THE DESIGN OF *SILENT TUTOR* HOMEWORK HINTS—
A TECHNOLOGICAL TEACHING ASSISTANT

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Overview: The Tools for Enriching Calculus (TEC) CD-ROM has been developed for use as a companion tool to a major calculus textbook. In addition to a collection of mathlets (interactive JAVA™-based applets) to explore important mathematical concepts in a point-and-click laboratory environment, a central component for each section of the text is structured homework hints for key exercises. These homework hints are designed to play the role of a "silent tutor" in guiding students toward constructing a solution without giving the actual answer. In this paper we discuss how looking carefully at student learning helped to shape the mathematical and technical issues in developing this technological teaching assistant, and the broader development issues of providing homework hints that engage the students in active learning while helping them to solve exercises. The general design issues and the approaches of the making of TEC are presented in "Design Issues in Using Electronic Media to Improve Learning," (Keynes and Olson, 2003),[1] and "Pedagogical and Mathematical Issues in Using Technology," (Keynes and Olson, 2003).[2]

Description: The "silent tutor" provides a carefully chosen collection of solution hints for five to eight representative exercises in each section of the companion text. Trying to emulate an experienced teaching assistant (TA) or tutor who provides sequential hints by asking questions, this technological TA *asks* questions which incrementally give more information as the hints for each exercise progress. The approach used to guide the student in developing a solution is generally helpful in similar types of exercises, and each question attempts to incrementally give slightly more specific hints by suggesting a technique or clarifying a key point or procedure that lead the student toward the solution. The hints are generally coupled to the complete solutions given in the student solutions manual, and are designed to help to reinforce understanding of the concepts necessary to solve the exercises rather than simply accepting the steps necessary to get the solution.

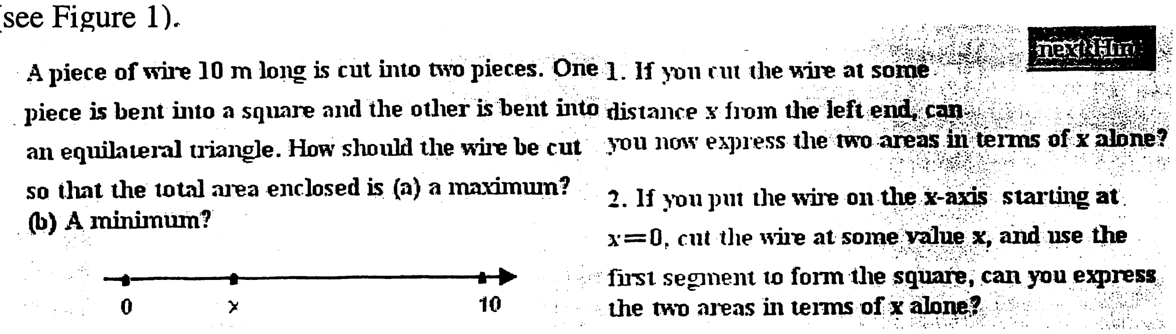
Design Features: The overall layout and navigation foster easy usage to ensure that the student will focus on the learning the mathematics rather then the technology. The exercise statement together with any graphics is always given on the left side of the screen. Thus, the student does not need to refer to the text when solving the exercise. On the right side of the screen, homework hints, which are given in sequential order, do not appear until the student clicks on the *Next Hint* button. After the last hint is selected, the student has the option of clicking on the *Reset* mode of this button to start the sequence over. As the student clicks through the sequence of questions, prior hints remain on the screen, enabling the student to reflect on the connections between the hints. To assist the student with focusing on the most current hint, its text is a brighter color that the rest of the series.

Students are directed to actively develop the ideas behind each hint with pencil and paper and to fill in many of the details and computations. The sequential design of the navigation allows students to focus on each hint, and to exit at any point in the sequence when they have obtained enough hints to solve the exercise on their own. Those who need just a single general clue can exit after just one general hint, while others can exit after a couple of hints or use the full compliment of three to five hints. The methods used for finding the solution emphasize general approaches rather than idiosyncratic techniques, and are frequently useful in solving similar exercises in the text. The hints attempt to clarify only key points or procedures, and generally require additional student effort to come up with a complete solution. This design allows the 'silent tutor' to meet its two primary objectives: 1) to encourage the student to successfully solve the exercises using techniques that will be useful in solving similar types of exercises, and 2) to provide enough knowledge so that the important, underlying concepts of the processes presented in the student solution manual will be better understood by the student.

