College Geometry: A Computer Activity Approach using The Geometer's Sketchpad®

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We are developing a college-level course in Geometry using The Geometer's Sketchpad. The focus of this course is on developing skills in writing mathematical proofs. Computer lab activities using The Geometer's Sketchpad introduce all topics. Exploratory activities set the stage for class discussions. Students work in teams on activities, making and testing conjectures. Class discussion leads to developing proofs (or refutations) of students' conjectures. We have each taught the College Geometry course using these materials: in fall 2002 and again in fall 2003 at Cardinal Stritch University, and in spring 2003 at Bellarmine University.

Brief Table of Contents

Goals of the Project

We identified four student performance goals for this project:
- Students engage in explorations, which lead to making and testing conjectures about geometric objects.
- Students improve their geometric reasoning skills.
- Students develop skill in constructing mathematical proofs.
- Students develop skill in articulating mathematical ideas, both orally and in writing.

Methods of Assessment

<u>By the instructors</u>:
- Pre- and post-test scores on tests of van Hiele levels of geometric reasoning
- Records of within-semester student performance

<u>By an external evaluator</u>:
- Interviews of several students five months after the end of the course
- Analysis of videotapes of student groups at work in class
- Analysis of student work in their Proof Book portfolios

Evidence of Effectiveness

In evaluating the effectiveness of this project, we used multiple indicators for each of the student performance goals. At the beginning and the end of the course, each of us gave a pre- and post-test on van Hiele levels of geometric reasoning to the students. During the course, we kept records of student progress and achievement – test and homework grades, copies of student work at the beginning, middle, and end of the semester. At the end of the course, each student turned in a portfolio of proofs, selecting work done during the semester to illustrate growth in and mastery of proof-writing skill. We videotaped two different student groups working on the computer lab activities during one class session. After the course was over, we engaged an external evaluator who interviewed several students, analyzed the videotapes, and reviewed portfolios of the work of several students.

- Goal: *Students engage in explorations, which lead to making and testing conjectures about geometric objects.*
  In both one-on-one interviews and videotaped interactions, all students constructed geometric objects using Sketchpad, explored these constructions dynamically, discussed their observations, and made an tested conjectures based on these explorations.

- Goal: *Students improve their geometric reasoning skills.*
  Pre- and post-test scores of van Hiele levels of reasoning of all students who remained actively engaged in the course either remained the same or improved. Within-semester test scores of students showed consistent improvement on test questions that focused on reasoning, conjecture and proof.

- Goal: *Students develop skill in constructing mathematical proofs.*
  Analysis of students' proof portfolios showed improvement as students revised proofs over the semester. The standard assessment rubric used in marking these proofs appears to have assisted and encouraged students in revising their proofs productively.

- Goal: *Students develop skill in articulating mathematical ideas, both orally and in writing.*
  All the students who were interviewed were able to articulate mathematical observations about their constructions and the geometric situations that resulted.

Videotapes of students at work in class showed them explaining, suggesting ideas and exploratory approaches, talking over results of computer activities, and then producing written answers to questions.

Grading Rubric for Proofs

Assessing student work on written proofs is somewhat subjective. Devising and using a standard rubric for grading written proofs not only helped us to be more objective in assessing student work, but also made it easier to communicate with each other throughout this project. The external evaluator observed that the use of a standard rubric seemed to help students focus on specific ways to improve their proof-writing skill.

Score	Component	Description
2 1 0	Logic	• Correct • Flawed • Seriously flawed
	Comments	
2 1 0	**Diagrams / Tables**	• Appropriate, accurate, clearly labeled • Flawed • Seriously flawed
	Comments	
2 1 0	**Complete** • Cases	• All appropriate cases considered • One case missing, trivial case • Significant cases missing
	Comments	
2 1 0	• Justification	• All significant steps justified • Some justification given • Few steps are justified
	Comments	
2 1 0	**Explanations**	• Clear • Unclear • Missing
	Comments	
	Total Score	

We anticipate that a textbook using these methods, *College Geometry: A Computer Activity Approach using The Geometer's Sketchpad*, will be available from Key College Publishing in September, 2005. Please contact either one of us if you are interested in piloting these materials in your own classes.

The Arbelos: An Exploration with Circles

ACTIVITY: Draw a circle *C*, with diameter *AB*. Construct a
point *P* on the diameter *AB* and construct
two additional circles with diameters *AP* and *PB*.
The region bounded by the semicircular arcs on one side of
diameter *AB* is called an *arbelos*. Calculate its area.
Construct a perpendicular to *AB* at the point *P*.
Mark one of the intersections of this perpendicular
with the circle *C* as *R*. Construct a circle with
diameter *PR* and calculate its area.

