NUMERICAL METHODS USING MATHEMATICA NOTEBOOKS AND WEB APPLETS

Nora Strasser
Friends University
2100 University
Wichita, KS 67213
strasser@friends.edu

Introduction

Traditionally, the Numerical Methods course at Friends University is taught for Mathematics majors at the junior/senior level. Most Numerical Methods courses require students to use programming languages to code the methods and then run these on a computer. However, some Numerical Methods courses are starting to use other means to allow students with less programming knowledge and skill to still investigate numerical methods using the computer. This Numerical Methods course has been enhanced by adding Mathematica Notebooks and using web applets to demonstrate methods and properties. Students do not need a deep understanding of programming to benefit from these computer investigations.

Blackboard

In this course, the Blackboard course management software is used to organize the material for student access. Blackboard allows the instructor to quickly and easily design a course web site. This web site was used to organize and present the many links that students needed to complete the assignments in the course. The course web site also includes information about the instructor, the course schedule, the syllabus, links to the web applets, and Mathematica notebooks. Using a web site to provide students with the links necessary to complete the course is a great benefit. It allows students to access the web site by a simple click of the mouse as opposed to typing in a long url. Students also have the benefit of being able to download the necessary files without going to a different location.

When students enter the Numerical Methods web site, they are presented with several navigation choices that are listed down the left side of the screen (See Figure 1). These choices include: announcements, syllabus, faculty information, lessons, and assignments. Students may choose any of these links and all the information that is provided on the syllabus is also provided on the web site. The links that we are most interested in are the assignments link and the course lessons link. The assignments link describes which lessons are due and where to find them. These assignments include both lessons involving Mathematica notebooks and web applets. The lessons link allows students to access both the web applets and the Mathematica notebooks that are needed to complete the assignments.

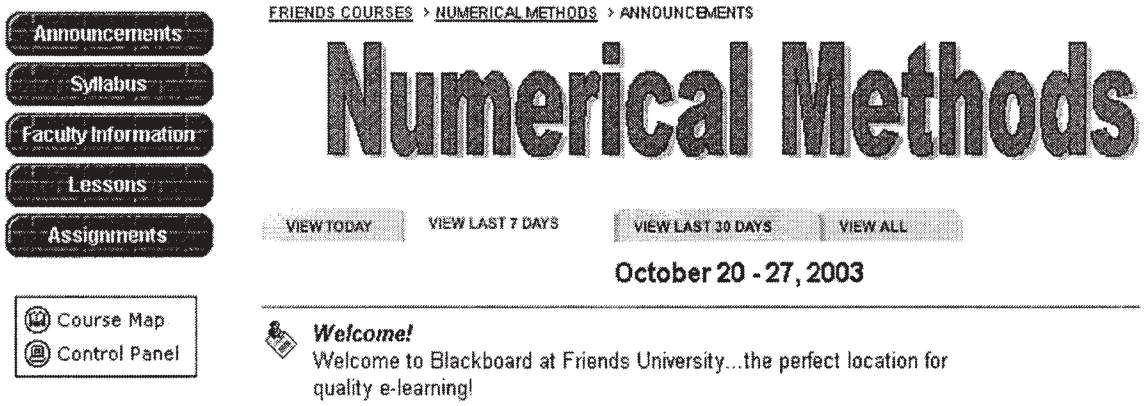

Figure 1 – Blackboard Web Site

Most of the lessons contain both links to java applets, and Mathematica notebooks. However, there are some lessons that have only java applets or only Mathematica notebooks. The following lessons are included on the site: Sample Lesson, Taylor Polynomials, Binary Numbers, Floating Point Numbers, Overflow Error, Bisection Method, Newton's Method, Secant Method, Fixed Point Iteration, Ill-Behaved Root Finding, Interpolation, Divided Differences, Spline Functions, Trapezoid/Simpson's Rule, Least Squares Data Fitting, Euler's Method, and Runge-Kutta Method. The Sample Lesson, Secant Method, and Divided Differences lessons contain only Mathematica Notebooks. The Overflow, Fixed Point Iteration, Ill-Behaved Root Finding, Least Squares Data Fitting, Euler's Method, and Runge-Kutta Methods contain only links to web applets.

Web Applets

Using web applets that already exist is a good way to demonstrate mathematical properties. The biggest problem is finding applets that will demonstrate the desired properties. To do this, a good search engine is necessary. Google was used to find the applets used for this Numerical Methods course. Google is an excellent search engine that produces good results with minimal effort. For example, to find a web applet that demonstrates Newton's Method, the following keywords were put into google: "Newton's Method", java applet. Note that the words Newton's Method were put in quotes so that Google would find only that exact phrase and not the two words separately.

The results of the search were then called up and examined to see which of the sites best demonstrated Newton's Method in action. By examining the web applets and creating criteria by which to judge them, many excellent applets were added to the course. Some of the criteria used for judging these sites included the amount of interaction with the user, whether basic or unusual aspects of the method were demonstrated, the visual appeal, and the ease of use. The best sites were then used in the lesson on Newton's Method. Links to the sites were established along with instructions on how the site was incorporated into the assignment.

Each of the topics included on the syllabus were treated in a similar manner. Google was used to search for related applets and then the applets were chosen for inclusion on the course web site. Excellent applets were found for most of the topics, however, there were some topics that were left out.

It is important to check and update these web sites both prior to the start of the semester and shortly before the assignment is given to the students. One problem with using links to existing sites is that these sites tend to be of a temporary nature. Links are moved to different servers, or disappear altogether. If the site has been moved, the links need to be updated. If the link has disappeared altogether, a new search will need to be made to find a new site that will demonstrate the desired property.

