Maple does numerical approximations of area under curves. Are you ready with the current technology to generate some numbers in front of the engineering majors?

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Mathematics and engineering majors learn calculus in the same integral calculus class after the mastery of differential calculus. Mathematics majors (abstract learners) are convinced with proofs while the engineering majors (visual learners) want some numbers in front of them to convince their conclusions. We, as teachers having different discipline set of students in the same class must able to handle this situation and be effective teachers of Mathematics and/or Engineering.

We will illustrate this situation using the numerical approximations of areas under curves. We will explore this through examples of nice functions over a fixed interval. We assume that the problem in front of the students is to figure out the best approximation of area under a curve of some given nice function defined over a given interval and will also assume that the available approximations are upper, lower and middle sum approximations.

Example: Finding the best approximation of the area under the curve of the nice function

$f(x) = x^2 + 1$, Over the interval $[0,1]$.

Typing "with" followed by student in parenthesis brings the student directory of Maple. The semicolon at the end denotes the end of the statement. When you hit enter the following Maple window will show up on your computer.

$>$ **with(student);**

$[$ D, $Diff$, $Doubleint$, Int, $Limit$, $Lineint$, $Product$, Sum, $Tripleint$, $changevar$,
$\quad completesquare$, $distance$, $equate$, $integrand$, $intercept$, $intparts$, $leftbox$, $leftsum$,
$\quad makeproc$, $middlebox$, $middlesum$, $midpoint$, $powsubs$, $rightbox$, $rightsum$,
$\quad showtangent$, $simpson$, $slope$, $summand$, $trapezoid$ $]$

Input the function f through the assignment operator, "colon" followed by "equal" sign.

The arrow sign shown below after the variable is obtained by typing "minus" sign

followed by "greater than" sign from the keyboard. The end of the statement is obtained by typing "semicolon" at the end.
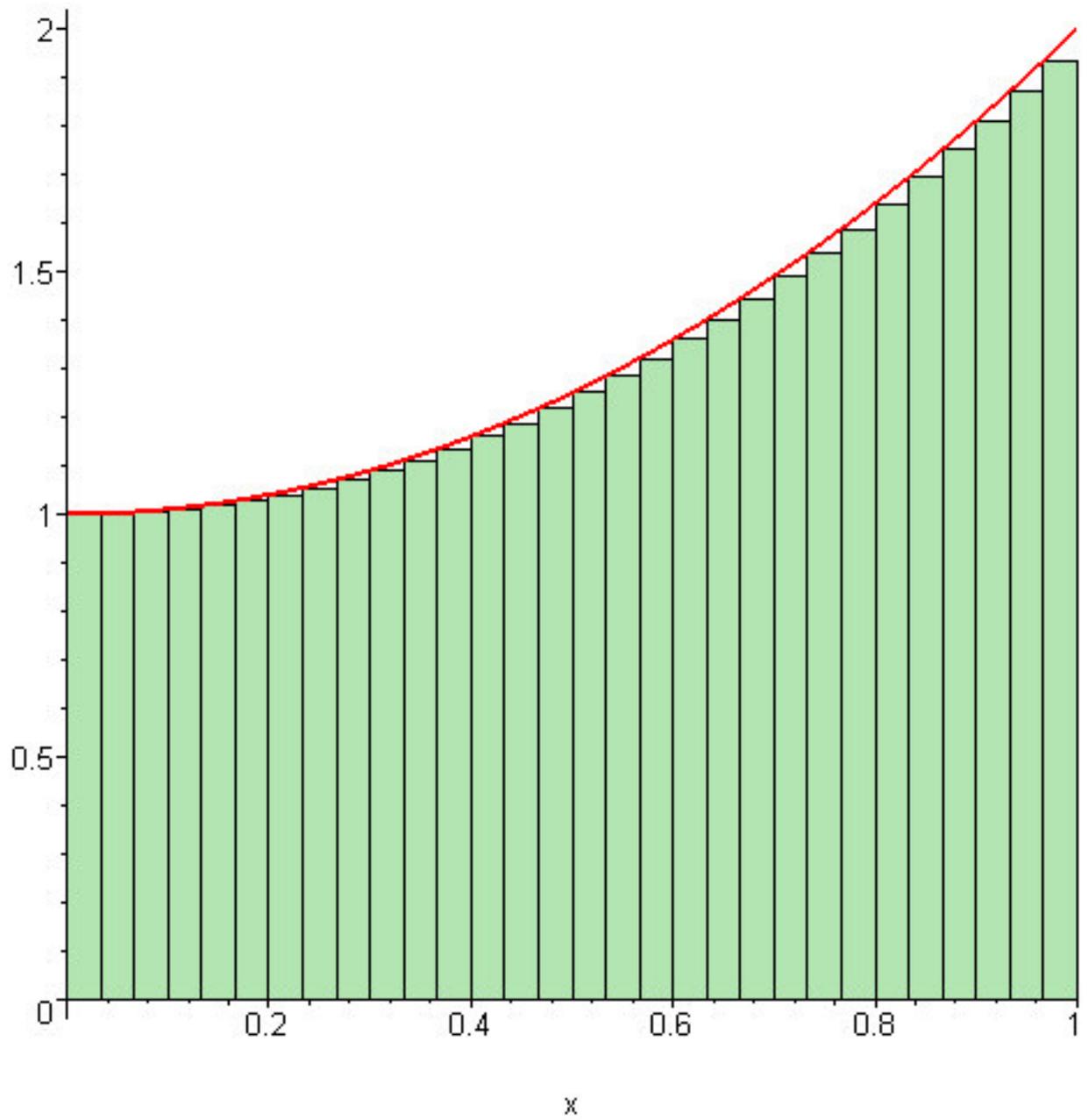
`> f:=x->x^2+1;`

