TEACHING NUMERICAL ANALYSIS WITH MATHCAD

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1. Introduction

This work is to expound a way to enhance the teaching of Numerical Analysis with technology, especially with PC and math software of Mathcad 2001.

Numerical Analysis is a course in the curriculum for students of science and engineering.It occupies a special position in the curriculum since it is the course to introduce students how to apply math.--the central subject of all applied math.; it is the course to reinforce and extend students' understanding of Calculus; also it is the course to require a lot of tedious and repetitive computation.

The feature of Numerical Analysis is to employ math and computational tool to implement enormous calculation. This feature made some difficulty for the teaching and learning of Numerical Analysis because of the lack of computational tool before the computer technology became popular. Since it is very time-consuming to perform a numerical method the instructor just can give simple problems in order to save time and students are often scared by complex calculations.

Now, the situation begins to change. First, the demand for using of numerical method to analyse, simulate and design enginerring process and system has been increasing at a rapid rate in recent year. Second, as technologies are growing the amazingly fast computers are commonplace and powerful softwares make it possible to solve highly complex problems. This development provides us with the demand and the possibility to improve the teaching and learning of Numerical Analysis.

On the other hand, although this computing power has enormous potential to enhance the teaching and learning of Numerical Analysis, it still is a problem how to use technology intelligently so that we can achieve desired results. When using technology the case often is that the teaching of technology replaces the teaching of mathematics, i.e., the secondary supersedes the primary so that the mathematics education is not enhanced but damaged.

This work is based on writer's experience from teaching Numerical Analysis and also it is an attempt to find an appropriate way to improve math education with technology.

2. PC approach for solving problem in Numerical Analysis

2.1 About solving problem in Numerical Analysis

It is not easy to work out a problem in Numerical Analysis because it often involves a lot of difficult aspects and requires not only mathematics background but also other skill to solve it. Now, the computer technology provides us with a powerful tool to overcome these difficulty.

In this part I'll describe an approach how to use PC to solve the problem in Numerical Analysis through an example--fixed point iteration.

2.2 Sample approach

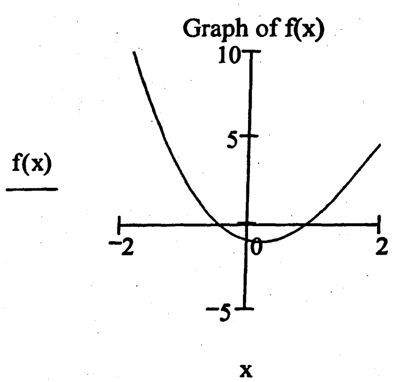
In this section a problem of fixed point iteration will be used to illustrate the PC approach to solve the problem in Numerical Analysis.

Fixed point iteration is often used to find the approximate solution of an equation.The procedure to perform a fixed point iteration includes following steps: To locate the solution of the equation; To determine a function of fixed point to convert the rootfinding problem to fixed point problem; To implement fixed point iteration to get desired approximate solution. In this procedure every step is not easy. To locate the solution is difficult; To determine an appropriate function of fixed point is difficult; to implement fixed point iteration which needs a lot of calculation is difficult. But, now the computer technology can ease the job. I'll use following example to illustrate.

Given: equation $3x^2 - e^x = 0$; Determine a function g(x) and an interval [a,b] on which fixed point iteration will converge to the positive solution of given equation with accuracy 10^{-5}.

Sol'n: First, To locate the positive solution by graph the function.

