USING DERIVE TO HELP SURVIVE IN MULTIVARIATE CALCULUS

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Introduction

DERIVE is a user-friendly piece of software that students are able to use independently in a relatively short amount of time. At our institution, DERIVE is used for the entire calculus sequence and all students are required to do DERIVE laboratory experiments on their own or with a partner outside the classroom environment.

Students who start out with DERIVE in Calculus I quickly get the hang of it, and what they learn about DERIVE while in Calculus I carries over into Calculus II, where they are also required to do lab reports. However, when students get to Calculus III (=multivariate calculus), they have a hard time using DERIVE to do "stuff" unless further instruction is provided. In the absence of additional guidance, most students find that using DERIVE in a multivariable/3D context turns out to be considerably more challenging than using DERIVE in a single variable/2D context.

In this article we will take an ever so brief look at how to use DERIVE to help students visualize curves and surfaces in 3D. As the title indicates, I will focus on many of the commands and procedures that a student needs to know in order to work through assigned homework exercises or do laboratory experiments independently using DERIVE. In other words, how students can use DERIVE to help them survive in multivariate calculus. Many of the examples and instructions that follow are taken from the *Survivor's Guide* that I put together, update each semester, and hand out to the calculus III students at the beginning of the term. (The *Survivor's Guide* is an unpublished 40 page packet of DERIVE information, examples, lab report guidelines, and a sample lab with the key attached.) This is a guide that I prepared with my students in mind and hence, some of the language is informal and light since I am explaining "to my students" how to use DERIVE to help them get the result they are after. Nevertheless, I invite you to listen in!

Graphing in the 3D Window

To produce a graph in DERIVE you must "Author an expression and then do something with it". The *expression* may be a function or an equation or a vector or a number of other things, but you will always "Author first and then do something". At this point, the cursor should be in the box at the bottom (=entry line), so go ahead and type in an expression and then hit enter. It will appear on "the stack" and highlighted. See the Algebra window in the figure below, where I've Authored a position vector.

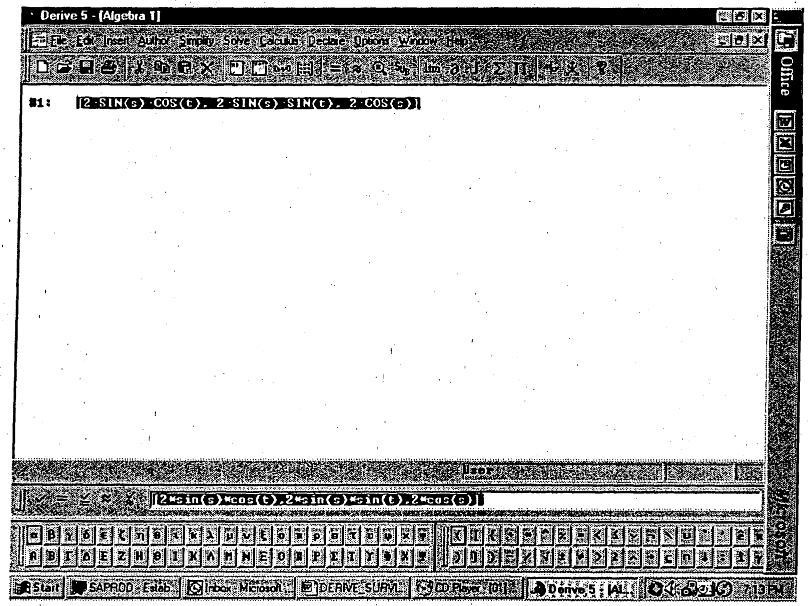

Figure 1: Authoring an Expression in the Algebra Window

Well, now that we Authored an expression (in this case a vector) I suppose we better *do something with it.* Since a lot of what we use DERIVE for in calculus III is to plot "stuff" for us, we will begin by having DERIVE plot the expression we just Authored. To this end, click on the button with the 3 axes (2[nd] from the right) to get into the 3D plotting window. Once you are in the 3D window, click on the 3D plot button to get the below graph, which should look like a sphere.

