USING CONCEPTESTS IN SINGLE AND MULTIVARIABLE CALCULUS

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ConcepTests were developed by Eric Mazur [1] as a method of improving student conceptual understanding and scores on concept examinations in physics. This has been replicated in other science areas (see http://galileo.harvard.edu), now including calculus.

Our ConcepTests are usually conceptual, multiple-choice questions, but include some True/False, and some free response questions as well. They are designed to promote the discussion and learning of mathematical concepts during a calculus class, and not as a means of testing students for a grade. A few examples from [3] are given in Appendix A. (Note that some questions have more than one correct answer.)

ConcepTests can be used in many ways -- here are three we have successfully tried:
1. Introduction to a topic. This is possible if the topic is closely tied to previous lessons or is something with which most students would have some familiarity.
2. After presentation of a topic. Here ConcepTests are used to see if the students have grasped the concept, or if the topic needs more discussion or examples.
3. A review of material.

To use a specific ConcepTest in class, we displayed it using an overhead projector, or sometimes distributed a copy of the ConcepTest to each student. Students are then given short time (one to four minutes, depending on the question) to think about the ConcepTest and then vote for the answer they think is correct. Providing almost all do not vote for the correct answer, the students are then given a few minutes to discuss the ConcepTest with adjacent students and then are given another chance to vote on the correct answer. Besides determining the correct answer, students also develop communication skills since they must support their answer in order to convince their peers. To reinforce the concept, the instructor then discusses the correct answer (or answers) and/or has students present their reasons.

A fourth way to use ConcepTests is to let the students take them home, and then vote on the correct answers at the beginning of class. A short discussion with peers might clear up any misconceptions students may have had. This would allow students to obtain feedback on their homework without grading by the instructor.

Using ConcepTests can be effective, even in a large lecture classroom, as students are actively participating in the concepts and ideas being presented. The discussion process greatly increases the students' familiarity with the subject and helps them formulate the

ideas in their own words. This increases the students' base of knowledge and enthusiasm for mathematics. Scott Pilzer [2] taught two sections of Calculus I, using ConcepTests with one section, and using standard lectures in the other. At the start of Calculus II, these students were given an examination that included both conceptual problems and standard problems. The students who were taught with ConcepTests outperformed the other section by 73% to 17% on the conceptual problems, and 63% to 54% on the standard problems. These dramatic results mirror the results found in physics: a large increase in conceptual understanding in addition to an increase in standard computational problem solving ability.

ConcepTests were originally developed for use in a large classroom setting, where some means of gathering data electronically was available. Currently you could use a Personal Response System. This system is similar to the technology used in the popular game show, "Who Wants to Be a Millionaire?" Using this method, students maintain absolute anonymity. The students not only input their answer, but they can also indicate the confidence with which they made their choice. Once all the students have input their choice, a graph of the results can be displayed. See Appendix B for a sample of the output. For this technology the questions must be multiple-choice or True/False. We note that the students are quite excited to use the technology. TI-Navigator is a less elaborate system that can supply similar information to the instructor. Expensive technology is not needed for small classes, as the show of hands or having students hold up colored cards to vote allows the instructor to quickly tabulate the vote.

We found that ConcepTests were equally effective in small or moderate sized classes where they promote "active" or "discovery" student learning. The ensuing discussion greatly increases the students' familiarity with the subject and helps them formulate mathematical ideas in their own words. This in turn, increases the students' base of knowledge and enthusiasm for mathematics. In his article, Scott Pilzer describes how this enthusiasm translated into more mathematics majors, and we have discovered the same to be true. Cornell University has been developing three types of ConcepTests for use in their calculus courses - namely - "Quick Check", "Probing" and "Deep". They call them "Good Questions", and are available from Maria Terrill's website at http://instruct1.cit.cornell.edu/courses/.

REFERENCES

[1] Mazur, Eric. "Peer Instruction - A User's Manual". Prentice Hall, Englewood Cliffs NJ. 1997.

[2] Pilzer, Scott. "Peer Instruction in Physics and Mathematics, PRIMUS Volume XI # 2, June 2001. pp. 185-192.

[3] Pilzer, Scott, M. Robinson, D. Lomen, D. Flath, D Hughes Hallet, B Lahme, J. Morris, W. McCallum, and J. Trash. ConcepTests - to accompany Calculus 3rd Edition, Hughes Hallet et al. John Wiley & Sons, Inc. NY. 2003.

#1. $\sum_{n=1}^{\infty} a_n (x-2)^n$ converges at $x = -2$ and diverges at $x = -4$. At $x = 6$ the series is

(a) conditionally convergent   (b) absolutely convergent   (c) divergent   (d) cannot be determined

#2. For each of the vector fields, $\vec{F}_1, \vec{F}_2, \vec{F}_3$, decide which of the properties the vector field has:

$$\vec{F}_1(\vec{r}) = \frac{\vec{r}}{\|\vec{r}\|} \qquad F_2(\vec{r}) = -\vec{r} \qquad \vec{F}_3(x, y) = \frac{\vec{i} + \vec{j}}{x^2 + y^2}$$

(a) All vectors point toward the origin.
(b) All vectors point away from the origin.
(c) All vectors point in the same direction.
(d) All vectors have the same length.
(e) Vectors get longer away from the origin.
(f) Vectors get shorter away from the origin.
(g) The vector field has rotational symmetry about the origin.
(h) Vectors that start on the same circle centered at the origin have the same length.

