TEACHING MATHEMATICS WITH LIVEMATH

Morteza Seddighin
Associate Professor of Mathematics
Indiana University East
Richmond, IN 47374
mseddigh@indiana.edu

LiveMath in Support of a Mathematical Pedagogy:

Cognitive oriented constructive approach in mathematics instruction has been found to be one the most effective ways of teaching mathematics. In this instructional model teachers emphasize exploration and discovery on the part of each learner. Learning is described as occurring in the following four phases:

Phase 1 The instructor provides the learner with the complete solution for some initial problems. At this stage not only the steps within each problem are identified for the student but also the detailed explanation of each step in the solution is carried out by the instructor.

Phase 2. The instructor provides the learner with a set of problems similar to the initial problems. For each problem, an outline of the steps within the problem is provided to the student, but it is the responsibility of the student to carry out each step.

Phase 3. The instructor provides the student with drill problems similar to those in phases 1 and 2, but does not provide him or her with an outline of steps necessary to solve the problem. Instead, the expectation is that the student him or herself first identifies the steps necessary in order to solve the problem and then carries out those steps without any intervention by the instructor.

Phase 4. At this phase students are expected to look back over their solutions to develop an abstract mental model for the type of problems they have solved.

Many students have difficulty solving the follow up drill problems in Phase 3 because they are unable to identify and/or solve the intermediate steps within a problem successfully. It is therefore helpful to develop interactive computer based tutorials with recalculation capability that not only show students how to divide a problem into intermediate steps and solve each step within a problem but also allow students to modify problems as many times as they wish and observe the effect of a change in the solution. Reviewing the intermediate steps and written instructions for multiple versions of each type of problem lead students through the thought process required as well as the mathematical steps. This type of interaction between students and the computer is very beneficial to students in reducing their anxiety towards mathematics and thus affecting the retention in mathematics courses.

With the help of a powerful software program called LiveMath Maker we can develop interactive web-based mathematical notebooks that not only support the implementation of various phases of cognitive oriented constructive instructional design but also foster important learning factors such as: modeling, coaching, articulation, reflection, exploration, communication, and collaboration. These interactive and dynamic mathematical notebooks allow students to perform mathematics in a manner similar to doing math on paper. Unlike a calculator, LiveMath Maker allows the students to record intermediate steps in a calculation sequence so that the spirit of mathematical thinking be coupled with the speed of computer. The dynamic recalculation features of LiveMath Maker allow students to propose hypotheses, perform mathematical explorations, draw conclusions, and learn by discovery. The recalculation features of these notebooks resemble those of a spreadsheet. With each change in an equation, the mathematics and all of the symbolic notations in each step of the problem are updated instantly. However, unlike a spreadsheet, the steps involved in problems discussed in these notebooks are not necessarily simple arithmetic computations. A step might include factoring, expanding, graphing, differentiation, integration, etc. Upon the execution of each step within a problem, the name of the step taken is displayed in a colored font to the right of the symbolic solution of the step. As an example consider the general problem of adding two fractional expressions such as $\frac{x+2}{x^2-5x+6}$ and $\frac{x-1}{x^2-6x+9}$. The steps involved in adding such fractions are: factoring the denominator of the first fraction, factoring the denominator of the second fraction, finding the least common denominator, finding the numerator of the sum, finding the denominator of the sum, and simplifying the numerator of the sum. The final result is $\frac{2(x^2-2x-2)}{(x-3)^2(x-2)}$. Once the solution and the explanation to each of the above steps are displayed, a student can modify each of the fractions above and the solution to each of the steps above will be updated instantly. The final answer will also be updated. For example, if the first fraction is changed to $\frac{x-1}{x^2-3x+2}$ and the second fraction is changed to $\frac{x+6}{x^2-5x+4}$, the final answer as well as each of the intermediate steps will be updated instantly.

LiveMath Maker is equipped with a complete palette containing all mathematical symbols and operations. This allows instructors and students to write mathematical texts easily and perform computations on them by the click of a button. For using other computer algebra programs one needs to know the syntax of the program but LiveMath is a WSWG (what you see is what you get) program. The operations on the palette include "Simplify", "Expand", "Collect", "Factor", and so on. By clicking on various symbolic and operational buttons on the palette one can create and execute mathematics. The files created can be saved as notebooks. Mathematics instructors are able to modify these notebooks on a regular basis to fit the mathematical ability of the learner. The notebooks can easily be modified so that all, some or none of the intermediary steps are shown. Once a student is advanced enough to be able to identify the intermediate steps him or herself, the steps can be eliminated (a practice called fading). On the other hand, for weaker learners we have the option of providing the steps within a step. In all cases,

however, the best approach is instructing the learner to carry out each step with paper and pencil himself and compare his results with those provided by the computer. Therefore, the tasks required by these notebooks become very similar to those used by a human mathematics tutor who implements a cognitive oriented constructive instructional design in training learners.