$$f := x \rightarrow x^2 + 1$$

To draw the graph of the function as well as the rectangular regions having the left end height of the sub interval, we use the Maple command "left box". This command followed by three arguments in parenthesis separated by commas will draw the graph of the function as well as the rectangular regions whose areas added will approximate to the desired area. The three arguments are the function , the range of the variable, and the number of rectangles. The range of the variable is put in a format: the variable followed by equal sign, followed by the lower limit of the variable, followed by two periods which is followed by the upper limit of the variable. The number 30 denotes the number of rectangles. End of the statement is obtained by typing semicolon.

See the following command for example.

`> leftbox(f(x),x=0..1,30);`

The graph and the rectangular regions are drawn by Maple and will be displayed on the screen.

Our engineering majors can visually justify the approximation by looking at the picture above. Maple is giving an effective way of gaining that visualization. Now to obtain the approximate area we use the "leftsum" command followed by the "evalf" command. The "left sum" command will write out the sum of the areas of the rectangular regions as the sum of a series and "evalf" command will compute the value of that series to at least eight decimal places.
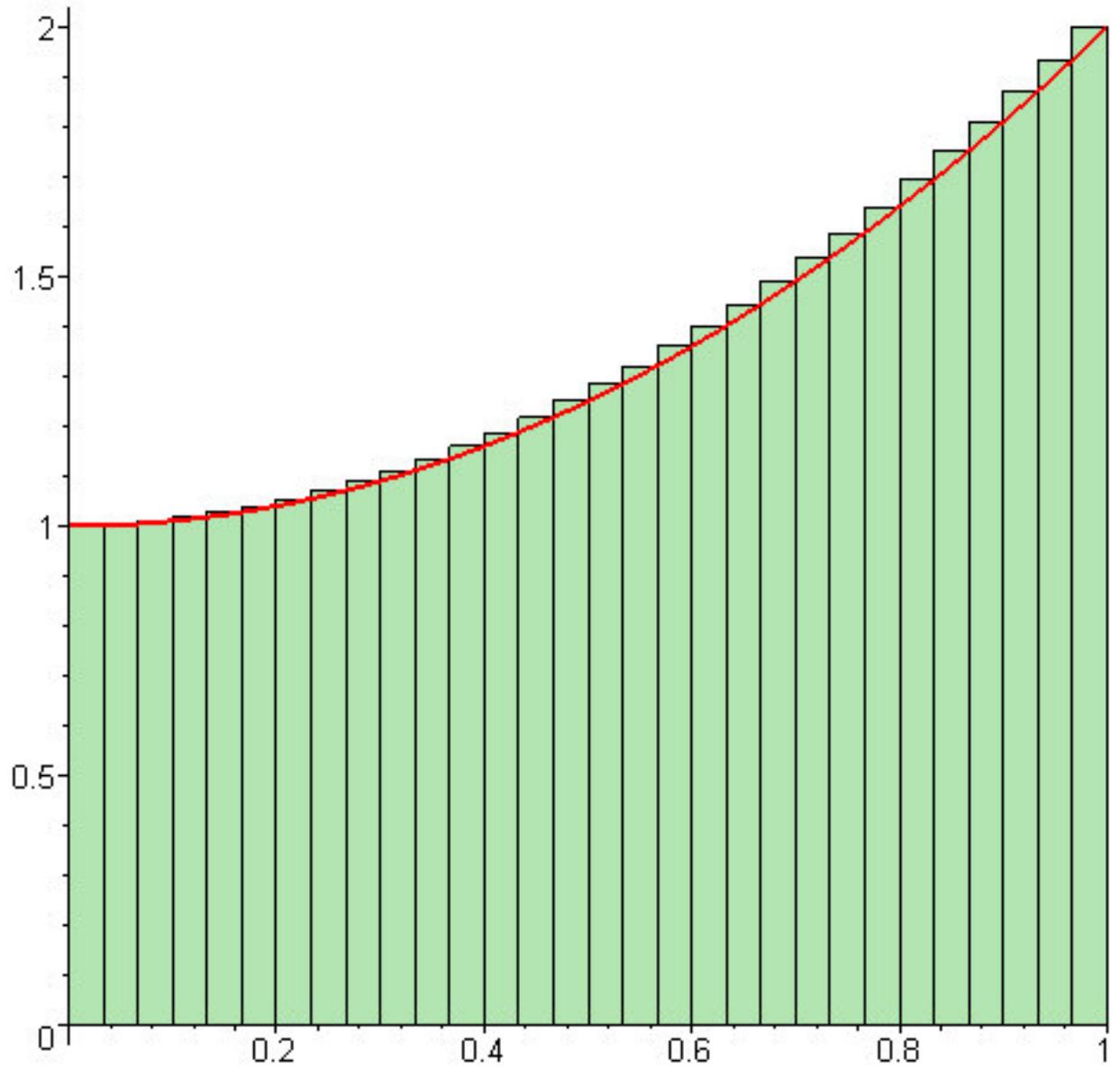
> **leftsum(f(x),x=0..1,30);**

