THE BASICS OF MATHEMATICA
FOR THE FIRST-YEAR CALCULUS STUDENTS

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This paper is a follow-up of our presentation at the 15th Annual International Conference on Technology in Collegiate Mathematics, in Orlando, Florida, November 1-4, 2002. Our goal in presenting that talk was to let our colleagues see how Georgia Perimeter College has incorporated technology throughout its Calculus courses in a way that is non-threatening to the students. We have been very fortunate in having taken part in the Mathematica experiment at GPC, and the necessity of the present paper has been previously suggested to us several years ago, for the simple reason that despite the actual widespread use of computers and Computer Algebra System, most Calculus classes are still taught in the traditional classroom, the old fashion way. In order for the attendants at the Conference or the readers of this paper to understand the setting of the experiment, they must know about our school.

1. Georgia Perimeter College
We are a Two-Year College, formerly known as Dekalb College, located outside of the city of Atlanta, Georgia. Georgia Perimeter College consists of five Campuses (Clarkston, Decatur, Dunwoody, Lawrenceville, and Rockdale), and one Center with about 200 students (Alpharetta). Geographically, the campuses are mostly on the eastern side of Atlanta, strategically near the main highways, the "perimeter," around the city.

The College presently has about 16,000 students, and we expect to grow significantly in the coming years. For example the Lawrenceville Campus, where GPC and the University of Georgia collaborate to constitute the Gwinnett University Center, is predicted to have 10,000 students within the next 10 years and about 15,000 later on.

Our student body is very diverse, we have traditional, international (from about 80 countries), working adult, transient, and joint-enrollment/Post Secondary students. We make it a point to stress that GPC has the largest number of international students in the state of Georgia. The Chancellor of the Georgia University System once said that "when GPC catches a cold some other Institutions of the System sneeze," that statement is a good pictorial description of the impact of any initiative at GPC on the other schools in the Atlanta area. Our students in their diversity have a common point: they are all technologically unprepared, they are not of the video-game generation that one would

expect, and one of the missions of the College is to bring them up-to-date on the technological realities of the twenty-first century.

2. Introduction to technology
At GPC, the initial training to use technology begins with the required Applied Technology courses, better known as ATEC, where the students are introduced to the basics of computers, word-processing and the wonders of the Internet. To finance the enormous amount of equipments made available to our students, the College charges a Technology Fee every term, similar to a Parking Fee for parking a car on-campus.

In all our mathematics courses, including the learning support courses, we require the students to be equipped with a graphing calculator. On the second and third weeks of the semester and also after the midterm, the Mathematics Department runs several workshops or seminars to familiarize any willing students, new or old, with their calculators.

In the same framework, the Department prepares the Calculus students in the use of Mathematica by special workshops for those who wish to learn it at the beginning of their courses.

3. Mathematica in Calculus
This is our 9th year of experimenting with Mathematica in Calculus, and the process is essentially settled for us at GPC. We run all our Calculus classes in computer labs where each student has a computer with Mathematica on it. Every computer on a given campus is connected to the M- and T-drives. The M-drive has directories for the various classes where Mathematica is used, and a faculty member teaching a particular class has his/her own subdirectory there, where he/she can loads Notebooks from his/her office, for the students to access; however, the students cannot save files on the M-drive. The T-drive is mainly for specific individual classrooms use only, and students can save files on it; sometimes, it is used to collect assignments electronically from them. It has been agreed among the faculty that any Notebook loaded on the M-drive is public domain and may be used by all colleagues. This arrangement saved a huge amount of time for us, it allowed the Department to continually improve the projects assigned, and fostered a closer collaboration among the faculty members.

In the classroom, the policy on the use of technology is entirely left to the instructor. He is the only one to decide on how to introduce Mathematica; he is free to choose the number of projects to assign, but in general that number is between two and five a semester. We have noticed that the strategy of assigning a few CAS problems in homework does work very well, because the majority will simply skip them. Depending on the teacher, Mathematica assignments are collected on diskettes or paper.

Some of us went all the way to the point of asking the students to write animation programs to illustrate a particular Calculus concept. But most of us understood that the subject of a Calculus course is Calculus, not programming, so we strongly suggest that

the teacher avoids requiring extensive programming as much as possible; however, we know that programming is implicit any time we use a computer in mathematics.

Very often, the students get an output that is incomprehensible to them, and our position on such a situation is that if they cannot understand it, the output is not valid, and they must figure out what they entered incorrectly; in other words, change their input to get an understandable result.

