BLACK BOX AND WHITE BOX CAS IN CALCULUS

J. Douglas Child
Department of Mathematical Sciences
Rollins College
Winter Park. FL 32789
child@rollins.edu

Computer Algebra Systems Provide Many Options for Learning Calculus

In the days of calculus reform, I hoped the mathematics community would make a clear statement about the focus of calculus learning. It seems that today's popular calculus books still stress algebra skills and by-hand symbolic calculations. What of all the talk about the importance of learning concepts, problems solving, and the development of careful, correct solutions to problems? I use a text whose title contains "concepts and contexts". But, in practice what students actually do is largely algebra and by-hand symbolic calculation. Little student learning energy is left for concept development or learning how to use theorems to solve problems. My impression is that very few instructors make use of powerful computer algebra systems even though they have been available for many years. If a student is actually going to use calculus to solve non-trivial problems, shouldn't the student know how to use appropriate tools to help him obtain correct solutions? This article discusses some of the ways I've used TI-92 Plus calculators in the last few years. The most frustrating fact is that my students must fit into the current world of calculus instruction and use. To be fair to my students I must still teach by-hand symbolic calculation and, therefore, underlying algebra and trigonometry.

TI-92 Plus, TI-89, and TI Voyage 200 calculators provide a collection of tools that instructors can use to focus on the aspects of calculus they believe to be the most important. The original TI-92 came with a capable Home screen computer algebra system which performs calculations and provides answers. This type of computer algebra system is usually called a "black box" CAS. For the last several years, Texas Instruments has been developing a "white box" CAS, The Symbolic Math Guide(SMG), that students control to develop correct step-by-step solutions to symbolic calculation problems. SMG also serves as an expert system that helps students look-up practical information about what they can legally do when solving a particular symbolic calculation problem.

To begin consider several ways a black box CAS might be used when learning about derivatives and their uses.

Derivative as a tool for solving problems

A black box CAS is very useful for getting information needed to solve optimization problems, informed graphing problems, and other problems that often require lots of symbolic calculation. Consider optimization problems. An outline of the work involved might be: obtain the model (use a constraint if appropriate), differentiate, find the zeros (and other critical points), determine the optimal value using suitable theorems and tools, and corroborate with graphs and other theorems.

Script for finding the maximum/minimum values of a function differentiable on a closed interval