$x := -2, -1.99 .. 2 \qquad f(x) := 3 \cdot x^2 - e^x$



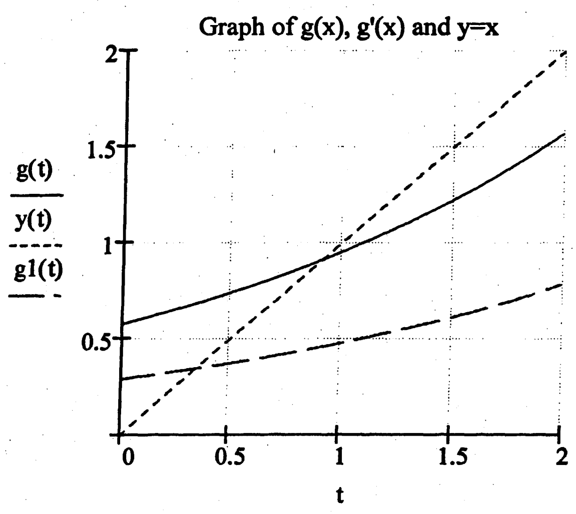
From graph we can see f(x) has a positive solution in [0,1]

Second, To determine a function g(x) which fixed point is the solution of f(x)=0 and an interval on which fixed point iteration converges to the solution. This is pretty tough job. But, from Mathematics we know: the g(x) and [a,b] are proper if they satisfy 3 conditions: g(x) is continuous on [a,b]; $a \le g(x) \le b$ and $\left|\frac{d}{dx}g(x)\right| \le k<1$. And, PC can help us to complete the job.

From f(x)=0 we can get 3 g(x), they are $g(x)=\frac{1}{\sqrt{3}}\cdot e^{\frac{x}{2}}$, $g(x)=\frac{e^x}{3x}$ and $g(x)=\ln(3x^2)$.

Which one is proper? It should satisfy 3 conditions mentioned above. How to verify if a g(x) satisfies these conditions? PC can help us to check if a g(x) satisfies these conditions through graphing g(x), g'(x) and line y=x. since fixed point is the intersection point of y=g(x) and y=x;
an proper interval must contain this point and on which g(x) is continuous, $a \le g(x) \le b$ and |g'(x)|<1.i.e. looking for such g(x) which graph and intersection point with y=x is located in a square having y=x as diagonal and |g'(x)|<1.
Through graphing these 3 g(x), we can determine the first one is proper choice. Below graph shows this fact.

$t := 0, 0.01 .. 2$ $\quad g(t) := \frac{1}{\sqrt{3}}\cdot e^{\frac{t}{2}}$ $\quad g1(t) := \frac{d}{dt}g(t) \rightarrow \frac{1}{6}\cdot\sqrt{3}\cdot\exp\left(\frac{1}{2}\cdot t\right)$ $\quad y(t) := t$



g(0) = 0.57735027

g(1) = 0.95188967

g1(0) = 0.28867513

g1(1) = 0.47594483

From above graph, we can see the fixed point is in [0.5,1], so intervals [0,1], [0,2] and [0.5,1] all are proper interval.we can choose. say, we use [0,1]. Also,g(x) and g1(x) are increasing on [0,1], so g(0)=0.577 is min, g(1)=0.952 is max, thus 0<g(x)<1; g1(0)=0.289 is min , g1(1)=0.476 ix max, thus |g'(x)|<1. Therefore, such g(x) is our desired.

Third, To determine the number of steps needed for desired accuracy and perform fixed point iteration to get approximate solution. Let n be the number of steps, k is max of $|g'(x)|$, a initial guess and m indicate accuracy, then from error formula of fixed point iteration we have: $n > \frac{\log(1-k) - \log(|g(a)-a|) - m}{\log(k)}$. And, we use a program to perform the iteration.

$a := 0.5 \qquad k := 0.5 \qquad m := 5 \qquad s := \frac{\log(1-k) - \log(|g(a)-a|) - m}{\log(k)} \qquad s = 15.55873412$

$$p(a,g,n) := \left| \begin{array}{l} p_0 \leftarrow a \\ \text{for } i \in 0..n-1 \\ \quad p_{i+1} \leftarrow g(p_i) \\ p \end{array} \right.$$

Number of steps: $n := \text{ceil}(s) \qquad n = 16$

The exact solution: $r := \text{root}(f(x),x,0,1)$

$r = 0.91000757$

The approximate solution is $B := p(a,g,n)$

$B^T =$

	11	12	13	14	15	16
0	0.90994807	0.9099805	0.90999525	0.91000197	0.91000502	0.91000641

3. References

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