#3. If the graph of f is given in Figure 6.9, then which of (a)-(d) is the graph of

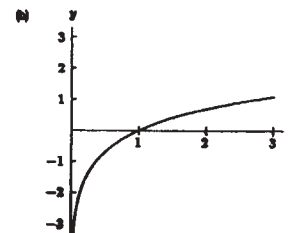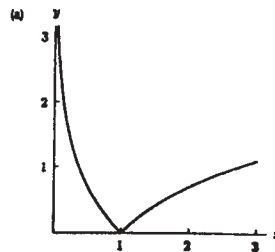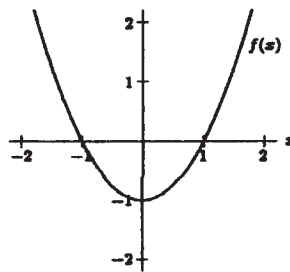$$\int_1^x f(t)dt \quad \text{for } 0 \le x \le 3 ?$$

Figure 6.9

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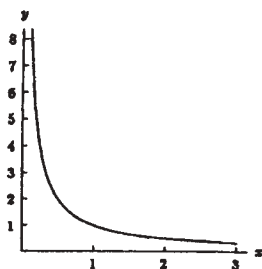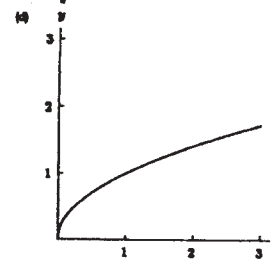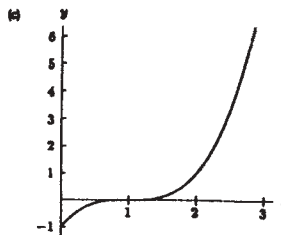
#4. Suppose $P_2(x) = a + b(x-1) + c\dfrac{(x-1)^2}{2}$

is a Taylor polynomial of degree two about $x = 1$ for some function f. Give the signs of a, b, and c, if the graph of f is as shown to the right. (Note: 0 is also a possible answer.)

#5. Figure 2.27 shows position as a function of time for two sprinters running in parallel lines. Which of the following is true?

   (a)   At time A, both sprinters have the same velocity

   (b)   Both sprinters continually increase their velocity.

   (c)   Both sprinters run at the same velocity at some time before A.

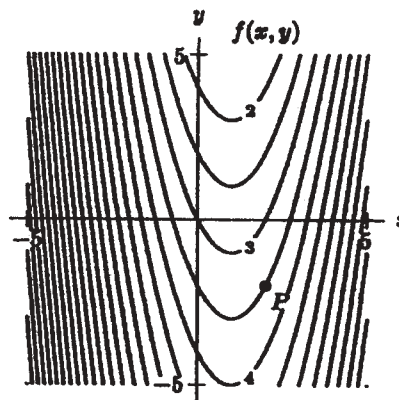   (d)   At some time before A, both sprinters have the same acceleration.
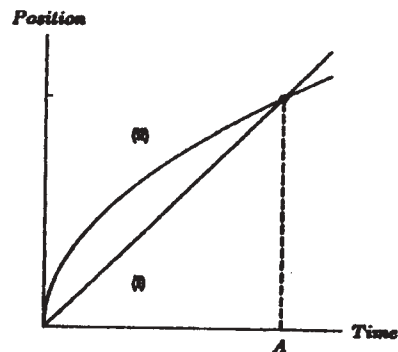
Figure 2.27

#6. Level curves of $f(x, y)$ are shown to the right. What are the signs of $f_{xx}(P)$ and $f_{yy}(P)$?

#7. True or false?

   (a) $\vec{v} \cdot \vec{w} = 0$ if and only if $\vec{v} = \vec{0}$ or $\vec{w} = \vec{0}$.

   (b)   The zero vector is orthogonal to any other vector.

   (c)   Any plane has only two distinct normal vectors.

   (d)   If $\| \vec{v} \|$ and $\| \vec{w} \|$ are large then $\vec{v} \cdot \vec{w}$ is large.

   (e)   The dot product of a vector with itself is its magnitude.

   (f)   Any vector normal to a surface has length one.

   (g)   Parallel planes share a same normal vector.

   (h)   If two planes are perpendicular, so are their normal vectors.

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#8. Consider the area between the two functions shown in Figure 8.4. Which of the following graphs (a)-(d) represents this area as a function of x?
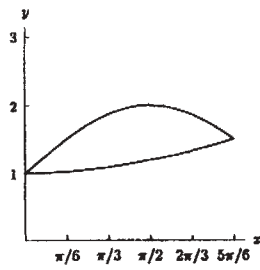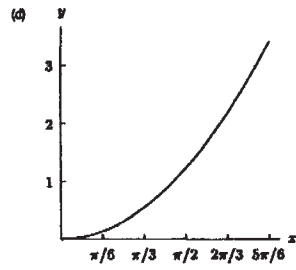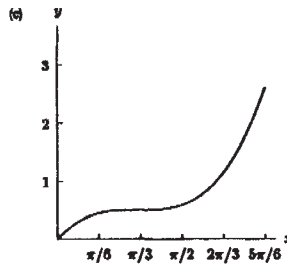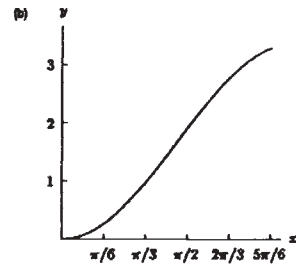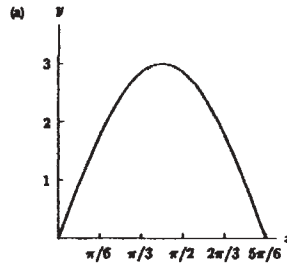

Figure 8.4

APPENDIX B
SAMPLE OUTPUT OF PERSONAL RESPONSE SYSTEM

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