MATHEMATICAL COMMUNICATION IN A WEB-SUPPORTED UNDERGRADUATE GEOMETRY CLASS

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Computing and modeling technologies are enriching classroom mathematical presentations, discussions, and inquiry in many ways. For instance, powerful modeling technologies now make it possible for teachers to introduce mathematical concepts using graphical representations that appeal to and make sense to all students. And, to the extent that they motivate and facilitate exploration, these same technologies empower students to formulate, test, and defend conjectures. It seems likely that advances in computer hardware and software will continue to facilitate mathematical inquiry in new and powerful ways. But what else should we expect from this revolution? For me, the answer is, "More and better mathematical dialogue": Dialogue that extends discussions begun but not finished in class; patient dialogue informed by careful reflection; dialogue that clarifies complex issues and converges on powerful truths. Unfortunately, out-of-class dialogue of this sort is difficult, if not impossible, for today's busy undergraduates to schedule. On the other hand, students are ready and eager to participate in discussions conducted in on-line forums. This paper discusses lessons learned concerning the use of WWW-based communication technologies in an undergraduate geometry course.

Geometry for Teachers

Geometry for Teachers is a technology-rich survey of geometries, including Euclidean geometry, transformation geometry, hyperbolic geometry, fractal geometry, and projective geometry. The overarching goal of the course is to provide future mathematics teachers a technology-rich overview of both traditional and modern geometry. As in traditional instruction, the instructor and students meet three times per week in a traditional face-to-face setting. For homework, students are assigned readings in and written assignments taken from a course textbook (Thomas, 2002). Unlike traditional instruction, each assignment spans about six class meetings and culminates in the writing of a comprehensive report by a collaborative team of four students. In these reports (typically 20+ pages of typed, carefully illustrated mathematics) students present and discuss the solutions to assigned exercises and investigative findings. In this approach, students probably do fewer problems or exercises than in a traditional course, but they are held to a higher standard in the formulation and communication of their findings.

Written communication for *Geometry for Teachers* is conducted via the *Idaho Virtual Campus*, a course management tool similar to *BlackBoard* and *WebCT*. Tools of this sort are particularly helpful with routine course administration: Managing student access to course materials; presenting course information; facilitating communication; and tracking student participation. Unfortunately, these tools offer little or no support for the specialized symbols and graphics essential to mathematical communication. For that, *Geometry for Teachers* relies on *MS Word* with *Equation Editor* and *NetTutor,* a licensed whiteboard and messaging system. *NetTutor* provides extensive support for textual, symbolic, and graphical communication, as well as real time application sharing and audio. And because chats are automatically archived by *NetTutor*, they may be reviewed after the fact by students and faculty as needed. Figure 1 shows a portion of a chat involving the author and four students. To protect the privacy of the participants, the actual names of the students, which are normally displayed in different colors, are replaced with anonymous designations.

