

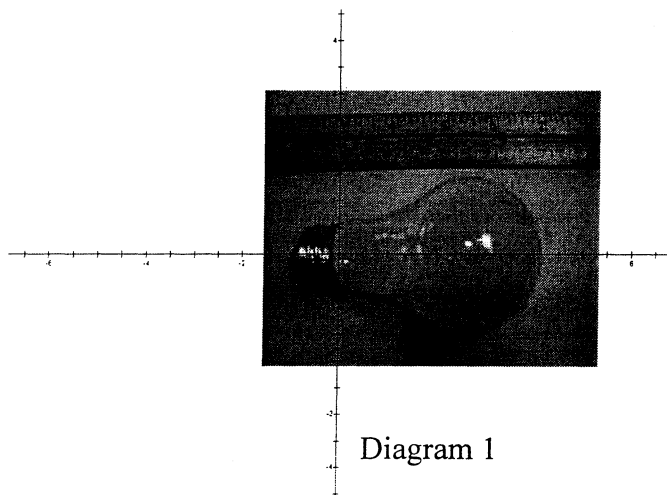
## MATHEMATICAL MODELING IN A SECONDARY MATHEMATICS PRESERVICE COURSE

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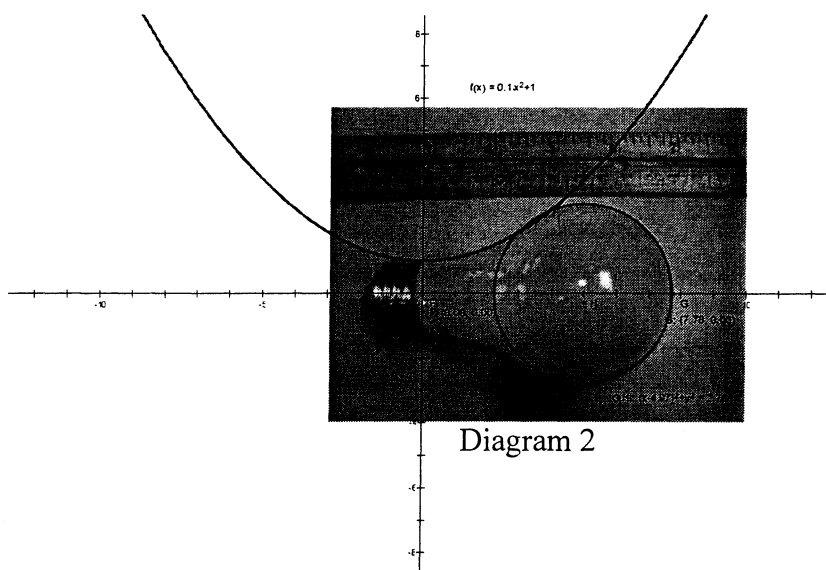
Northern Michigan University is a comprehensive state supported institution of approximately 8,500 students and 300 faculty located in the city of Marquette on Lake Superior. NMU has been designated as an IBM "Thinkpad" University where every student is issued an IBM Thinkpad laptop for their course-work. The Thinkpad initiative provides students and teachers with easy access to technology for the classroom. This easy access in turn, affords classroom opportunities to explore various technologies appropriate for the classroom. This presentation investigated the use of *The Geometer's Sketchpad* in a mathematical modeling course to develop piecewise continuous functions which model the contour of a standard light bulb. These equations were then used to find the volume of the light bulb.

The mathematical modeling course at NMU is primarily taken by preservice secondary students and is focused on understanding mathematical models these students may encounter in their teaching careers. The course is divided into three units, with one of the units focused on calculus concepts. Within this unit, students are required to give one presentation incorporating the use of *Sketchpad*. The following presentation is a description of one such student activity.

Activity: Students were given a jpeg (Diagram 1) in *Sketchpad* of a standard light bulb and were asked to consider finding the volume of the light bulb.



To find the volume using integration techniques, it was necessary to derive equations which model the contour of the light bulb. This particular student chose to use *Sketchpad* to derive the equations for a parabola and a circle (Diagram 2).



One process in *Sketchpad* to derive the equation for the parabola is as follows:

1. Under "Graph" menu, choose plot new function.
2. Type in a reasonable guess in the form of  $y = a(x-h)^2-k$ . Based on our axis and picture a reasonable guess may be  $y = 1(x-.5)^2+1$ .
3. Assuming this is not an exact match, manipulate parameters  $a$ ,  $h$ , and  $k$  until your equation models the 1st portion of the light bulb. The dynamic nature of *Sketchpad* allows for interactive graphing. When these parameters are changed, the graph of the equation will change accordingly.
4. For our purposes,  $y = 0.1x^2 + 1$  models the section of the light bulb we are interested in.

One process in *Sketchpad* to derive the equation of a circle is as follows:

1. With the "point" tool, estimate the center of the circle.
2. With this point selected, select a second point on the parabola close to where the circle and parabola will intersect (inflection point).
3. Manipulate (with the pointer tool) the center and radius of the circle until the circle adequately models the 2nd portion of the light bulb.
4. Once this is done, select the center and a point on the circle. Under the "measure" menu, choose "equation". *Sketchpad* will display your desired equation.
5. For our purposes,  $(x - 4.97)^2 + y^2 = 2.76^2$  models the section of the light bulb we are interested in.

To calculate the volume, it is necessary to define the coordinates of the following three points: (i) the left endpoint, (ii) point of intersection, and (iii) the right endpoint. These points are as follows in their respective order (0, 1), (2.92894, 1.85787), and (4, 0).

Using these points, the following equation yields our desired result.

$$\begin{aligned} \text{Volume} &= \int_0^{2.92894} \pi (.1x^2 + 1)^2 dx + \int_{2.92894}^4 \pi \left( \sqrt{-(x - 4.97)^2 + 2.76^2} \right)^2 dx \\ &\approx 15.8184 + 16.5861 = 32.4045 \text{ cubic inches} \end{aligned}$$