What Students Learn: Math Modeling vs Traditional Precalculus

Florence S. Gordon
New York Institute of Technology
Old Westbury, NY 11568
fgordon@nyit.edu

For the last decade, the reform of mathematics education at the college level has been accompanied by the so-called "math wars". One dimension of this struggle has involved calls to prove that non-traditional courses are at least as effective as the courses they replace. In this article, we look at the results of a comprehensive, multifaceted study comparing student performance, success, retention, and attitudes in a reform version of precalculus to that in a traditional version of the same course. Another component of the study involves student performance, success, and retention in the follow-up calculus course based on the type of precalculus experience the students had.

Background The precalculus offering at NYIT is a college algebra/trig course stressing traditional drill-and-skill techniques presumed to be necessary for calculus. Graphing calculators are used to augment the usual algebraic topics. For science and engineering majors, this course has been followed by a reform calculus sequence based on the Harvard calculus materials. Recently, some faculty decided to teach a reform version of precalculus based on mathematical modeling with an approach that emphasized conceptual understanding. Several other faculty members objected to this change, believing that such a course would irreparably damage the students in terms of their perceived weaknesses in algebraic manipulation. To resolve this issue, the department decided to offer two sections of the course in the traditional, algebra skill development-oriented way and two sections using the modeling approach and then to compare the results between the two groups. Four experienced faculty members, all of whom are considered excellent instructors, taught the four sections.

While the core topics covered in both precalculus groups were the same (linear, polynomial, rational, exponential, logarithmic, and trig functions), the emphasis and sequence of topics were quite different. The traditional course was mainly lecture-based and stressed routine algebraic manipulations to improve student skills. The modeling course, based on [1], stressed conceptual understanding, problem-solving, and realistic applications. In the modeling course, algebraic manipulations arose only in the context of problem solving, not as lists of drill-and-skill exercises. Students learned by applying the mathematical ideas and methods to real-world data.

Student backgrounds prior to precalculus A departmental placement test used to assesses algebraic proficiency was administered to all students before the start of classes. The students in the two modeling sections scored a mean of 12.47 with a standard deviation of 3.93. Those in the traditional sections scored a mean of 13.58, with a standard deviation of 3.36. Statistically, there is no significant difference.

Performance in precalculus To compare student performance, ten common questions, primarily manipulative in nature, were given on the final examinations for both precalculus groups. The content of all the questions was agreed to by the four faculty members who were confident that their students could handle the problems with relative ease. The ten questions were worth a total of 66 points. Students in the modeling sections scored a mean of 49.69 with a standard deviation of 9.32, while those in the traditional sections scored a mean of 43.63 with a standard deviation of 12.03. The means are significantly different statistically (p-value = 0.0266), so that the students in the modeling sections out-performed those in the traditional sections on the algebraic manipulation questions. This result is particularly striking when one considers that the students in the modeling sections started with weaker algebraic skills on average (as measured by the placement exam), took a course that did not explicitly emphasize such skills, and ultimately outscored their peers on questions involving precisely those kinds of skills. A detailed analysis of the results, along with the author's speculations about the reasons for this outcome, are in [2].

The most striking difference in the two groups is revealed by a comparison of the responses to part of one question that asked the students to interpret the meaning of the slope of a line in the given context (enrollment at a college in different years). 35 of the 37 students in the modeling sections gave a meaningful response, indicating their understanding of the significance of the slope. A typical response was: "*This means that for every year the number of students increases by 78*". The remaining two students wrote an appropriate statement for the meaning of slope, but calculated the slope as $\Delta t/\Delta y$. In comparison, only 9 of the 27 students in the traditional sections gave a meaningful response. Five left that portion of the question blank and 3 simply rephrased the algebraic formula $\Delta y/\Delta x$ for the slope in words. The remaining 10 students wrote statements that made no sense. The actual student responses for both groups are in [2].