$$\frac{1}{30}\left(\sum_{i=0}^{29}\left(\frac{i^2}{900}+1\right)\right)$$

> **evalf(%);**

$$1.316851852$$

We do the similar computation using the "rightbox" and "rightsum" commands as follows.
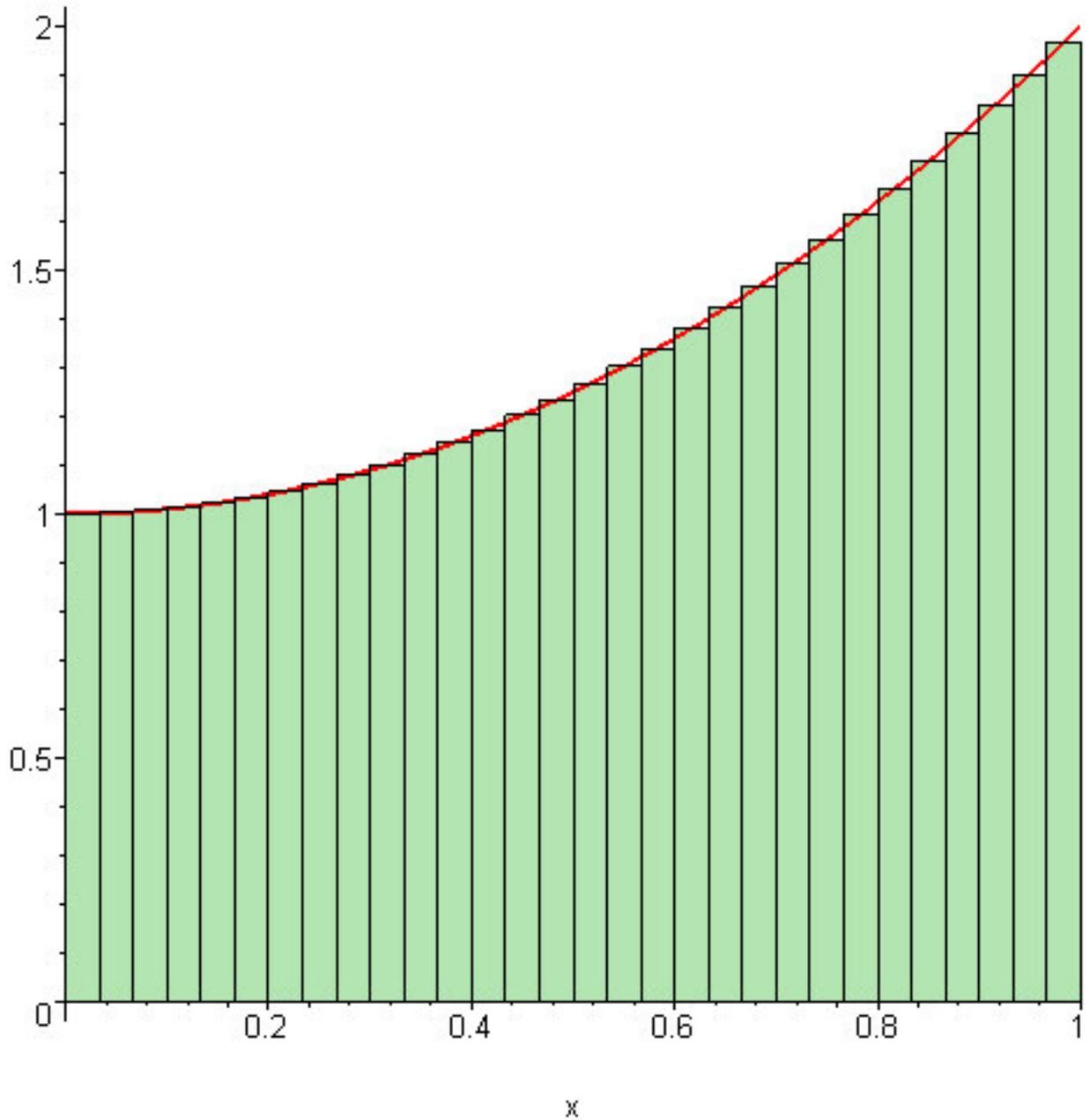
> **rightbox(f(x),x=0..1,30);**

```
> rightsum(f(x),x=0..1,30);
```

$$\frac{1}{30}\left(\sum_{i=1}^{30}\left(\frac{i^2}{900}+1\right)\right)$$

```
> evalf(%);
```

$$1.350185185$$

Lastly, we shall use the "middlebox" and the "middlesum" commands to get another approximation shown below.

> `middlebox(f(x),x=0..1,30);`

> `middlesum(f(x),x=0..1,30);`

$$\frac{1}{30}\left(\sum_{i=0}^{29}\left(\left(\frac{i}{30}+\frac{1}{60}\right)^2+1\right)\right)$$

> `evalf(%);`

1.333240741

The upper, lower and middle sum approximations were displayed using the Maple commands available in the "student" directory of Maple. We did this by making use of the Maple commands that allowed us to draw the graph of the function involved in the problem along with the rectangular regions that covered part of the area in question. This also allowed the visual learners (engineering majors) to see the pictures and make some understanding of these area approximations. Some numbers were generated in front of the engineering majors using some Maple commands. These engineering majors compared the upper, lower and middle sum approximations via the calculated numbers, which were calculated using Maple, and finally came up with some comparisons of these three approximations for the particular problem.

Conclusion: The middle sum approximation was the best approximation of the three approximations: Lower sum approximation, Upper sum approximation, and the Middle sum approximation. This was a visual aided conclusion using Maple.

References:
(1) ICTCM Proceedings: 14th Annual International Conference on Technology in Collegiate Mathematics
(2) Calculus, Seventh Edition, by Larson Hostetler Edwards
(3) Calculus I with Maple V, by Joe A.Marlin with Hok Kim