

# ANIMATIONS: A TEACHING TOOLS FOR MULTIVARIABLE CALCULUS

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## Introduction

An increasing number of educators are using animations in teaching multivariable calculus and others upper division mathematical courses such as differential equations. See for example Blanchard [1], Putz [3], and Stroyan [4]. Animating multivariable concepts enhance the ability of the student to comprehend the dynamics of three dimensional mathematical concepts. With this philosophy in mind we have created a package that contains animations for most of the concepts covered in a multivariable calculus course.

## Background

We have developed a package that makes use of animation for the Multivariable Calculus course, Flores [2]. The animations are generated using *Mathematica*® and then exported into *Micromedia Flash*®. We used Flash to create the buttons and slider bar with an ActionScript code that gives the interactive capabilities to the animation.

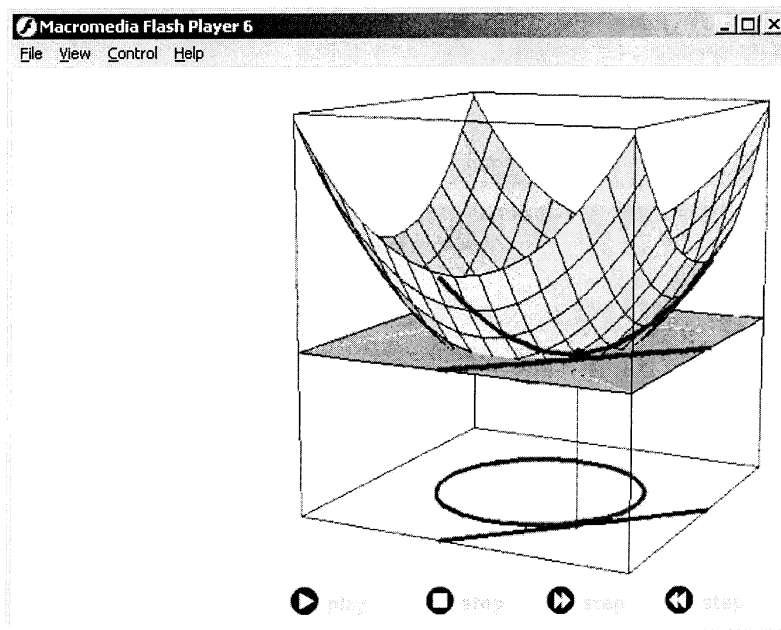


Figure 1: Lagrange problem

## Motivation

To motivate this presentation we refer to two examples that we cover in the syllabus of the multivariable calculus course.

Example 1: The Lagrange Theorem, Figure 1. In general students learn the manipulations to calculate the extrema of a given optimization problem, but a significant number of them do not really understand the concept behind what they are computing. The animation of The Lagrange problem shows the constraint curve at base of the system and it also views the image of the constraint curve mapped onto the surface. The horizontal plane moves up and down and as the plane intercept the surface it creates the contour curves. That is, the animated elements here are the plane and the contour curves. It is precisely at this point where you can explain to the students the equation of Lagrange. When the contour curve becomes tangent to the constraint curve "the gradient vectors are parallel", and this occurs only when the horizontal plane has reached the extrema point. The interactive animation of the Lagrange problem gives the students an intuitive idea of the problem that they will need to compute.

Example 2: Students can experiment when covering the concept of directional derivatives, Figure 2. With the slider bar the user can rotate the direction vector and discover that by selecting different direction vectors, the rate of change will vary. Therefore, they will be able to visualize the direction of the maximum rate of change instead of merely calculating them by hand.

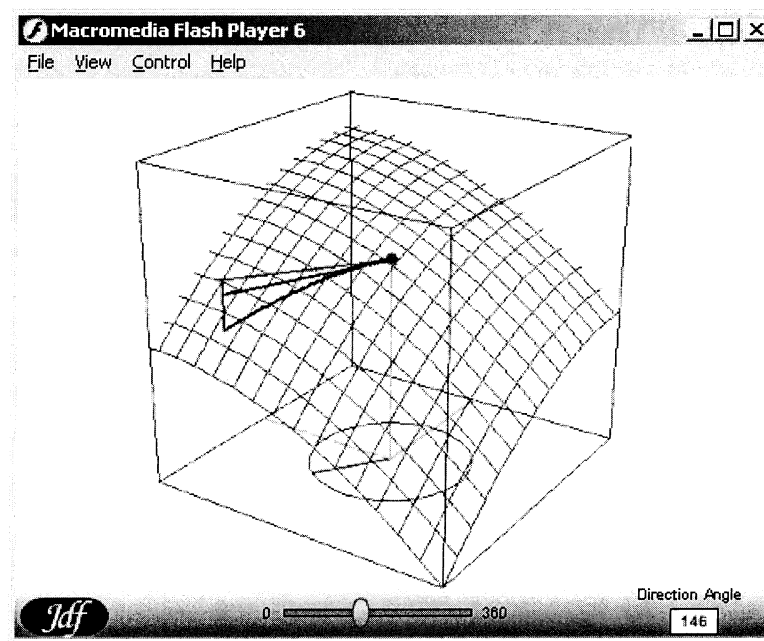


Figure 2: Directional derivatives

## Summary

The use of animations as part of the lectures has brought to the classroom the dynamics of three dimensional objects to create a more active way of teaching calculus. Animations bring the mathematical concepts alive, and with interactive animations students can manipulate the animations to understand mathematical concepts.

We believe that the incorporation of computer animations together with the understanding of basic hand computations will help the students to achieve the traditional goals more successfully.

## References

- [1] Blanchard, Paul. "*Multivariable Calculus*"  
[http://math.bu.edu/people/paul/225/ictcm\\_2001.html](http://math.bu.edu/people/paul/225/ictcm_2001.html)
- [2] Flores, José. "*Multivariable Calculus Project*", <http://www.usd.edu/~jflores>
- [3] Putz, John. "*Maple Animations*", CRC Press, May 2003.
- [4] Stroyan, Keith. "*Interactive Multivariable Calculus*",  
<http://www.math.uiowa.edu/%7Estroyan/multicalc.htm>