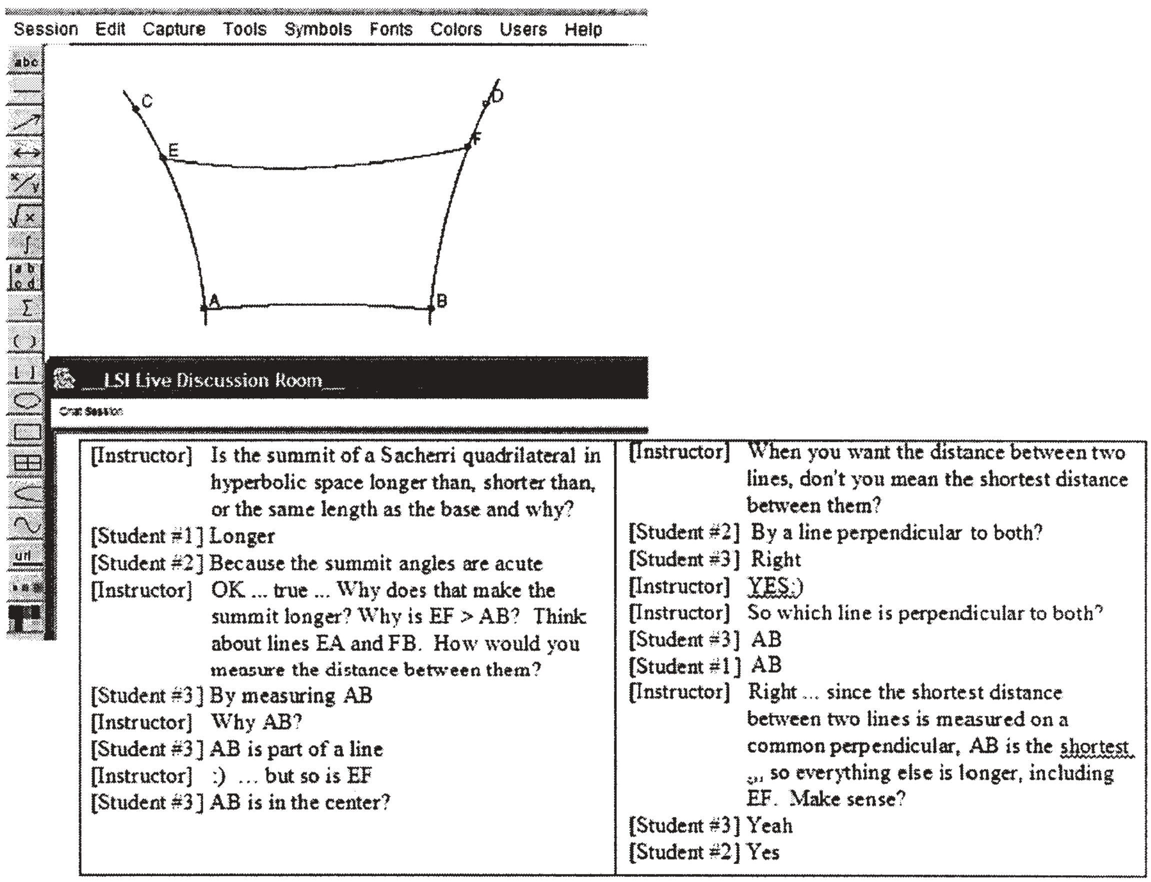

Figure 1: *NetTutor* Chat

Between chat sessions (and outside of class), students communicate with one another and the instructor using the *Idaho Virtual Campus* bulletin board system. Students use *MS Word*, *Equation Editor*, and a variety of geometric modeling tools to compose their thoughts. The resulting documents and models are then attached to messages posted in the *Idaho Virtual Campus* discussion area. Unlike the synchronous discussions, which tend to focus on main ideas, asynchronous discussions are cyclical in nature, with students contributing, editing, and revising each others' homework solutions. During this process, students do more than check each others' answers. They negotiate meaning, justify their reasoning, and learn to collaborate in high level, on-line, mathematical dialogues. More than any specific mathematical concept or skill, this process is often the most meaningful and rewarding aspect of the course for students. A portion of a typical exchange of messages converging on the development of a team homework report is shown in Figure 2. All messages appear bottom-to-top in the order they were sent. That is, the message listed at the bottom was sent first. The symbol ✉ indicates that the message includes an attachment, files normally created by a word processor or geometric modeling tool.

- Draft of Report #1 (3) by: Student #1 ✉
  - Second Draft (1) by: Student #1 ✉
    - WEB by: Student #2 ✉
  - Third Draft by: Student #1 ✉
  - Final Draft???? by: Student #1 ✉
- Final Draft! by: Student #1 ✉
- trisection by: Student #3 ✉
- A different trisection by: Student #1 ✉
- More of #5 by: Student #2 ✉
- 1.4_6 by: Student #2 ✉
- 1.4 #6 GSP by: Student #2 ✉

Figure 2: Sample Message Log

<u>Lessons Learned</u>

The following lessons learned summarize the author's principal observations and recommendations concerning the use of WWW-based communications.