In addition to the symbolic features discussed above, the dynamic graphing and animation capability of these notebooks allows students to study two dimensional and three dimensional graphs of functions experimentally. The student can change a coefficient in a function and see the effect on the graph. Once a function is altered, its graph will be altered instantly to reflect the effect of a change in the function. This is a very effective way of teaching properties of functions visually. The author has used the graphing capability of LiveMath in his multivariate calculus course to help students understand the concepts of level curves and contour diagrams. These concepts can not be taught effectively without creating explicit three and two dimensional graphs. By studying the contour diagrams generated by LiveMath students were able to solve many problems related to differentiation, integration, and optimization numerically.

It should be mentioned that the program comes with an extensive library of prewritten notebooks called the "Starter Library" which includes notebooks on pre-calculus, college algebra, trigonometry, single variable calculus, multivariable calculus, geometry, differential equations, linear algebra, and statistics. Instructors can tailor and modify these notebooks to fit their needs. For example, the author has used a statistical package from the Starter Library to create a notebook which enables students to build confidence intervals and conduct tests of significance. Among other functions, this package contains the following:

AverageOf() function, which computes the mean of a data set X.

StdDevOf() function, which computes the standard deviation of a data set X.

VarianceOf() function, which computes the variance of a data set X.

To create a notebook capable of performing inferential statistics, one has to copy and paste the above statistical package into a blank notebook. This empowers the notebook with these features and functionality. After empowering the notebook with these functions, one can for example compute a confidence interval. Recall that a confidence interval can be expressed merely based on the mean, standard deviation, and the size of a sample (for a given level of confidence). The author has taught elementary statistics for more than ten years and has used famous data analysis programs such as SPSS in his statistics courses. When using these programs students enter the data and get an out put without understanding the actual process. However, by using LiveMath, the manner described above, students actually write the formulas and then execute them. This allows students to understand the process. Furthermore, since expressions in LiveMath are

"live", a student can learn about the effects of various variables on a statistical analysis by experimentation.

How to Implement LiveMath in Mathematics Courses:

There are two interfacing programs involved in writing and using LiveMath notebooks, LiveMath Maker and the LiveMath Plug-in. The first, LiveMath Maker, allows an instructor to develop mathematical workspaces, or "notebooks." These notebooks contain symbolically correct, mathematical canvases that students can interact with as they learn problem-solving step by step.

The notebooks are converted to web pages using some of the built-in features of LiveMath Maker. The second program is the LiveMath Plug-in, which allows instructors and students to view and interact with the notebooks on the internet as web pages. Students are able to add text and graphics within a notebook. Also the fonts can be easily sized and colored.

These notebooks can be customized to meet varied teaching and learning styles. The author implemented the notebooks into his courses in the following four modes:

A. He integrated the web enhanced notebooks into IU's online course management system Oncourse. (It is also possible to integrate these web pages with other course management systems such as WebCT, BlackBoard, and IntraLearn. They can also be integrated in streaming applications such as the Tegrity WebLearner system.)

B. He embedded the notebooks into PowerPoint presentations for use in the classroom. This increased students' familiarity with the process of using the notebooks and provided an interactive and dynamic environment for demonstrating problem solving.

C. He installed the notebooks on computers located in the campus's computer labs.

D. In some courses he put the notebooks on CD-ROMs and made them available to all students.

These notebooks fostered collaboration between students. Usually students solved various steps of the problems collaboratively and posted their completed notebooks on the web for classmates to examine and interact with. Students could submit their assignments to me with a click of a button and receive feedback.

The problems included in these notebooks were selected from the problem sets in the adapted mathematics textbooks used at IU East. Knowing that working the problems on the computer will assist them with specific class assignments, further motivated students

to make use of these notebooks and proved to be another advantage to these notebooks rather than using existing commercial computer tutorial aids.

Outcome:

To measure the effectiveness of LiveMath notebooks we utilized IU's course management system Oncourse in providing students with anonymous surveys and self-assessment instruments for assessing the usefulness of the notebooks. We also gave students similar anonymous instruments for classroom and lab implementation of the notebooks. In some courses such as pre-calculus, we gave students pre-tests and post-tests on the course content before and after the use of the specific notebooks to measure their effectiveness. The data clearly indicated that students' scores were improved after the use of notebooks. In general, students felt that LiveMath had helped them to understand the concepts better. I have also noticed that the scores of students in courses were I use LiveMath have improved significantly since I have implemented this program. Let me mention that student evaluations of my courses have also been considerably improved since I have used LiveMath in my courses. For example, in the fall of 2004 in my multivariate calculus course students rated me "excellent" on every question included in the student evaluation form. These are the highest ratings I have ever received in my multivariate calculus courses.

The Theorist, the manufacturer of LiveMath Maker has created a site named LiveMath Board, a web-board that allows the developer of LiveMath notebooks post their products. This allows instructors post the LiveMath notebooks on the web so that students and educators can comment on them.