A script on a TI-92 can streamline the process of getting basic information. To use the script below, edit the function definition and the values for xmin and xmax. Then execute each line to generate the basic information. Pay attention because you may need to edit other lines.

```
C:NewProb
 :Optimize differentiable function
on a closed interval [xmin,xmax]
C:Define f(x)=(x^2-x+1)/(x^2+1)
 :Specify the interval
C:0→xmin
C:5→xmax
 :Find critical numbers
C:d(f(x),x)
C:ans(1)→df(x)
C:zeros(df(x),x)
 :Choose the optimal value
C:f({1, xmin,xmax })
 :Check results
C:Graph f(x)
C:zoomfit
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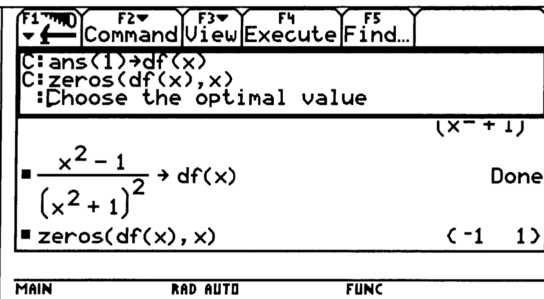
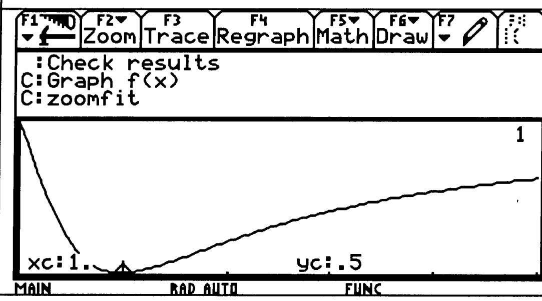


Students make a conclusion about the solution, reference the theorem they use as well as its hypotheses.

Completed solutions contain supporting graphs and sign charts.



A student can easily modify the script to find additional information for application of other theorems.

There are different methods of obtaining information from a black box CAS. Students can type commands one by one (e.g. use the Home screen CAS of a TI –Voyage), students can edit and execute a script, or students can execute a program that produces all of the information. The author's preference is to have students enter commands for several problems then to save a script for remaining problems. With scripts students remain in control, can make changes on the fly, and use the same commands they use on the Home screen.

Teaching options for optimization problems

1. To focus on translating word problems to mathematical models, use graphing to obtain the optimal solution.

2. To focus on how calculus applies to solving optimization problems, let the black box CAS generate all of the information about derivatives, zeros, and values of the function necessary to locate and prove the result. Require the student to correctly use theorems to make conclusions about the solution. If you let (require) the student generate the information by-hand, he may make a mistake and attempt to reason with incorrect information. For even slightly complicated functions, when the student is finished obtaining the necessary information after translating from words to math model, he has little energy left to think about theorems or to corroborate his solution. A middle position would be to require students to be able to complete the process for simple functions and to be able to use a black box CAS for more difficult problems. In either case each solution should be corroborated.

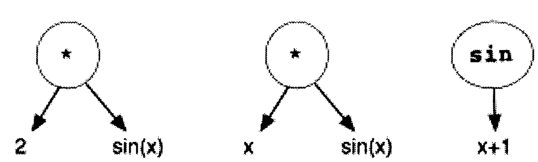
Learning to differentiate

Given the state of calculus education today, we must teach our students to compute derivatives by-hand. Most mathematics departments teach it and our Physics Department does not allow the use of TI-92 Plus calculators for tests and even for homework. The Symbolic Math Guide, SMG, is designed to speed-up the process. It is especially useful for our classes because about half of the students already know how to differentiate and SMG can serve as a patient learning assistant for those who need extra help.

Learning to differentiate consists of learning derivatives of basic functions and learning to apply derivative rules to various kinds of functions constructed from basic functions. Students often have difficulty properly selecting and using the scalar product rule, the product rule, and the chain rule. The author uses derivative structure trees to help students see the structural differences among scalar products, products, and compositions so they can chose appropriate rules and SMG to help students learn to apply the chain rule properly.

Derivative Structure Trees