- *Technology-supported team investigations.* Almost without exception, students are drawn to collaborative, computer-based, interactive, exploratory activities. Investigations should begin with observations that lead to the formulation and testing of conjectures. For most students, this experience, rightly called discovery, is both new and exciting. Sharing the discovery with teammates enriches this experience. And collaborating on the development of thoughtful, thorough reports enriches their mathematical dialogue. Best of all, the conviction that a personal discovery is true provides an authentic, sustaining motive for seeking a proof.
- *Technology learning curve.* Getting comfortable with the communication and modeling technologies takes three to four weeks. During this time, students with limited technical backgrounds and a low tolerance for technical difficulties are apt to experience significant levels of stress. Be patient, purposeful, and flexible.
- *Synchronous chats.* Chats generally focus on expectations, conceptual and technical difficulties, and challenging exercises. Limit participation to four students, allowing the students to set the agenda based on their immediate needs.
- *Asynchronous messages.* This is where the real work of the course is done. In general, continuous group progress should be evident in the exchange of messages. If it isn't, something is wrong with the team dynamics.

- *Reports.* Since one objective of the course is to train the students to edit one another's work, a high standard should be used in evaluating their reports. Most teams respond constructively to this challenge, becoming better and better writers as the term progresses. Assuming each team member does his/her "fair share", all receive the same grade on the report.
- *Term projects.* The objective in assigning a term project is to provide an opportunity for each student to develop and demonstrate specialized knowledge relative to one or more of the geometries and technologies studied. Many students are inclined to attempt too much. Project proposals should be developed with the instructor's advice and assistance. Still, in general, students give far more time and effort these projects than is normally asked. Samples of student work may be seen at the URL http://www.math-ed.com/Resources/mgnew/
- *Examinations.* From the beginning, I inform my students that each student is individually responsible for meeting all course objectives. The collaborative groups exist for one purpose, to facilitate individual learning. Consequently, all quizzes and examinations must be written individually.

Discussion

From the perspective of the student, the principal difference between *Geometry for Teachers* and traditional mathematics courses is the emphasis on collaborative learning and reporting. From the perspective of the instructor, the most interesting aspect of this approach is the process by which a class of individuals evolves over time into cohesive, reflective teams of learners. What are the implications of these changes? First, as students learn to function as members of a team of mathematicians, they engage the course content and one another in a dialogue that is fundamentally different in nature than that observed in traditional courses. Their dialogue is sustained, purposeful, and reflective. And the reports generated by that dialogue are mathematically and technologically mature. This dialogue is made possible by the course communication technologies. It is not difficult to imagine a better set of communication tools. Where *Geometry for Teachers* cobbles together multiple communication technologies, a next-generation suite of math-friendly tools might bundle all of our mathematical communication technologies into a single easy-to-use package. The development of such a tool would undoubtedly reduce the stress on new users.

Second, students learn what mathematical inquiry is and what its rewards are. In this endeavor, the modeling technologies used in *Geometry for Teachers* both motivate and facilitate many investigations that otherwise could never occur. What would make mathematical modeling more appealing to teachers? Currently *Geometry for Teachers* makes use of many modeling technologies in order to address a variety of geometrical topics. Learning each technology takes time. How much better it would be if there was a general purpose, easy-to-use modeling tool that could address meaningful questions in several branches of mathematics. The development of such a tool would simplify instruction and make it possible to engage more students in mathematical inquiry emphasizing observation, discovery, and the formulation of mathematical conjectures.

Seen in a broader frame of reference, the same technologies and procedures used to support an on-campus mathematics course could be used to deliver an on-line course or to foster on-going dialogues among K-12 teachers, university based mathematicians, and other interested parties. The only requirements are enabling technologies (which are becoming more powerful and convenient every year) and people who are prepared to interact in WWW-based learning environments. In my opinion, these are skills that preservice teachers should acquire as a routine part of their undergraduate education. That knowledge would empower them to pursue their life-long professional development agendas with confidence.

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