

MORE STRUCTURED MAPLE PROJECTS FOR DIFFERENTIAL EQUATIONS

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BACKGROUND – THE GRANT

In 1999 the author and a colleague (M. Dillon) received a grant from the University System of Georgia to develop and implement a course in differential equations which would use technology—including, but not limited to, the TI-89 calculator and other computer algebra systems—in an essential way. This grant covered release time, and some equipment and software for personal use. In addition, enough funds were obtained to equip the Mathematics Department at Southern Polytechnic with a destination system room (computer, two large monitors, speakers, software, and Internet access). As outlined in the grant proposal, each of the authors taught a “traditional” section of differential equations in the Fall of 1999 and team-taught the technology intensive section during Spring Semester, 2000. The grant called for the authors to make full use of multimedia presentations, the Web, the TI-89 and other computer algebra systems in classroom presentations.

LESSONS LEARNED

The team-teaching aspect of the course went quite well. The author’s skills sets complemented each other nicely with Professor Fadyn taking the lead with the TI-89 and Maple and Dr. Dillon taking the lead with Power Point presentations and web-related issues. It was clear that the TI-89, although a valuable tool, was somewhat underpowered for this course and required considerable supplementing via user-defined functions and programs. It was also clear that a good source of projects which did not require students to have a substantial knowledge of Maple was needed. In addition, these projects needed to be original, since many projects out there are quite “well-worn” and indeed in a number of cases, solutions can be found published to the web and accessible to students.

FACULTY DEVELOPMENT AT GEORGIA INSTITUTE OF TECHNOLOGY

The author was accepted into the Faculty Development Program at Georgia Tech for the Fall Semester 2000 and the Spring Semester 2001. My proposal was to write a number of structured Maple projects for differential equations. These projects would be “structured” in the sense that they would be written in the form of a Maple worksheet which would guide the student through completing the project. This guidance would allow for a minimal knowledge of Maple by the student since in many (although not all) cases, examples of the Maple syntax would be blended into the presentation and would precede the questions asked in the project. Other questions asked in these projects might, for example, produce a plot or

an animation and require detailed explanations of these by the student. The idea was that students should learn some Maple, but not be overburdened by the Maple syntax since this would certainly take away from the main idea of using the technology to aid in understanding the mathematics.

BEYOND GEORGIA TECH

Using my experience at Georgia Tech as a springboard, I have written a number of additional Maple projects for differential equations. The projects were written in Maple V release 5 mainly because this is the version of Maple which is accessible to students at my home institution. However, for purposes of this talk, I have updated these projects to Maple 8 as well. Thus, the disk which I will be distributing (described below) will be available in two versions: Maple V, release 5, and Maple 8.

THE DISK

During my presentation at ICTCMXV, I will be distributing a disk which contains a set of nine structured Maple projects for differential equations, as well as complete solutions to each project. Although I retain the copyright to this material, I hope that those who receive the material will be able to use some of it in their differential equations classes. I would ask, however, that you do not provide students copies of the project solutions or publish any of these solutions to the web. A brief description of each of the projects follows:

1. FALLING BODIES--PART I: To obtain a realistic model of vertical motion under the influence of gravity we must, of course, take air resistance into account. If we designate air resistance by $F[r]$; then the appropriate differential equation has the form: $m \cdot \text{diff}(v(t), t) = F[g] + F[r]$; . In many cases it is appropriate to assume that $F[r]$; has the form: $F[r] = k \cdot v^p$; , where $1 \leq p \leq 2$, and k depends on the surface area and shape of the object and the density of the air. In many cases it can be assumed that $p=1$ or $p=2$ and we obtain a reasonably accurate model which has the advantage that the corresponding differential equations are solvable relatively easily in terms of elementary functions. Indeed, according to Borrelli, resistance proportional to v (called viscous damping-- $p=1$) is used for bodies of low density and extended rough surface (eg. a feather or a snowflake) whereas resistance proportional to v^2 (called Newtonian damping-- $p=2$) is used when a relatively dense body (eg. a raindrop, baseball or meteor) falls through the air. In this project we consider only the case of viscous damping. In another project (Part II) we look at Newtonian damping.
2. FALLING BODIES--PART II: Considers objects which fall under the influence of gravity and are subjected to Newtonian damping--that is, resistance proportional to the square of the object's instantaneous velocity (see 1 above). Such damping is appropriate when a relatively dense body (for example, a raindrop, baseball or meteor) falls through the air. This project is completely independent of Falling Bodies Part I.
3. PURE RESONANCE: Discusses the concept of pure resonance in a simple mass-spring

system. We consider a simple mass-spring system which is set into a back and forth motion by stretching or compressing the spring from equilibrium and perhaps giving the mass an initial velocity to the right (positive) or to the left (negative). In addition, we will assume that an outside force acts on this system--that is, the motion is "forced". Further we assume that the "forcing function" has the form $F(t) = F[0] \cdot \cos(\omega t)$. The case of pure resonance requires that the natural frequency of the system (usually designated by ω_0) be equal to that of the forcing function (that is, ω), and that there be no resistance in the system. This case results in a solution function whose amplitude increases to infinity. This is the case which we consider in this project.

