TEACHING MATHEMATICAL PROOF WITH TECHNOLOGY TO FIRST YEAR STUDENTS

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Motivation. "*In order to evaluate the validity of proposed explanations, students must develop enough confidence in their reasoning abilities to question others' mathematical arguments as well as their own. In this way, they rely more on logic than on external authority to determine the soundness of the mathematical argument.*" (NCTM, pp. 345-346)

This NSF-funded project (DUE-0230755) is focused on the development of pedagogically sound computer-based tools for teaching and learning mathematical proof. The distinction between "content material" and "logical process" is a very subtle one in mathematics, especially in the context of reasoning and proof. We believe that the contructivist teaching philosophy is correct for this setting as well, and that the nature of mathematical proof makes the use of technology an obvious vehicle for implementing this approach.

Even though, as Steen writes, "Nothing divides research mathematicians and mathematics educators from each other as do debates about the role of proof in school mathematics" (Steen, p. 275), the NCTM has long been an advocate of reasoning and proof in K-12 instruction. The NCTM Principles and Standards for School Mathematics specifically states that, "Reasoning and proof are not special activities reserved for special times or special topics in the curriculum but should be a natural, ongoing part of classroom discussions, no matter what topic is being studied." (NCTM, p. 342) Steen further suggests that, "The important question about proof may not be whether it is crucial to understanding the nature of mathematics as a deductive, logical science, but whether it helps students and teachers communicate mathematically." (Steen, p. 275)

Certainly for the teaching of mathematical proof to be successful in the K-12 curriculum, the teachers at that level should be confident about mathematical reasoning and communication in many forms. A teacher must be able to listen to a mathematical argument, in any form from the empirical to the precise, and give adequate feedback as to the argument's correctness and clarity. To do this, it is essential that teacher's know how to read and write mathematical proofs in a variety of styles.

It is therefore an important time for mathematicians to supply curricula and methods for best preparing teachers to understand proof at many levels so that the objectives of these standards can be achieved. To accomplish this ambitious goal, this Educational Materials

Development grant is currently developing computer-based activities to supplement an innovative discrete mathematics textbook, also co-authored by Doug Ensley and Winston Crawley, which is the first college course in which students at Shippensburg University encounter formal mathematical proof. The goals and objectives of the development of this material are as follows:

- To strengthen students' understanding of the logic of implicational ("if..., then...") statements.
- To teach students to read a formal proof as an interactive dialog.
- To train others to use the technology tools to implement the same ideas in other courses.
- To develop new tools for teaching students how to write proofs.
- To contribute to the literature on teaching and learning mathematical proof.

The nature of the material itself is fairly simple. It is based upon the basic idea that students need to learn to understand logic and read proofs before they can write proofs.

Applications under development. The following summarizes the actual types of activities being developed and currently being tested. The interested reader is directed to the website reference to try out the activities or for more information.

CounterExamples: This is a collection of mathematical statements that a student must read critically and decide if each statement is true or false. If the statement is false, the student must provide a counterexample. This encourages students to construct their own understanding of truth or fallacy, and it establishes a point of view from which to write proofs of statements they believe are true.

ProofReader: To understand mathematical proof, one must be able to first effectively read mathematical proofs. Under this simple premise, we will develop a second Flash application in which students "trace" through a formal proof and respond to each statement therein. This process is not unlike the debugging process that is done in computer programming, but our main objective is for students to see mathematical proof as an interactive dialog between author and reader. Within this general format, we have "incorrect proofs" of false statements as well as "incorrect proofs" of true statements. This twist connects with the type of discovery practiced in the *CounterExample* exercises, and it also gives students a tool by which they can discover particular errors in a proposed proof.

Shuffled Proof: To further encourage students to read proofs for mathematical content and logical understanding, we have created some simple examples of proofs where the lines of the proof are presented in a scrambled order. The student must "drag" the statements into the correct order to complete the exercise.

Mathematical induction: "Proof by induction" is a fairly specialized technique that is very important in the discrete math course. Once again we take advantage of technology to help students create their own understanding of the process. The technology is used first to reinforce and enhance the students' ability to reason recursively, both for recursively-defined sequences and for summations. Students proceed from concrete examples to generic (variable-based) extensions of those examples. Finally, in connection with the coverage of mathematical induction, students check statements one at a time in a table format, looking at the direct connection between a statement and the previous statements for many numerical examples, until they can finally argue from one generic step to the next.

Developer's material. Material developed under this grant should be open source with sufficient instructions for other instructors to modify the problems to suit their own needs. Our first step in this direction is posted on the course web site (see references). It is a complete set of templates and Flash source code with detailed instructions to create the "Shuffled Proof" problems discussed above.

Conclusion. As the first year of this grant period comes to a close, we are very pleased with the preliminary results and we are looking forward to further development as well as continued research into the way students learn mathematical proof. Our longitudinal assessment should have preliminary results by the end of the academic year, and we are anxious to share our results with the mathematics community. The full content of this ICTCM session addressed each of these activities, including aspects of design and implementation. The full set of material as well as a sample of the development templates are given in the references below.

References

Ensley, D. E. and J. W. Crawley, Discrete Math course web site with links to Flash applications for teaching mathematical proof and developer's material (zipped). http://www.ship.edu/~deensl/DiscreteMath/

Ensley, D. E. and J. W. Crawley, *Introduction to Discrete Mathematics: Mathematical Reasoning with Puzzles, Patterns, and Games*, under contract with John Wiley and Sons.

NCTM Principles and Standards for School Mathematics, National Council of Teachers of Mathematics, 2000.

Steen, L., "Twenty questions about mathematical reasoning," in *Developing Mathematical Reasoning in Grades K-12*, 1999 Yearbook, National Council of Teachers of Mathematics, 1999, pp. 270 - 285.