lab 5: Shape of the Universe

In the final lab, students examine possible shapes for our universe, real-life attempts to discover the geometry, such as NASA's WMAP (Wilkinson Microwave Anisotropy Probe), the idea of a fourth physical dimension, and the management of data using higher dimensions [3]. To help students visualize a finite universe in three dimensions, they can watch the *Futurama* episode *I, Roommate* (Season 1 DVD) [2]:

> *Fry and Bender are looking for housing. Leela, Fry, Bender and the manager enter an apartment that resembles Dutch graphic artist M.C. Escher's Relativity print* [1].
> ***Fry***: *I'm not sure we wanna pay for a dimension we're not gonna use.*
> *Bender, the robot, falls down the staircase and continues to fall "down" the other staircases in many different directions.*

Students can use Bender's position in each of the frames to give gluing instructions and explain which openings are identified.

Conclusion

The quest to understand the precise geometry and shape of our universe began thousands of years ago, when mathematicians and astronomers used mathematical models to try and explain their observations. While there seem to be some irregularities in the WMAP data that throw the conflicting analyses and conclusions from recent news articles into doubt, there is hope that the data from the proposed 2007 Planck satellite will ultimately lead us to the answer. In the meantime, students can be exposed to this exciting topic in a classroom module [3]. Many students state that the geometry segment is their favorite module. The theme of Mathematics Awareness Month in 2005 is the *Mathematics of the Cosmos* [4] and so these labs are especially timely.

References

1. Escher, M.C., *Relativity*, 1953.

2. *Futurama* TM and copyright Twentieth Century Fox and its related companies, http://www2.foxstore.com/detail.html?item=606.

3. Greenwald, Sarah J., *Classroom Activities on the Geometry of the Earth and Universe*, http://www.mathsci.appstate.edu/~sjg/talks/earthanduniverse.html.

4. Greenwald, Sarah J., *Shape of the Universe Theme Essay,* 2005 Mathematics Awareness Month on the Mathematics of the Cosmos, http://www.mathaware.org/essays/shape.of.universe.html

5. Henderson, David and Daina Taimina, *Experiencing Geometry: Euclidean and Non-Euclidean with Strands of History*, Prentice Hall, 2004.

6. Key Curriculum Press, *The Geometer's Sketchpad*, http://www.keypress.com/sketchpad/index.php.

7. Weeks, Jeffrey R., *Exploring the Shape of Space*, Key Curriculum Press, 2001, http://www.keypress.com/catalog/products/supplementals/Prod_ShapeOfSpace.html.