SWING, PENDULUM, SWING

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Introduction:

When a pendulum is pulled back and released it portrays the concept of simple harmonic motion for a short period of time. We will use the CBR with TI-83 plus to collect time and distance data from a swinging pendulum and relate amplitude, period, phase shift, and vertical shift to this real life example. Each person will hang a fishing bob on a string from a dowel, pull it back and release it to start the harmonic motion. They will take actual measurements of the distance of the bob from the CBR in rest position (vertical shift from the x-axis), measure the distance the bob is pulled back from rest position (amplitude), and time 3 to 4 periods of swing with a stop watch in order to average for one period of the cycle. Phase shift will be determined from the scatter plot formed on the calculator. The student will trace the graph transferred to the calculator to find maximum and minimum points from which they can calculate amplitude, period, phase shift and vertical shift. Comparisons are made between measured data and calculated data to emphasize what these terms actually mean in real-life. From this analysis, each person is able to find an appropriate model for the data as a sine or cosine function. Follow-up will be to use sine regression to find the "best fit" model. Discussions include the question "Which model is better, yours or the sine regression model?"

Pedagogically speaking:

A primary goal is to engage our students in a more investigative learning environment. To have students collect their own data and analyze it, gives the student more ownership to the problem. Relating their math to a real-life situation influences the student to value their mathematics because they can finally answer the typical student question, " What is this good for?" The Standards wants our students actively engaged in problem solving and logical reasoning. This activity offers excellent opportunity for these goals. Secondly, we want our students using technology. Using the CBR gives a table of data points; connecting the CBR to the TI-83p transfers that data to the calculator where graphs and tables can be studied in a more timely manner. In this activity we are analyzing a function of time with distance data. Because the CBR has also collected velocity and acceleration data, this activity can be extended and used in calculus.

Equipment Needed:

String or yarn	Stop Watch
Fishing Bob	CBR
Meter stick	T-83p
Masking Tape	Calculator link cables
Wooden Dowel	

Part A: Almost Undamped Data

Set-up Instructions:

1. Tie one end of the string to the fishing bob. Tie the other end of the string to the center of a wooden dowel. Hang the dowel between two flat desktops and adjust the string so the ball hangs about an inch above the floor.
2. Use masking tape to mark the rest position on the floor of the fishing bob.
3. Use the meter stick and the tape to mark the floor a specific distance away from the bob. Note the CBR works best if you are at least .5 meters away.

Place the CBR here on top of a book. Have the CBR directly in line with the bob, facing the bob. The CBR is acting like the x-axis line. The distance between the CBR and the rest position of the bob is the vertical shift.

4. Connect the CBR to your TI-83p. Go to the APPS key on the calculator and select #2 CBL/CBR. Press any key and select #3 Ranger. Select #1 SETUP /SAMPLE. Adjust this window to look like:

MAIN MENU	START NOW
REALTIME:	No
TIME(S):	10
DISPLAY:	DIST
BEGIN ON:	[ENTER]
SMOOTHING:	LIGHT
UNITS:	METERS

Figure 1

5. Arrow up to start now and press enter. Pull bob toward CBR and measure distance of bob from rest position. 10 – 12 cm usually gives a good wave. Have a person ready with a stopwatch to time four cycles. We will average the 4 cycles to get the period. Let the bob swing and start the CBR with enter. You should get a nice almost undamped motion like Figure below. Repeat until you are satisfied that you have a good wave. It may take several tries. No flat tops are accepted!

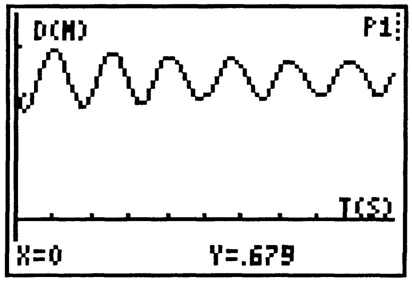

Figure 2

6. Record the following measurements using a meter stick and the stopwatch:

   Amplitude (distance you pulled bob back before letting it go) in meters ______
   Vertical Shift (distance between CBR and bob) in meters ________
   Period of one cycle _______

7. Calculator Measurements:
   Quit the program and trace to record the following points:
   Recall x = time in seconds; y = distance in meters between the CBR and bob.