4. **PRACTICAL RESONANCE:** Discusses the concept of practical resonance in a simple, damped, mass-spring system. See also 3 above. In practice, there is always some damping in the system, so that "pure resonance" cannot occur. In the case of "practical resonance", we assume that some damping is present. Although in this case it is not possible for the amplitude to increase in an unbounded fashion, large and destructive amplitudes can occur when the frequency of the system and that of the forcing function bear a certain relationship to one another. This project is completely independent.

5. **PURE AND PRACTICAL RESONANCE:** As we indicated 4, since some damping is always present, it is not possible for the amplitude to increase in an unbounded fashion as is the situation in "pure resonance". Even so, large and potentially destructive amplitudes can occur when the frequency of the system and that of the forcing function bear a certain relationship to one another. One of our objectives here is to uncover this relationship between the frequencies which is said to yield "practical resonance". The elementary ideas in the cases of pure and practical resonance are considered in separate projects in this series (see 3 and 4 above). In this project we review these ideas briefly and then look at some deeper questions involving practical resonance. This project is completely independent.

6. **TANK TIME PROJECT I:** We discuss some problems related to a cascade system of three brine tanks. This investigation leads immediately to a system of first-order linear equations. The theory of linear first-order systems and Maple provide us with a number of ways to solve these systems. One objective of this project is to show students how to apply some of these methods using Maple. We will actually use five different methods of solving the appropriate system of differential equations. The basic problem which will consider can be found in the text *Differential Equations and Linear Algebra* by Edwards and Penney as Example 2 on page 493; however, we state the problem in a more general form. First we discuss various ways to use Maple to help us solve the basic problem as it appears in Edwards and Penney. Later, we try to answer some deeper questions about a more general form of the problem using Maple to help us. The focus of the latter part of this project is obtaining a complete answer to the following question: at any given time $t_0 > 0$ and any salt content c_0 what conditions on t_0 and c_0 are necessary to make it possible for all three tanks have content c_0 at time t_0 by appropriate choices for the initial salt contents a , b , and c of the three tanks?

7. **TANK TIME PROJECT II:** As in Tank Time Project I (see 6 above), we consider a

cascade of three brine tanks. To be more specific, we consider the following problem: Consider a system of brine tanks containing $V[1]$, $V[2]$, and $V[3]$ gallons of brine, respectively. A brine solution containing k pounds of salt per gallon flows into tank 1 at a rate of r gallons per minute, while mixed brine flows from tank 1 into tank 2, then from tank 2 into tank 3, and then out of tank 3, all at the same flow rate of r gallons per minute. Assume initially that tanks 1, 2 and 3 contain a , b , and c pounds of salt respectively. Let $x[i](t)$ represent the amount (in pounds) of salt in tank i at time t , for $i = 1, 2, 3$. Find $x[i](t)$, for $i = 1, 2, 3$. In this project, we fix the initial salt contents a , b , and c of the three tanks and the flow rate r . We allow k , $V[1]$, $V[2]$, and $V[3]$ to vary and attempt to find conditions on these four variables which will give us 50 lb of salt in tank 1 after 1 minute, 100 lb of salt in tank 2 after 2 minutes and 150 lb of salt in tank 3 after 3 minutes. This project is completely independent of the project Tank Time Project I.

8. MECHANICAL VIBRATIONS -- THE CASE OF FREE UNDERDAMPED MOTION

I: A detailed analysis of the free, underdamped motion of a mass-spring system. We consider a simple mass-spring system which is set into a back and forth motion by stretching or compressing the spring from equilibrium and perhaps giving the mass an initial velocity to the right (positive) or to the left (negative). We will assume that no outside forces act on this system--that is, the motion is "free". This project develops general formulas for the pseudoperiod, times the mass passes through equilibrium, the time between successive extrema, and the familiar "cosine" form of the solution. Some analysis of three-dimensional surfaces is required by the student.

9. MECHANICAL VIBRATIONS -- THE CASE OF FREE UNDERDAMPED MOTION

II: Refer to 8 above for the basic problem of free underdamped motion of a mass-spring system. In Part I we developed formulas for the solution of the free underdamped motion of a mass on a spring and various quantities related to this motion. In this project (which is completely self-contained) we continue this investigation. However, in this project, many of the required formulas from Part I will be stated without proof or derivation. If you require more details about these formulas, you might want to refer to Part I. However, this project is completely independent of Part I. After some preliminaries and an example, our main focus in this project is to investigate the effect of holding all parameters in the solution constant except one. We will concentrate on the three cases of: (i) varying the mass m ; (ii) varying the damping coefficient c ; (iii) varying the spring constant k .

THE WEB SITE

The web site given below contains a sample of the differential equations Maple projects as well as some additional Maple worksheets. Please feel free to access this material and to use whatever you find appropriate in your own classes. The copyright on this material is, however, retained by myself.

<http://www2.SPSU.edu/math/fadyn/index.html>

The material can also be found by beginning at the Southern Polytechnic State University home page and then navigating to Professor Fadyn's home page. The home page for Southern Polytechnic State University is located at: <http://www.spsu.edu/>