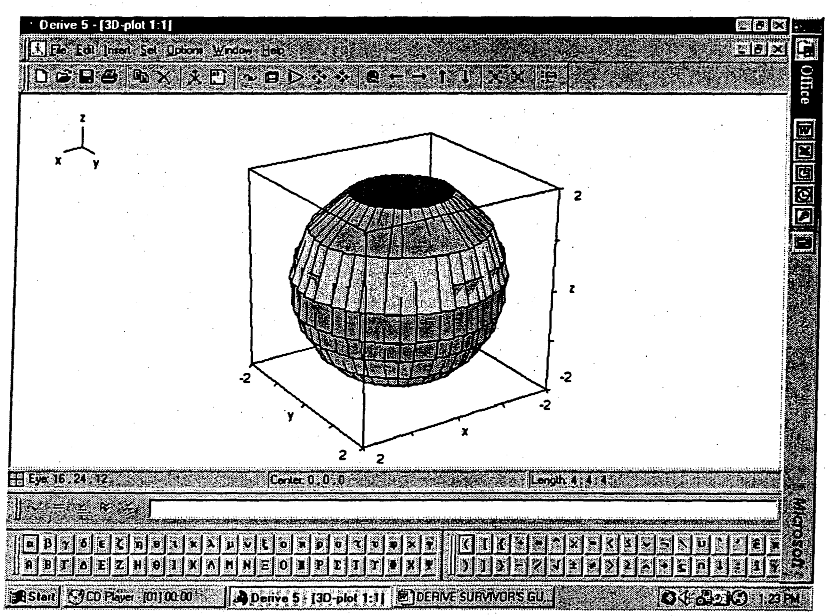

Figure 2: Graphing in the 3D Plotting Window

Ouch. This hurts my eyes! It's pretty pathetic looking for a sphere. We need do some major work here, namely, we need to "make it smooth". To do this, *right* click anywhere on the sphere and select Edit. This produces a panel in the middle of the window. The numerical values in all the boxes are the default values, which aren't working too well for us right now. So, change the Number of Panels from 20 to 40 (both of them) and change the values for s and t to run from 0 to pi and 0 to 2pi, respectively. Once you've done this, click OK and DERIVE will re-plot the surface.

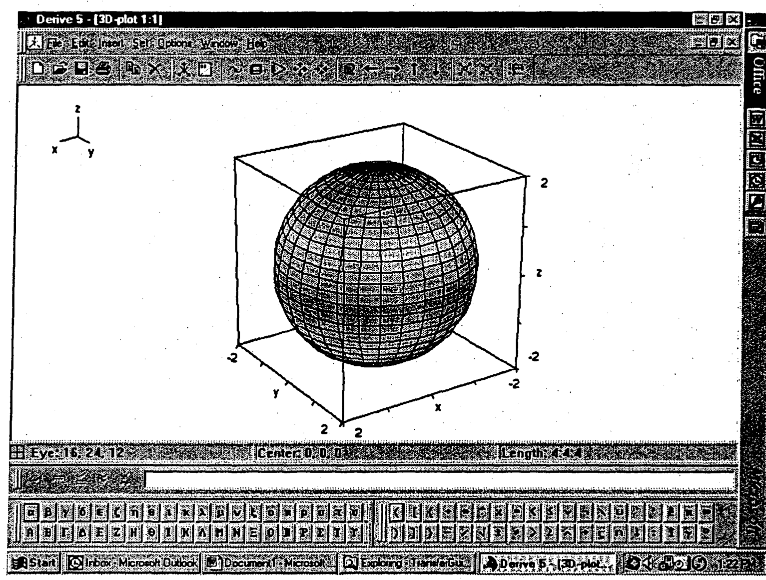

Figure 3: A "Nice" Sphere in the 3D Plotting Window

Notice the nice colors and how smooooooth it looks! You'll have to put that on a post card and send it home to mom!

<u>"Clipped" Surfaces</u>

When an expression is plotted, sometimes you get what is referred to as a "clipped" surface. For example, in the Algebra window, go to the entry line, Author the expression $z = x \wedge 2 - y \wedge 2$, and hit Enter. This expression should appear highlighted on the stack, so go to the 3D plot window and hit the plot button. This should produce the graph in Figure 4A below.

The graph in Figure 4A looks like it was "clipped", i.e. that part of it was "lopped off". If you want to get rid of the clipped look, use Set, Plot Range (in the 3D window) to change the range of the variables. For instance, I left x and y running from –5 to 5, but reset z to run from –25 to 25. The graph in Figure 4B below is the resulting graph, which does not have the "clipped" look like the graph in Figure 4A.