It's useful to have a representation for mathematical expressions that unambiguously displays the structure of expressions. Structure trees show the structure by putting basic functions such as polynomials, log(x), sin(x),.... in leaves and operators and function applications in non-leaf nodes. The three trees below represent 2*sin(x), x*sin(x), and sin(x+1). Structure trees are useful for determining which students are not properly classifying expressions and for helping them to learn to choose suitable derivative rules.

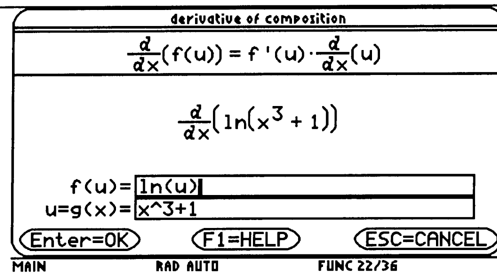


Using SMG to help students apply derivative rules

Consider the problem of differentiating ln(x^3+1) wrt. x.

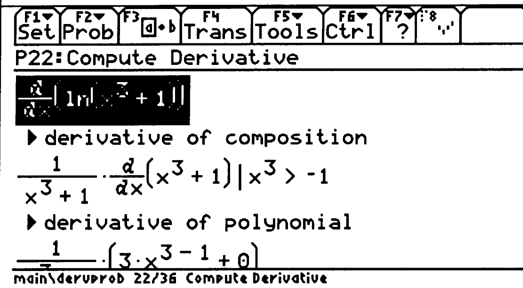
After a student decides to use the chain rule, SMG displays the dialog box on the right.

The student enters definitions for f(u) and g(x). If the student doesn't know what to do, he can ask for help. SMG will correctly fill-in the missing information.



Note that SMG correctly produces the derivative with a constraint that makes the expressions equivalent.

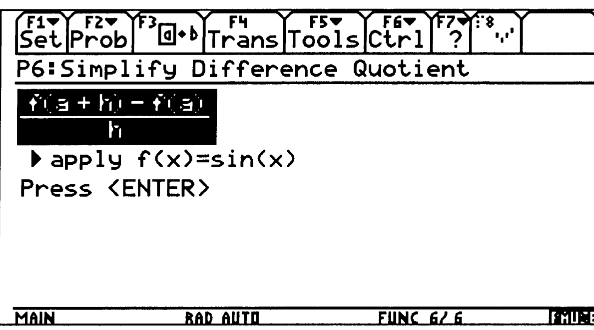
What if a student tries to apply the product rule to this problem? Once the student asks for help he receives the message "No help available. The product rule may not be useful for this derivative."



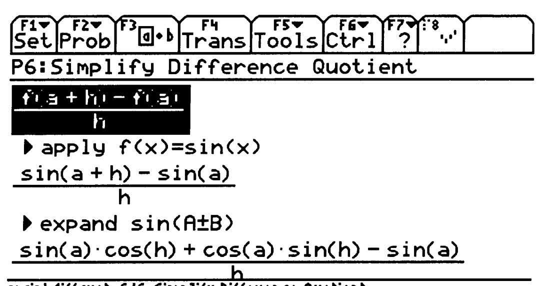
Definition of derivative

I still want students to know how to use a version of the definition of derivative. This can be accomplished without seeing the details by using scripts. If you want students to know how to do the algebra, Symbolic Math Guide may be useful.

Address student issues such as function notation. What does f(a+h) mean for various functions such as constant functions,....



SMG helps students fill-in and reinforce trigonometric and algebraic background.



Using SMG as an Expert System for Symbolic Computation

Students often do not remember key facts that are needed while performing symbolic computations. SMG knows quite a bit about basic algebra, laws of exponents, trigonometric transformations, solving equations, factoring, computing derivatives, and finding indefinite integrals. It knows what steps are legal to take when solving a particular problem. Suppose a student is solving the equation ln(x+1)/(ln(x-1) = 2 for x.

The problem is to solve an equation for x.	P6: Solve for x $\frac{\ln(x+1)}{\ln(x-1)}=2$
Start solving by asking SMG what can be done. The full list of options is shown on the right. Since many students have imperfect knowledge of log properties, some will be surprised not to see a step involving ratios of logs.	SELECT TRANSFORMATION $\frac{\ln(x+1)}{\ln(x-1)}=2$ 1: add ? to each side 2: multiply each side by ? 3: switch sides 4: cross multiply 5: $\frac{\ln(A)}{B} \to \ln\left(A^{\frac{1}{B}}\right)$ MAIN RAD AUTO FUNC 6/11
After choosing to cross multiply and then a property of logs, the student sees how to get rid of the logs to obtain a simpler problem.	▸cross multiply $\ln(x+1)=\ln(x-1)\cdot 2$ |... ▸$U\cdot\ln(A) \to \ln(A\text{^}U)$ $\ln(x+1)=\ln\left((x-1)^2\right)$ |... ▸$\ln(A)=\ln(B) \to A=B$
The rest of the problem involves solving a quadratic equation. Once this is done, a student can ask SMG to verify the solution. In this problem, verification may save a mistake.	VERIFY SOLUTION $\frac{\ln(x+1)}{\ln(x-1)}=2$ x = 0 false x = 3 true MAIN RAD AUTO FUNC 6/11

A few questions to consider

Given the availability of low cost computer algebra systems, how important is by-hand calculation, especially for more difficult problems? How much calculus time do your students spend doing algebra, solving equations, computing derivatives, computing integrals? Can students learn more (of the calculus you want them to learn) if they can think about what to do and what mathematics applies without being distracted by low-level computational details? Which students should learn to use powerful tools like CAS? When?