SHORTCOMINGS AND MISUSES OF TECHNOLOGY (MAPLE,TI CALCULATORS) IN UNDERGRADUATE MATHEMATICS

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Introduction

In the past decade using technology, specifically graphic calculators and Computer Algebra Systems (CASs), has revolutionized the teaching of the mathematics. The symbolic manipulation, computational and graphical power of these tools is intended to help students to explore mathematical ideas through experimentation. Although, many books and manuals are available to familiarize students with these tools, there are few references to the limitations and shortcomings of both graphing calculators and CASs. This paper is a modest attempt to highlight some of the inadequacies as well as misuses of these tools. The first part of this work is an extension of a previous paper [1] by the first author. We will present examples where Maple9 produces incorrect or misleading results. The second part of the paper is devoted to the examples demonstrating the shortcomings of TI-89 and Voyage200 calculators. Of equal interest to the authors are cases where these tools produce correct results that are often misinterpreted by novice users of CASs or calculators. We have included examples to demonstrate some of the common misuses of these tools. The authors have many years of experience in using technology as a teaching tool and wholeheartedly endorse the technology-based mathematics instruction. Some examples presented here are based on our classroom experiences. Other cases have been reported by our students, by our colleagues and in various newsgroups devoted to discussions on CASs and graphing calculators such as *sci.math.symbolic* and *comp.soft-sys.math.maple*. We have only used the most common features of Maple, TI-89 and Voyage200 in our examples. We have also tried to limit our examples to those topics, which are normally covered in an undergraduate mathematics and statistics curriculum.

Computer Algebra Systems- examples using Maple9

The earlier versions of Maple (up to Maple6) contained many software bugs. The reader may want to consult an earlier paper by the first author [1] for some of these examples. For example version V3 and V5 of Maple produced a result of ½ for the sum of infinite terms of the well known and divergent! series $(-1)^{(n+1)}$, n=1 to ∞. However, most of these algorithmic deficiencies have been corrected in the recent versions of Maple. Following are examples of some the persistent software bugs and also examples of problems where the results are misinterpreted by the user.

Example 1: The following example which is reported in [2] demonstrates a software bug that produces the incorrect result of $x=b^2$ as a solution to the simple algebraic equation b+sqrt(x)=0. Clearly for b>0 we have no real solution. However, if we tell

Maple (using the *assuming* command) the sign of parameter b, then Maple produces the correct result.

```
> eqt:=b + sqrt(x)=0;
```

$$eqt := b + \sqrt{x} = 0$$

```
> solution:=x=solve(eqt,x);
```

$$solution := x = b^2$$

```
> # substitute the solution back into the equation
> simplify(subs(solution,eqt));
```

$$b\,(1 + \mathrm{csgn}(b)) = 0$$

```
> # Note that csgn(b) returns 1 if Real(b)>0 and -1 if
Real(b)<0. Try b=5
> simplify(subs(b=+5,%));
```

$$10 = 0$$

```
> # Assume b<0 and then solve the equation
> solve(eqt,x) assuming b<0;
```

$$b^2$$

Example 2: Here is another example [3], which shows an algorithmic flaw in Maple. Maple returns no solution for the seemingly trivial equation: $x^{1/3}+12=0$. However Maple returns the correct real solution $-a^3$ for the general equation: $x^{1/3}+a=0$, then by substituting a=12 we get the correct solution –1728. It appears that Maple solves the initial equation in the complex domain and finds the principal branch of the cube root, namely $12\exp(\pi i/3)$, which is not real and of course not equal to $(-1728)^{1/3}$.

```
> solve(x^(1/3)+12=0,x);
> # Maple returns no result. Consider the general equation
> solve(x^(1/3)+a=0,x);
```

$$-a^3$$

```
> # Specify Real Domain and then solve the equation
> use RealDomain in solve(x^(1/3)+12=0) end;
```

$$-1728$$

Example 3: In the following example [4], first Maple fails to find the antiderivative, and then a simple algebraic change from sqrt(xy) to sqrt(x)sqrt(y) solves the problem. It appears that while Maple's integration algorithm(s) can find anti-derivatives, which are elementary functions, it does not work well when the results are special functions such as error or gamma functions.

```
> int(sqrt(exp(-x)*x),x);
```

$$\int \sqrt{e^{(-x)}\,x}\,dx$$

```
> # Clearly Maple fails to find the anti-derivative. Let's
try a different form:
> int(sqrt(exp(-x))*sqrt(x),x);
```

$$2\sqrt{e^{(-x)}}\,e^{\left(\frac{x}{2}\right)}\sqrt{2}\left(-\frac{1}{2}\sqrt{x}\,\sqrt{2}\,e^{\left(-\frac{x}{2}\right)}+\frac{1}{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)\right)$$