   1st maximum: x = _______, y = ________

   1st minimum: x = _______, y = ________

   2nd maximum: x = _______, y = ________

   2nd minimum: x = _______, y = ________

8. While the graph screen is displayed, select 2nd PRGM, 4: vertical. Use this line to locate the beginning of a cycle. Record the x value. Use this same line and trace to the end of the fourth cycle. Record the x value. Subtract these two values and divide by 4 to determine the average time for one cycle.

Period = ___________
How does this value compare with your stopwatch measurement?

9. Use your first maximum x value to determine the phase shift for a cosine function or use the first minimum x value to determine the phase shift for a reflected cosine function.

phase shift for cosine = ___________

10. Use your flow chart to determine A, B, C, and D for a cosine function in the form $y = A\cos(Bx + C) + D$.

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11. Type this function in your calculator on Y1 = and overlay this graph on your scatter plot. How well does it fit? Graph link a picture into your write up and label.

12. Find a phase shift for a sine function by either of the following methods:

a. Draw a midline onto your graph (y = D). Trace to see how far to the left or right the point that is normally at (0, 0) for sine has moved over (x value).
b. Average the x values of your first maximum and minimum.

Phase shift for sine = ____________

13. Use your flow chart to determine A, B, C, and D for a sine function in the form $y = A\sin(Bx + C) + D$.

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14. Type this function in your calculator on Y2 = and overlay this graph on your original scatter plot. How well does it fit? Graph link a picture into your write up and label it.

15. Do we really have the best fitting function for our data? Your calculator offers a "best fit" function for sine. It is called sine regression. Keystrokes are: STAT, CALC, C: SinReg L1, L2, enter. Paste this equation into Y3 = with the following keystrokes: Y =, Y3, VARS, #5: STATICS, arrow right to EQ, 1: RegEQ. Graph this function to see how well it fits. Which one of the three functions do you think fits your data the best?

Part B: Damped data

The data we collected was actually damping which means as time passes the waves are diminishing in amplitude.

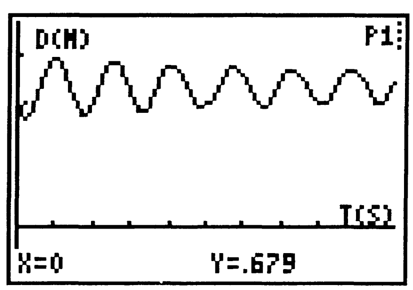

Figure 3

1. Trace to find all the maximum points and put in L3 and L4 on your calculator.
This can be done by tracing to the highest point, go to STAT, EDIT and put cursor in first slot in L3, ALPHA X, cursor in first slot in L4, ALPHA Y. Trace to the next maximum

and repeat for second slots in L3 and L4. Repeat for as many maximums as you have on the screen.

2. These points closely approximate an exponential decay curve. Recall from college algebra an equation of the form, $y = a * b^x$. You could guess values for a and b or you can have your calculator perform an exponential regression by STAT, CALC, 0:ExpReg , enter, L3, L4, enter. Paste this equation into Y4. Graph Y4 to see if this curve touches all your maximums.

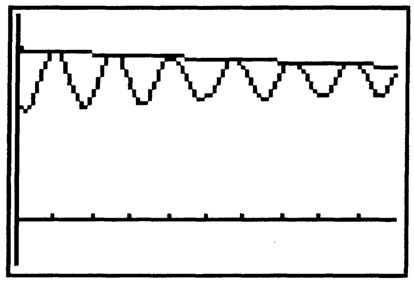

Figure 4

3. This Y1 could be your changing amplitude except for the fact your graph is not centered on the x-axis. Rewrite your Y3 equation and change your amplitude to (Y4 – the vertical shift) or (Y4 – D). Put your cursor where the amplitude should go in Y3. Then go to VARS, Y-VARS, 1:Function, 4:Y4. Adjust Y4 to (Y4 – D). Overlay the graph of this equation onto your scatter plot and graph-link a picture into your write-up. How well does this function fit your original curve?

Write-up:

Write how this project has helped you better understand the concept of using math to model " real situations." In terms of the physical aspects of the swinging pendulum, explain what the amplitude, period, phase shift, and vertical shift are. Show how you calculated to determine A, B, C, and D. Did you have any problems with this activity? Hand in with graph-link pictures and answers to questions in 1-15.

Credits:

This activity was adapted from Explorations, Modeling Motion: High School Math Activities with the CBR, Linda Antinone, Sam Gough, Jill Gough, "Swinging Along", pp. 73 – 76, Texas Instruments Incorporated, 1997