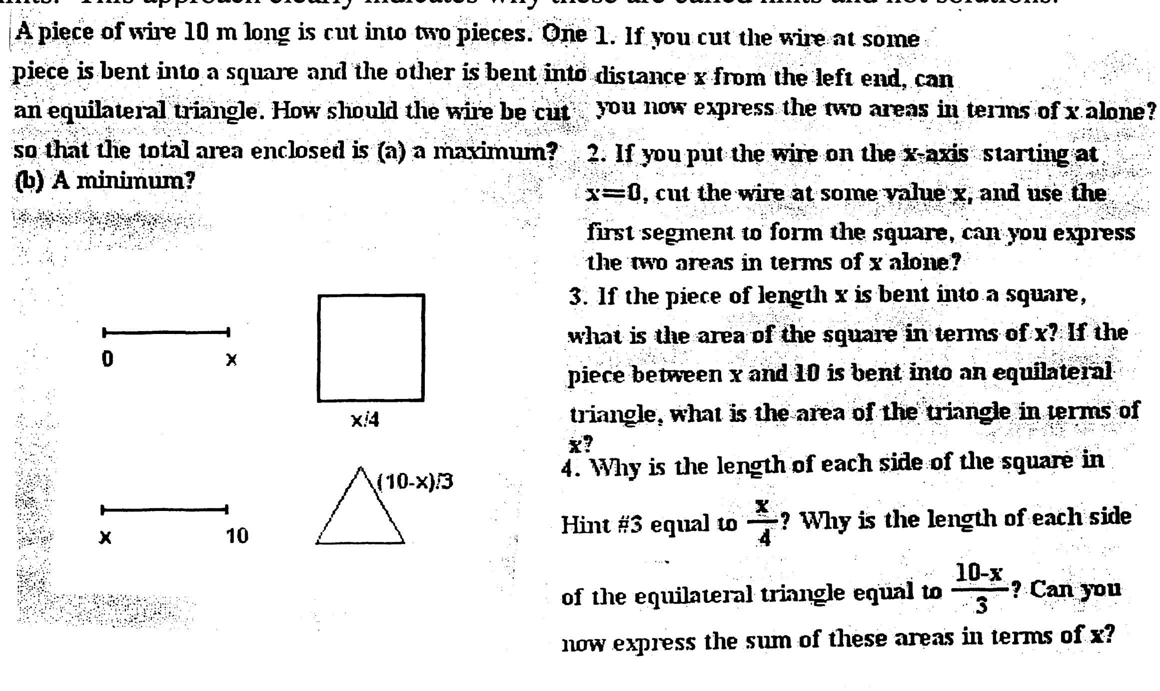
TEC's overall approach encourages students who have already solved the exercises to use the hints as a way to check their methods and understanding, and as a way to increase their problem solving abilities. In some of the exercises, the best approach to providing hints involves additional graphic information as part of the questions. To present the graphic information incrementally the graphics' display is updated as subsequent hints appear. The graphics are displayed on the left side of the screen directly under the problem statement so that they do not interfere with the sequential presentation of the hints. Finally, global navigation buttons are provided on the upper right corner of the screen to provide easy access to the prior exercise, next exercise, or the entire list of exercises within a section, or to return to the main menu for all sections.

Development Issues: In developing TEC in general and the homework hints in particular, we needed to balance several features. We wanted the homework hints to be useful in helping students to learn mathematics and to solve most types of homework exercises even if their instructor's interest in the use of technology was minimal. On the other hand, we wanted to provide homework hints for some of the more challenging geometric and conceptual problems as well as the standard computational problems. We wanted faculty to value these homework hints as learning tools and not just electronic step-by-step solutions to an exercise in order to encourage them to support students' use of TEC. Thus, we needed to provide some hints to straightforward problems that would be assigned in virtually all calculus classes as well as hints for some problems usually reserved for stronger students. The following principles were used in developing hints for all levels of problems: 1) the initial hint is always very general and does little more than start the student thinking about the way to get to a solution, and 2) subsequent hints do not give the next step in a solution, but *ask* why a certain function or integral might be useful to consider in working toward the solution, suggest the assignment of a variable or ask if a certain computational technique, or some geometric principle might be useful. This method is in contrast to other types of electronic homework hints or solutions, which frequently parse out step-by-step complete solutions in an electronic version that is very similar to printed solution manuals. In this step-by-step approach, students are not required to actively participate in developing any aspects of the solution or to consider its underlying concepts or its connections to similar techniques. Typically, they simply read or copy information off the screen. In the TEC homework hints, each hint requires further thinking or pencil-and-paper development for the subsequent hints to be helpful. Reading the last hint without making some progress toward understanding during the prior hints will generally not help the student to find a solution. If the student is still unable to complete a solution after using all of the hints, the effort is still valuable when reading a complete solution in the printed manual. Working on the hints will usually provide a better understanding of the steps in the complete solution guide, and help the student to understand the approach as well as the answer.

Examples: A good example of the use of graphics is provided by the following problem (see Figure 1).