The ability of both groups to calculate the slope of a line was comparable. However, any graphing calculator can do that. What should be more valuable to our students is the ability to understand what the slope means in context, whether that context arises in one of their other courses in mathematics or courses in one of the quantitative disciplines or eventually on the job. These results suggest that, unless explicit attention is devoted to emphasizing conceptual understanding of what the slope means, a majority of students are not able to create meaningful interpretations on their own. Lacking conceptual understanding, they are not able to apply the mathematics to realistic situations in new contexts or in other courses.

Many of us have heard complaints from colleagues in other disciplines about students who appear not to have learned key mathematical ideas and techniques, such as finding or using the equation of a line. In most other disciplines, linear functions do not arise in the form usually taught in a traditional math course: *Find the equation of the line through the points (1,3) and (5,11)*. Instead, one typically faces a collection of data relating two quantities that follow a roughly linear pattern and one has to find and use the (regression) line that best fits the data. If students have such difficulty just giving meaning to the slope of a line, it is no wonder that they are unable to connect what they learn about lines and linear functions in their math classes to what they are expected to do in their other courses. Moreover, if students are unable to make their own connections with an elementary concept like the slope of a line (which they have encountered previously), it is unlikely that they will be able to create meaningful interpretations and connections on their own for more sophisticated mathematical concepts, such as: What is the significance of the base (growth or decay factor) in an exponential function? What is the meaning of the power in a power function? What do the parameters in a realistic sinusoidal model tell about the phenomenon being modeled? What is the significance of the factors of a polynomial? What is the significance of the derivative of a function? What is the significance of a definite integral?

On the basis of this study, it is clear that we cannot simply concentrate on teaching mathematical techniques and skills. It is at least as important to stress conceptual understanding and the meaning of the mathematics. This can be accomplished by using realistic, contextual examples and problems that force students to think, not just to manipulate symbols. If we fail to do this, we are not adequately preparing them for successive math courses, for courses in other disciplines, and for using mathematics on the job and throughout their lives.

Assessing student attitudes in precalculus Another component of the student involved determining student attitudes in four areas, both when they entered the course and when they completed it. One group of questions dealt with whether mathematics is an active, open-ended, discovery-oriented process or a passive, closed-ended, memory-based procedure. Students in the modeling sections displayed increases in positive attitudes while those in the traditional sections displayed substantially more negative attitudes and experiences toward mathematics.

A second group of attitudinal questions dealt with the usefulness of mathematics and whether the students viewed it as connected to situations beyond math courses. The external evaluator concluded that students appear to respond much more positively to the reform/modeling approach than to the traditional approach. A third group of questions dealt with the importance of technology in learning mathematics. The students in the traditional sections displayed a considerable drop in their attitudes toward the value of technology while most of those in the modeling sections displayed a substantial improvement.

The follow-up study in calculus A follow-up study was conducted on student performance and retention in the Calculus I course for science and engineering majors based on whether the students had taken a traditional or the modeling precalculus course. There were two sections of calculus taught by the same professor, and each section included students with both types of precalculus background. In total, there were 52 students enrolled in the two sections of Calculus I. Thirteen of them had the modeling precalculus background while 39 had a traditional precalculus background. However, only 5 of those 39 students came through the traditional precalculus course during the preceding Fall. The remaining 34 students had taken a precalculus course in a variety of ways; some were transfer students, some had taken a traditional precalculus course during an earlier semester, some had taken traditional precalculus in high school, and some had completed a two-semester traditional precalculus track. The calculus study included comparisons of students' performance on quizzes, class tests and the final examination, as well as student success rates, retention rates, and persistence in the course.