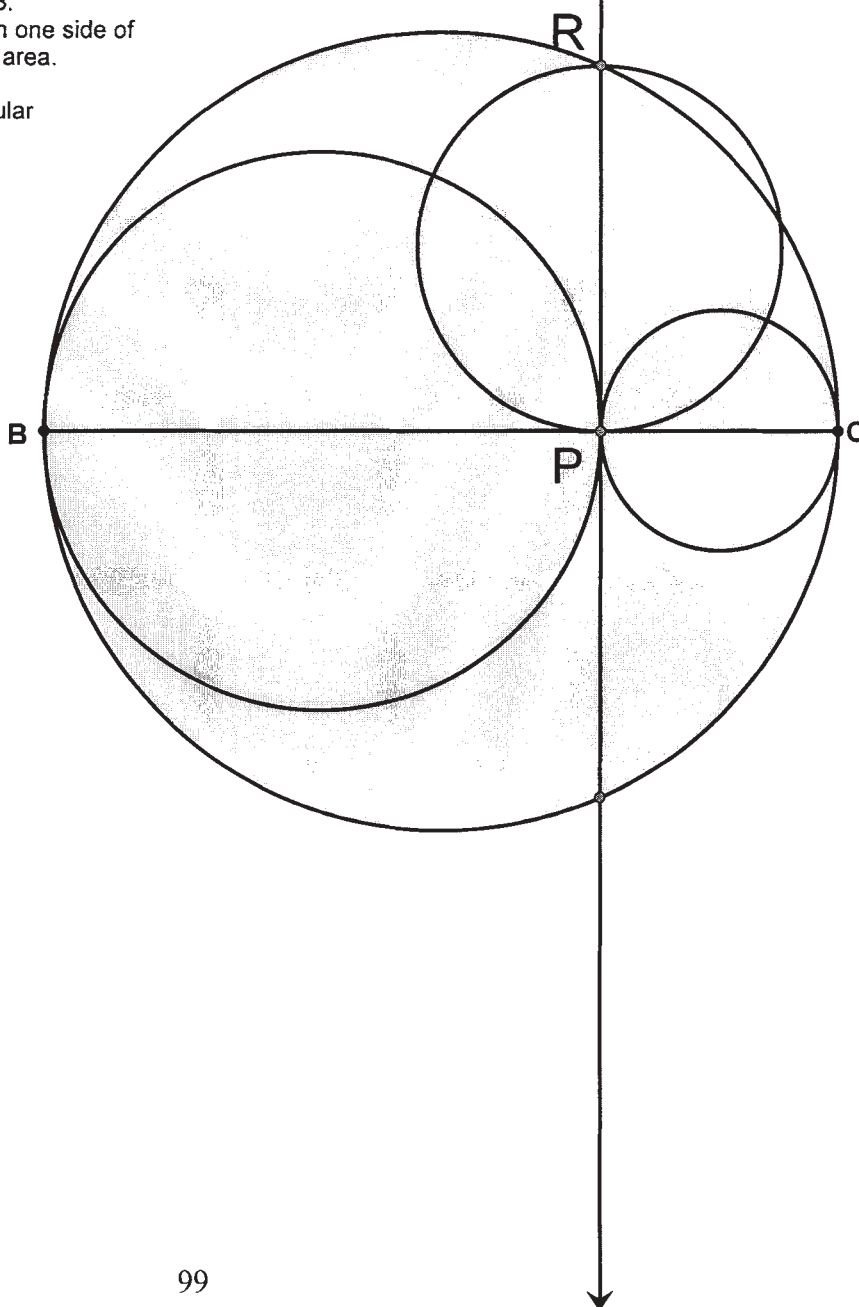Make a conjecture about these areas.
Prove your conjecture.

Area of circle AB = 148.67 cm^2

Area of circle AP = 72.81 cm^2

Area of circle PB = 13.40 cm^2

Area of arbelos = 31.23 cm^2

Area of circle PR = 31.23 cm^2

This activity appears in Chapter 3 of our text.

99

Rose Curves: An Exploration in Polar Coordinates

ACTIVITY: On a polar grid, plot the following functions, one at a time. These are called *rose curves*.

$r = \sin(3\theta), \quad r = \sin(4\theta), \quad r = \sin(5\theta), \quad r = \sin(6\theta)$

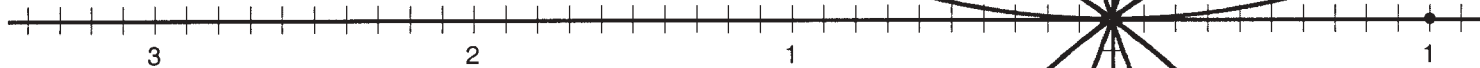For $r = \sin(n\theta)$, how many petals will the rose have?

Create a rose with sixteen petals. Create a rose with seventeen petals. Create a rose with eighteen petals.

$f(\theta) = \sin(3\cdot\theta)$ | Show the graph. |

$g(\theta) = \sin(4\cdot\theta)$ | Show the graph. |

$h(\theta) = \sin(5\cdot\theta)$ | Hide the graph. |

$q(\theta) = \sin(6\cdot\theta)$ | Show the graph. |

To try a different coefficient for θ, double-click on an equation and edit as desired. To change the domain, right-click on the graph and go to Properties, Plot.

This activity appears in Chapter 4 of our text.

100