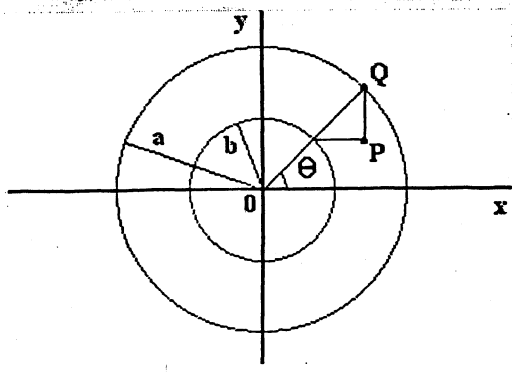


One important feature is the assignment of the length variable x, which is illustrated in hint #2. A second feature is finding equations for the square and equilateral triangle, which is illustrated in hint #4 (see Fig. 2). Note that the hints end after helping the student find these equations. The actual mechanics of finding when their sum is a minimum or maximum is viewed as a straightforward computation not covered by the homework hints. This approach clearly indicates why these are called hints and not solutions.



Another interesting geometric problem (see Figure 3) involves curve generation and parametric equations.

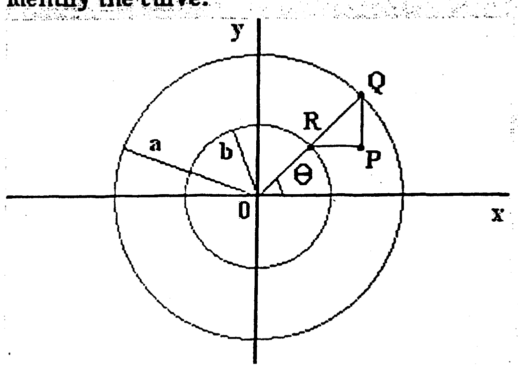
If a and b are fixed numbers, find parametric equations for the set of all points P determined as shown in the figure, using the angle Θ as the parameter. Then eliminate the parameter and identify the curve.



1. If Q is a point on the circle of radius a (see left), what are the x and y-coordinates of Q in terms of Θ and a?

Here the key strategy is to identify the coordinates of the intersection points of the two circles involved. Hints #1 and #2 (see Figure 3) suggest finding these coordinates and the other hints suggest using a trigonometric identity to eliminate the parameters (see Fig. 4). Once again, the non-standard use of the identity $\sin^2\theta + \cos^2\theta = 1$ is not covered by the hints, but left for the student to develop. Thus, this problem could still be part of a homework assignment even with the hints.

If a and b are fixed numbers, find parametric equations for the set of all points P determined as shown in the figure, using the angle Θ as the parameter. Then eliminate the parameter and identify the curve.



1. If Q is a point on the circle of radius a (see left), what are the x and y-coordinates of Q in terms of Θ and a?

2. If R is a point on the circle of radius b (see left), what are the x and y-coordinates of R in terms of Θ and b?

3. How can you write the x-coordinate of P in terms of the x-coordinates of Q and R? How can you write the y-coordinate of P?

4. Can you use a trigonometric identity relating sine and cosine to eliminate the parameter Θ?

An example of a more conceptual problem requires solving an integral equation (see Figure 5).

Find a function f and a number a such that

$$6+\int_a^x \frac{f(t)}{t^2}\,dt=2\sqrt{x} \text{ for all } x>0.$$

1. How can you get an equation for f(t) which does not involve definite integrals?

2. What happens if you differentiate both sides of the original equation?

3. How does differentiating both sides of the original equation lead to the equation $\frac{f(x)}{x^2}=\frac{1}{\sqrt{x}}$? Does this help you to compute $\int_a^x \frac{f(t)}{t^2}\,dt$?

4. What is f(t)? Why can you replace $\frac{f(t)}{t^2}$ with an expression not involving f(t)? If you recognize that $\frac{d}{dx}(2\sqrt{x})=\frac{1}{\sqrt{x}}$, can you now determine a?

While the approach of differentiating both sides is stressed, the actual integration is not discussed in the hints, nor is the method to determine the number *a*. Again, there is ample work for the student to complete even when using the homework hints.

Concluding Comments: The idea of using "silent tutor" homework hints has been generally well received, especially by faculty. This same approach has been used in several other calculus texts written by the same author, and hints have been developed for multivariable calculus problems as well as single variable calculus. While these hints are not always easy to develop, they appear to be useful as a general approach in many mathematics courses.

References:

1. H. Keynes, A. Olson, "The Making of the Tools for Enriching Calculus (TEC) CD-ROM: Design Issues in Using Electronic Media to Improve Learning," *ICTCM Proceedings: 14th Annual International Conference on Technology in Collegiate Mathematics*. Pearson Education, Inc., 2003, p.127-132.

2. H. Keynes, A. Olson, "The Making of the Tools for Enriching Calculus (TEC) CD-ROM: Pedagogical and Mathematical Issues in Using Technology," *Quaestiones Mathematicae: Journal of the South African Mathematical Society*, 2001, no. 1: 169-176.