DYNAMICAL SYSTEMS USING *MATHEMATICA*

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In this article, we would like to take the opportunity to expand some of the concepts in the original conference abstract and to report on the actual conduct of the course, which concluded on 19 December, 2002, with an oral examination of 14 students' structured projects submitted electronically in *Mathematica* notebook format.

AM14 course details

The course AM14, Dynamical Systems and Chaos, is compulsory for bachelor students of Applied Mathematics and optional for students in the International Masters program in Computer Systems Engineering. In its current format, it consists of 30 hours of lectures and 30 hours of tutorials, although this will change as a result of the experience with the 2002 edition. 16 students took the course, 2 in the Applied Mathematics program, 13 in the Computer Systems engineering program and 1 in the Mechatronic Engineering program which is based at our Sønderborg campus, which is about 200 Km by road from the Odense campus, where the course was held physically.

The course has run for the last 10 years, most recently using the book by Arrowsmith and Place as the course text. In practice, this has been supplanted by electronic material produced by the author, which the students have been able to download free of charge from the course home page. In future versions of the course, the home page will be replaced by the Blackboard learning management system.

The topics treated in the course are first order differential equations, non-linear and linear planar dynamical systems, limit cycles, non-planar dynamical systems and chaotic difference equations. The course presupposed freshman calculus and linear algebra, but students are recommended to have taken the course AM12 in Analytical Mechanics.

Until 2001, the course was evaluated on the basis of a compulsory project and a traditional written examination. In 2001 and 2002, the written examination was replaced by an oral examination of a take home examination project, which the students had about a week to complete. The oral examination, conducted by the teacher and an accredited external examiner (Dr. Poul Hjorth, Department of Mathematics, The Technical University of Denmark, Lyngby), had the main aims of ensuring that each student had worked independently and to give the high flyers a chance to distinguish themselves.

Course home page

All course information (news, learning material, examinations) was disseminated using a teacher maintained home page [1]. This was a very simple document, written in Microsoft Word and saved as a web page. Next year, this administration will be accomplished using the Blackboard learning management system.

Course modules

The theory of dynamical systems and some applications are in the form of 10 *Mathematica* notebooks, using the Classroom style sheet [2]. When printed out, they can be read as a traditional textbook. But as they contain *Mathematica* symbolic and graphical code fragments embedded in mathematical text and graphics, they are dynamical entities which can be changed by the reader to experiment with other scenarios. Navigation within the course modules is using hyperlinks from a table of contents to the section headers, resembling navigation in a hard copy textbook.

Toolboxes

The code fragments have been designed to be generically reusable provided the user follow certain protocols, which mean that the same tools can be used for analysing a wide variety of applications. The participants are encouraged to file these tools in an electronic toolbox from which they can be pasted into applications projects. These toolboxes can be in the form of a notebook, but we have experimented with storing them in palette buttons to allow even more rapid insertion into project reports. As these tools automate the technical procedures for analysing different kinds of dynamical systems, they render obsolete the original written examination, which mainly tested the ability of students to perform these operations competently under time pressure.

Structured projects

The students solved a number of these projects during the course. These included an analysis of a first order linear equation, the analysis of the Lottka-Volterra model of competing ecosystems, a graphical analysis of the limit cycle in the Van der Pol equation, the Holling-Tanner model, a more realistic version of the Lottka-Volterra system and an analysis of the isothermal and adiabatic motion of a piston enclosing a volume of ideal gas in a cylindrical pipe. The last two were the compulsory and examination projects.

Structured projects consist of 3 or more layers, of which only the top two are visible to the student. This is easily implemented using the *Mathematica* notebook folding editor. The top level is the project background, which describes the problem domain and establishes a notation. The next layer, also visible to the student, is the project summary, consisting of a list of the deliverables needed to complete it. These should be unambiguous so that the examiners can quickly check whether they are present in the report submitted by the student. The remaining layers consist of a folded model project report in which only the top level headings, corresponding to the deliverables, are visible. The student doing the project in a tutorial situation is able to open the folds to obtain clues as to how to proceed. In an examination situation, the folded report is accessible by the student on a hint-server, a piece of software developed by one of the authors as his masters thesis [3], which keeps track of how many clues have been opened by each student. In many ways, structured projects are an electronic implementation of a tutorial

Teaching methodologies

Lectures

The lectures were conducted with the teaching loading the appropriate learning module into the *Mathematica* technical computing environment running on a laptop. The screen image is projected using a data projector, with the teacher sitting down facing the audience. In all but one case, the students had had access to the learning module before the lecture. The teacher then uses the screen picture much the same as an overhead projector, executing the code fragments and highlighting text and equations using the mouse cursor. Experiments were also done with color-coding equations to make them easier to associate with the explanatory text. Acceptable quality sound is achieved using a cheap microphone. Another option is to use a webcam to film either the audience or the teacher.