Example 4: Some of the reported "bugs" are not really bugs; they are simply the result of improper assumptions on the part of the user. Consider the following example [5], note that the user (incorrectly) expects that all terms involving exp(-x/t) vanish as x approaches ∞. However, that is only true if we specify t >0.

```
> f:=(x-d)*(1/t)*exp(-1/t*x);
```

$$f := \frac{(x-d)\, e^{\left(-\frac{x}{t}\right)}}{t}$$

```
> int(f,x=d..infinity);
```

$$\lim_{x \to \infty} -e^{\left(-\frac{d}{t}\right)}\left(e^{\left(-\frac{x-d}{t}\right)} x + e^{\left(-\frac{x-d}{t}\right)} t - e^{\left(-\frac{x-d}{t}\right)} d - t\right)$$

```
> expand(%);
```

$$-e^{\left(-\frac{d}{t}\right)}\left(\lim_{x \to \infty} \frac{e^{\left(\frac{d}{t}\right)} x}{e^{\left(\frac{x}{t}\right)}} + \frac{e^{\left(\frac{d}{t}\right)} t}{e^{\left(\frac{x}{t}\right)}} - \frac{e^{\left(\frac{d}{t}\right)} d}{e^{\left(\frac{x}{t}\right)}} - t\right)$$

```
> simplify(%) assuming t>0;
```

$$e^{\left(-\frac{d}{t}\right)} t$$

Example 6: A common misunderstanding in using the integration commands of Maple is the use of *int* and *Int* .in the following example [7], use of the command *int* does not produce an answer, mainly because the anti-derivative of the given length function is very complicated involving elliptic functions. However, the command *Int* bypasses anti-derivative and numerically evaluates the integral

```
> f:=x->4*x^2-x^3;
```

$$f := x \to 4\,x^2 - x^3$$

```
> fprime:=diff(f(x),x);
```

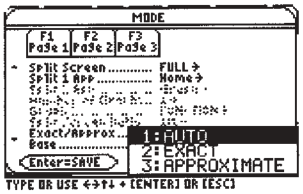
$$fprime := 8\,x - 3\,x^2$$

```
> L:=int(sqrt(1+fprime^2),x=-1..4);
Warning, computation interrupted
```

```
> # After a few minutes, we stop the computation. Now
let's use Int in place of int.
> NewL:=Int(sqrt(1+fprime^2),x=-1..4);
```

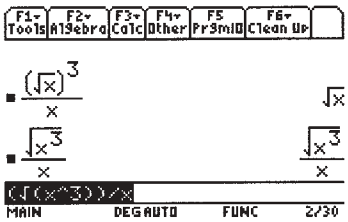
$$NewL := \int_{-1}^{4} \sqrt{1 + 64\,x^2 - 48\,x^3 + 9\,x^4}\, dx$$

```
> evalf(%);
```
24.87458872



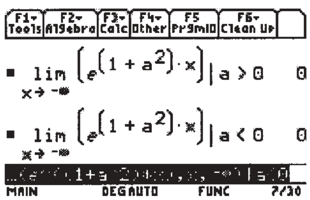
The AUTO setting will render EXACT or APROXIMATE results: approximate when one of the operands has an eplicit decimal point, exact results otherwise.

There are some genuinely quirky things that the TI-89 (the Voyage 200 gives identical results). Consider the following:



The student in middle school algebra has learned that the two expressions on the left side of the display are equal and both simplify to √x. But a glance at the right hand side indicates that the top expression simplifies to √x, but the bottom expression cannot be simplified by the calculator.

As a final example consider the following limit problem. The calculator will show, that with 'a' set to the value 0, $\lim_{x \to -\infty} (e^{(1+a^2)*x}) = 0$. With a < 0 and a > 0 the limit also converges to 0.



Let's review: this limit is zero when 'a' equals zero, when 'a' is less than zero, and when 'a' is greater than zero. You would think then that the limit is zero in all cases, but if a range for 'a' is note specified the calculator returns "undf" or undefined for the limit. Try it.

There are multiple additional examples that could be demonstrated except for page restrictions. The examples that have been sited are sufficient to indicate that the calculators and algebra systems still hold some surprises for the unwary. But we do not want to leave the impression that we view these tools hopelessly flawed. Our attitude is quite the contrary. We have had to look hard to find these pathological examples. Day in and day out our students use these tools and explore more problems than they could (or would) every try if they were restricted to hand computations. These are wonderful tools and we will continue to use them (and their upgrades) with our students.

REFERENCES

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