Mathematica Notebooks

The Mathematica notebooks were written specifically for the purpose of demonstrating numerical methods for this course. Mathematica is very powerful and can be used both as an interface and a programming language. The notebooks were written to give students an introduction to the methods and provide students with a walk through of how to use the Mathematica program to perform the method. Built-in Mathematica functions were used sparingly since the calculations tend to be transparent to the user and do not give insight into the process. Instead, programs were written to show at least some of the steps that are required for the method. These steps give insight into the process.

Each notebook has four sections: Description, Problems, Example, and Program (See Figure 2). The Description section contains a description of the method used in the notebook. It also compares the method to others that have been previously studied. The Problems section describes the assignment and what the students are expected to do. The Example section walks through similar problems using a step-by-step process. Finally the Program section contains the Mathematica code used to implement the method. Each section is in closed form so that students need to open each section before viewing its contents.

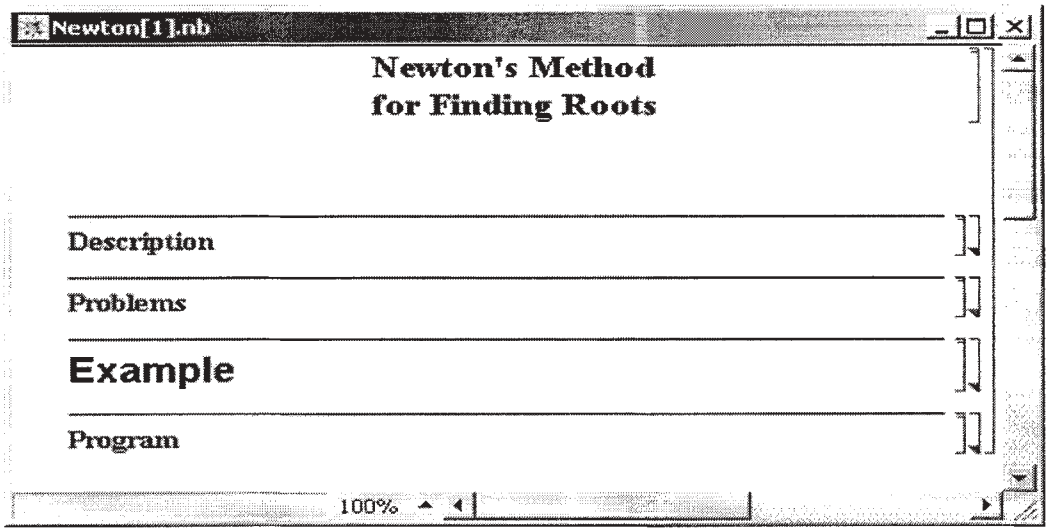

Figure 2 – Mathematica Notebook

Students are required to modify the Mathematica code and use Mathematica to perform the indicated processes. The students are then asked to print out the results and turn these in as a part of their grade. Knowledge of Mathematica is required, however, very little programming skill is actually needed. Students are able to use the example and the given program to investigate the Numerical Method. For example, the program in Newton's Method is as follows:

```
Input["f[x_]:="];
Input["error, e="];
Input["initial guess, a="];
g[x_]=D[f[x],x];
b=a-(f[a]/g[a]);
While[Abs[b-a]>e,a=b;b=a-(f[a]/g[a]);Print[N[b,15]]]
```

If you execute this program, the three input statements allow students to change the function, the error, and the initial guess. The only skill required from the students is entering these things using proper Mathematica notation. If the function f[x_]: = x^3 – x^2 – x – 1 is entered with an error of 10^{-5} and an initial guess of a = 1.2, the output from the program is as follows:

```
2.45968
2.02051
1.86153
1.83968
1.83929
1.83929
```

This allows the students to see the intermediate steps and also to see how long it took to reach the desired accuracy. In this case, it took longer to reach the actual root since if one had been used as the initial guess Newton's method would fail since the derivative is zero at the initial guess.

After opening the Example section, students will be presented with a step-by-step walk-through of problems similar to those assigned. In fact, students may be able to simply edit the Mathematica code from the example section to perform their own calculations. Again, some skill with Mathematica is required, but it is not extensive. Students at Friends University are introduced to Mathematica during the Calculus sequence and use it extensively in most of their courses. Therefore, students are familiar with Mathematica and do not have difficulty using the notebooks in Numerical Methods.

Conclusion

The students seemed to appreciate being able to use both the Mathematica notebooks and Web applets. Some of the comments collected from students included: "The Web sites really helped me visualize how the method works."; "Using Mathematica to perform some of the calculations really took the drudgery out". Overall, the student response was

positive and almost no class time was lost to the project. In class the traditional lectures were given and students were asked to do most of the homework by hand. However, the computer assignments were added as further homework. Students did these on their own time. One class period was spent in the computer lab introducing students to the tools needed to do these assignments.

The only drawback to using these activities with a Numerical Methods course is the amount of time and effort the instructor must use to prepare these items. Once the initial set-up is complete, an instructor still must update the links every semester. There is also the concern that using a new book may require you to rearrange assignments to correspond better.

Overall, using Mathematica Notebooks and Web applets appears to be a successful approach to take in a Numerical Methods course. Students can more freely investigate without being bogged down with the programming skill required. They can concentrate more fully on the mathematics required for each method. Visualization is drastically improved since the applets allow students to see what is actually happening geometrically as the method is applied. It was concluded that this resulted in better learning about the mathematics and the numerical methods involved.

References

Atkinson, Kendall, *Elementary Numerical Analysis*, 2nd ed., John Wiley & Sons, 1993.

Blackboard 5, Version 5.0.1, Blackboard Inc., 2000.

Google Search Engine, http://www.google.com/

Wolfram, Stephen, *The Mathematica Book,* 5th ed., Wofram Media, 2003.