The laptop was attached to the internet via a fast Ethernet connection. Contact was established with the student at the Sønderborg campus using Microsoft NetMeeting, either through a direct point to point connection or via a Multiple Conferencing Unit (MCU) maintained by the Danish Research Ministry. By using a head set and sharing the *Mathematica* application with NetMeeting, the dynamic screen picture was visible and the sound audible to the student in Sønderborg, who also had sound communication back to the main lecture site in Odense. NetMeeting also contains a chat utility and a shared whiteboard for exchanging drawings. NetMeeting is being replaced by Exchange, which has much the same functionality

This system proved to be pretty unstable, especially in the beginning, with communication going down randomly. As a backup, we started recording the sound and dynamic screen picture using the capture program Camtasia. In the event that the Internet had let us down, the recorded lecture was burnt onto a CD-rom (a 45 minute lecture requires about 120 Mb) and sent by internal post so that the student in Sønderborg had the recorded lecture by the next morning.

Because of the technical difficulties, the quality of these lectures was uneven, so they have not been widely distributed. In the next edition of the course, the lectures will be systematically recorded and made available to students from a streaming video server.

Tutorials

The tutorials turned out to be somewhat superfluous, as the structured projects were modelled on the events in a tutorial situation, so that the presence of a physical tutor was not required. A couple of experiments were performed , where the teacher and the remote student cooperated to generate an application report over the internet. The tutorials were held in a terminal room at the Maersk Institute containing about 30 Windows XP systems running *Mathematica* 4.2.

The participation at the tutorials was low, because the International Masters students had a lot of work in connection with their compulsory subjects. The 4 domestic students were all extremely good ones, so that the tutorials were used to explore the material more fully and how the teaching methodologies might be improved. In one case, one of these

The compulsory project

The students carried out a structured project, without hints, handing it in at the end of November. All but one student handed in a satisfactory project (one of the International Masters students experiencing overload). The project was quite a challenging on, to derive the differential equations for the Holling-Tanner model of competing ecosystems and showing that it exhibits a limit cycle. This model is a decoration of the Lottka-Volterra model, taking account of limits to growth of the prey, limits to the appetites of the predators and minimal nutritional requirements of the predators for survival. This model contains 6 parameters, so that the main technical task is to reduce the dimension of this parameter space in such a way that the symbolic calculations become manageable and the student can navigate in the parameter space to find a region where the model exhibits limit cycle behaviour. One student failed to complete the project and later withdrew from the examination.

The examination

The students were able to download the examination project at noon on Friday 13 December, and were required to deliver their report to the teacher by email before 0800 on Thursday 19 December. The hint server was available from 0900 on Wednesday 18 December. These were stored in a directory awaiting the arrival of the external examiner at 0930. The output from the hint server was available at this time, showing which examinees had opened which hints. The number of hints opened varied from 0 to 4. The first oral examination was scheduled for 1030. The first 5 minutes of each oral examination was devoted to scanning the report to check the presence of the deliverables. Points deserving closer scrutiny were highlighted in the electronic report. Each examinee's report was projected onto a screen, and the examinee was required to answer questions on their report. Two of the students had also sent in their electronic toolboxes. The external examiner took notes on the report and performance of each student during the oral examination which lasted about 15 minutes. The examination concluded at 1620, with the internal and external examiners negotiating a grade for each student. The oral examination of the student at Sønderborg was successfully carried out over the Internet with two way sound communication (the student actually achieved the top grade). The only piece of paper generated by the course was the examination protocol, provided in hard copy form by the examinations office and signed by the examiners. The distribution of grades was suitably peaked around the mean.

Perspectives and further experiments

Experienced teachers will recognise that the amount of time required to evaluate the projects and examine them was very much shorter than usual. This is encouraging, because although there is no doubt that project reports are a superior method of evaluating competence development compared with the traditional written examination, they are normally regarded as much more time consuming to grade. In the real world, a report with errors is worthless, so that the structured tutorial project with deliverables which are either correct or not, more resembles a realistic working situation than the written examination, which more tests the ability of students to carry out routine manipulations competently under time pressure, a situation which does not occur very much in the real world.

number of structured projects throughout the course would probably be the best one, and becomes feasible if the costs of grading them are as low as seems to be the case.

The ability to generate recorded lectures was a kind of bonus. In an extreme case, a teacher would only need to generate these lectures and allow the students to download and view them in their own time. Of course, the communication becomes entirely one way, which, unfortunately, is also the case with real time lectures to large classes. The real value of these is the opportunity for students to experience classes they have missed for more or less good reasons, or to repeat points they did not understand the first time. However, the distance learning potential is obvious.

This course was carefully chosen for the experiment, because it traditionally has a very high pass rate, so that a disaster was unlikely. In the next semester we will be conducting a trial with the course AM12 on Analytical and Computational Mathematics, which will be taught using the same methodology. This course is compulsory for all students of Applied Mathematics and Computer Systems Engineering, and optional for students of Mechatronic Engineering in Sønderborg, where it is anticipated that there will be a tutorial class (with the student from AM14 as a teaching assistant). The number of students will thus be around 80.

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