Performance and persistence on calculus quizzes The modeling group consistently outperformed the full traditional group on the seven quizzes given during the semester in the sense that the mean score on each quiz was substantially higher for the students with the modeling background. Moreover, a close examination of the quiz data reveals a striking pattern in terms of student persistence in calculus depending on the students' precalculus background – the persistence levels among the students with a traditional background were considerably lower and a large component of this group gave up on calculus and stopped attending. Also, the percentage of students with the modeling background who took each of the seven quizzes was consistently higher than the percentage of students with a traditional background. The comparison of performance on the last few quizzes is distorted by the disproportionate number of students with a traditional background who stopped attending calculus. Since they were already doing poorly, their scores on later quizzes, had they persisted, would undoubtedly have lowered the average for their group. That is, the lowest performing students were effectively removed from the traditional group when they stopped attending, but the lowest performing students from the modeling group completed the entire course. Had all these students persisted to the end of the semester, there would almost certainly have been an even wider discrepancy in the averages between the two groups. The actual data is shown in [2].

Performance on class tests in calculus Three class tests were given in calculus. On the first test, the group with a traditional precalculus approach had a mean grade of 64.36 with a standard deviation of 19.23. The students with the modeling background had a mean of 90.62 with a standard deviation of 8.96. On the second test, the group with a traditional background scored a mean grade of 59.58 with a standard deviation of 21.82. The students with the modeling background scored a mean grade of 77.42 with a standard deviation of 14.26. On the third test, the students with a traditional background scored a mean grade of 62.3 with a standard deviation of 17.31. The students with the modeling background scored a mean grade of 76.1 with a standard deviation of 18.13. In all cases the difference of means is statistically significant ($p = 0.0000257$, $p = 0.01249$, and $p = 0.035$, respectively.)

A careful comparison of the above numbers might suggest that the results for the two groups were converging as the semester progressed. Again, this is probably misleading because of the considerably higher percentage of students with a traditional background who gave up and stopped attending the course. Had they taken the later tests, the difference in the means likely would have been even more dramatic.

Performance on the final exam in calculus On the calculus final exam, the students with the modeling background scored a mean grade of 69.8 with a standard deviation of 26.21. The students with a traditional precalculus background scored a mean grade of 55.8 with a standard deviation of 23.09. The students with. The resulting difference of means test gives $p = 0.1$. Note that 12 of the 13 students, or 92.3%, from the reform/modeling background who started the calculus course completed the course and took the final exam. In comparison, 24 of the 39 students, or 61.5%, of the group with a traditional background who started the course took the final exam. Finally, 10 of the 13 students from the modeling background who started the course received passing grades in calculus, for a success rate of

76.9%. In comparison, 16 of the 39 students in the group with a traditional background who started the course received passing grades in calculus, for a success rate of 41.0%. The difference in the proportion of students who passed the course is statistically significant ($p = 0.025$).

Discussion of results The issue of student persistence and retention in calculus is perhaps the most critical factor for an institution such as ours. To summarize, only one of the 13 students from the modeling background stopped attending the calculus course, no other student from this grouped missed a single test or the final exam and, in fact, only two of the other students missed any quizzes. On the other hand, over 40% of the students from the traditional background stopped attending; six of them did not take the first test, eight of them did not take the second test, 16 did not take the third test, and almost half missed more than one quiz.

I can only speculate about the reasons for this. There is no doubt that the students with the modeling background in precalculus were better prepared to handle the intellectual demands of a reform calculus course. They were comfortable with the need to understand the meaning of the mathematical concepts, not just to manipulate symbols by rote. They were used to non-routine problems, both conceptual and realistic, that required them to think and understand. In contrast, many students coming from a traditional skills-focused precalculus experience may have brought with them higher levels of manipulative skills as measured by the departmental placement test. But, they evidently were not prepared for a balanced emphasis on conceptual understanding, realistic applications, and algebraic manipulation.

As mentioned earlier, at the start of our experience, some faculty expressed concern that the modeling approach in precalculus would inflict irreparable damage to the students. In retrospect, it appears that it is the traditional precalculus course that harms the students.

References

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2. Gordon, Florence S., *Assessing What Students Learn: Reform versus Traditional Precalculus and Follow-up Calculus,* in *Rethinking the Road Toward Calculus*, Nancy Baxter Hastings, et al, editors, MAA Notes, 2003 (to appear).