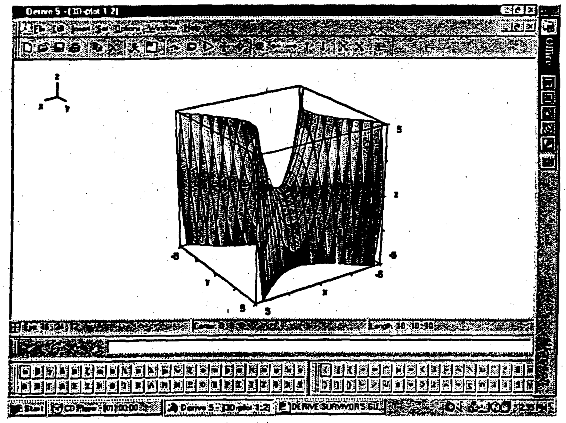

Figure 4A: Clipped Saddle

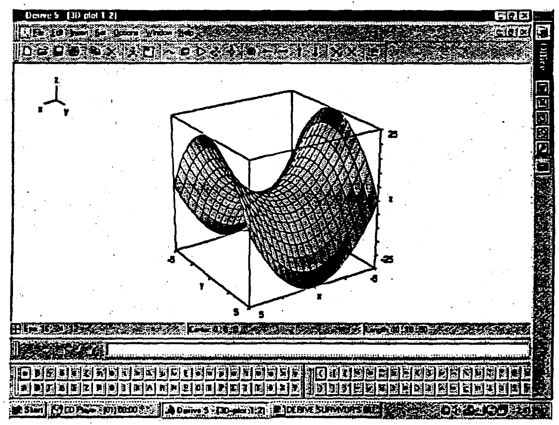

Figure 4B: Unclipped Saddle

Plotting Space Curves in 3D

To plot a space curve you must first Author the position vector and then plot it in the 3D window. For example, to plot the helix $x = \cos(t)$, $y = \sin(t)$, $z = t$ you would first Author $[\cos(t), \sin(t), t]$ and then plot it in the 3D plotting window. This yields the following graph in the 3D plotting window.

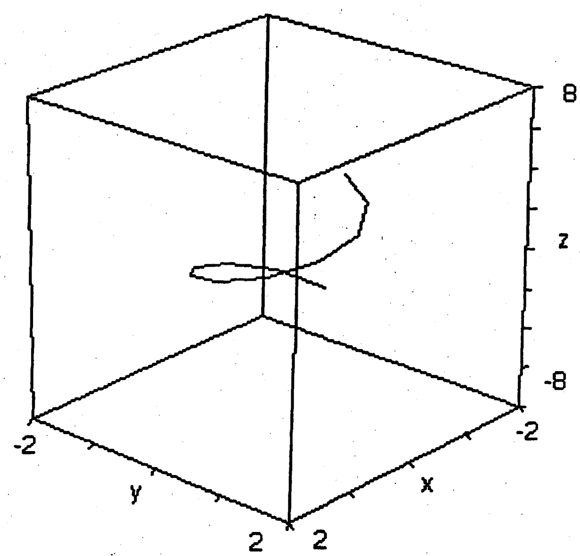

Figure 5: A Less Than Satisfying Helix

This is not the best-looking helix I've ever seen. It's kind of jagged and it's not all there! So, once again, we go to the Plot Parameters panel, BUT this time we get there by going to the Edit pull-down menu and selecting Plot. (Right clicking on the helix to get the Plot Parameters panel doesn't work.) Changing the t parameter to run from -2pi to 2pi and changing Number of Panels from 10 to 40 gives us a much better looking helix. See for yourself!

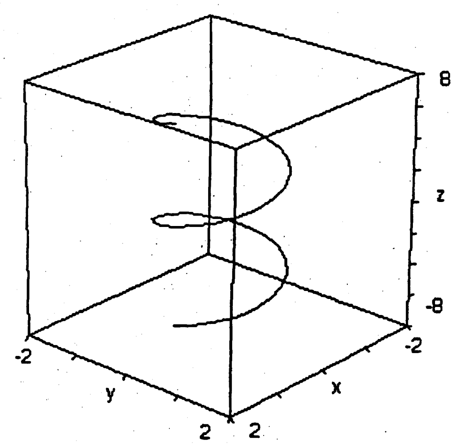

Figure 6: A Much More Satisfying Helix

Limitations of DERIVE

There are a few things that one might think DERIVE can do, and alas, it is not so. For instance, DERIVE cannot plot constant vectors in 3D. DERIVE can do vector algebra in the Algebra window, but if you try to plot a constant vector like [2, 3, 4], you end up getting a point plotted instead of a vector.

Another thing that DERIVE does not do in 3D is implicit plots. This is one of the difference between 2D and 3D plotting in DERIVE. For instance, if you Author x^2+y^2=1, go to the 2D window, and hit the plot button, you will get a nice looking circle. If we try to do the same kind of thing in 3D, DERIVE balks. For instance, if we Author x^2+y^2+z^2=1, go to the 3D window, and hit the plot button, we get an error message that states, "The highlighted expression cannot be plotted". Translation: Sorry, I don't want to play this game anymore!

Finally, DERIVE does not always get the right answer when finding the limit of a function of two variables. It does get the right answer when finding the limit "along a path", but DERIVE is not able to allow both variables to vary simultaneously, and hence sometimes gives erroneous results. My students are disheartened for the same reason as I am delighted over this. Namely, it forces the students to use DERIVE as a "tool to investigate" a limit rather than as a "magical black box" that can find the limit for them.

Conclusion

DERIVE is a useful tool in multivariate calculus because it helps the students to see what the algebraic expressions they are working with "look like". It is also a versatile tool for plotting many different kinds of surfaces, space curves, and contour plots and is quite useful as an investigative tool when finding limits. My students generally "like" DERIVING!