Mathematica is allowed in tests and exams! The concerned teacher might wonder whether the students would directly use the power of the software to get most of the answers; surprisingly during our many years of observation, generally, they will use the methods taught in class to solve the test problems. We have seen "smart" students in two or three classes that have decided to do the minimum work and expected to use the computer to pass the tests; unfortunately for them, soon they all realized that they could not do anything much, because they did know enough; they ended up withdrawing from those classes by the midterm of the course. In any case, if a teacher does not want the students to use the computer on a specific question, he should write the test accordingly; it can be an elaborate problem that cannot be solved except by hand computations; or it can be as simple as adding the statement "The computer is not allowed here."

We have 24 sections of Calculus I, 13 Calculus II, and 4 Calculus III throughout the entire College for Fall 2002. One question raised during the presentation in Orlando, was "Ok, they can use the computer in tests and exams, and everything is up to the teacher, what about if a student came from a class where Mathematica was not used?" This is not a problem at all since, our new students (mostly computer illiterates) enter at all levels from Learning Support to Differential Equations; at each entrance level we do not expect anyone to know anything about Mathematica, even though the faculty is expected to bring each and everyone of them to the level of the class, regardless of their background. The key to that lies on the workshops and on how we stacked up the Notebooks according to topics in an introductory file.

4. Topics with Mathematica
At GPC, Mathematica is used daily, on all the topics in Calculus; during the Conference, presenting the various topics took half of our session time. We provided diskettes containing sample Notebooks covering the most common commands on the following topics: Algebraic Manipulations, Functions, Limits, Derivatives, Implicit Differentiation, Newton's Method, Integration, Power Series, Parametric and Polar Equations, Euler's Method, Surfaces and Curves in Space. Each topic has a story to be told, but since evidently we can not do that here, we will restrict ourselves in describing briefly how they have been developed, and how they are used.

These Notebooks have been developed collectively by the Mathematics Faculty (If there are any errors in them, we, the presenters, are the only ones responsible for our carelessness). At that time the school was in the Quarter System, we had four courses in the Calculus sequence, and the participating faculty members were each assigned to one

of the four groups of Calculus I, II, III, and IV. Then the topics in each course were assigned to subgroups of people, and before you can say "Mathematica" the entire thing was finished. That's collegiality at work!

Of course, at first, we all had to learn the program using materials such as [1] and [3]; the trouble with those software-dependent books is that they were quickly discarded as newer versions of the software came out. Once the original Notebooks were developed, they were put on the M-drive for use in class, and the individual faculty members could then adapt them to their own tastes. These live files can be upgraded easily as Mathematica is improved from year to year. Among these files, the "How-To Notebook" is the introduction to Mathematica in each course. In Calculus I, it consists of the basic Algebraic Manipulations such as defining a function, plotting a graph, etc...; in Calculus II, it is the same file, but with all the commands used in Calculus I appended to it; and so on, the files from the previous classes are stacked-up that way into the "How-To Notebook". When a new student enters one of our Calculus classes, he is updated immediately by reading that introductory file!

With everybody doing what he wants, the assignments came out varying widely. Essentially, we have two categories, firstly the large projects where the computer is used to solve an involved problem, and secondly, short assignments designed to acquaint the students with some specific commands. Regardless of the type of assignments used, the teachers usually rely on the students to teach each other by organizing them in small groups of three or four. Those small groups are very effective in turning around simple difficulties that the students encounter regularly.

The most frequent obstacle the students have is in the distinction among the signs ( ), { }, and [ ]. When one thinks about it, they and Mathematica are right, it is tradition and us, the professional mathematicians, who are inconsistent, we should start using $f[x]$ instead of $f(x)$ as function notation. This remark is especially addressed toward the mathematics textbook writers.

The reader realizes at this point that using technology in mathematics raises some fundamental questions in pedagogy. For instance, during those years of experimenting we used the textbooks [2], [4], [6], and [7], they are voluminous; but right at the start it was clear that, referring everything to the technology, the topics on limits, derivatives, integrations, and convergence/divergence of series could each be dealt with in a stroke. At that time, the battle cry of the new proselyte was: "I don't teach Integration by Parts any more!" Yes, it was then tempting, it still is today, to envision a one-hundred-page new technology based Calculus textbook, to be covered in one semester. Yet the Calculus text writers all erred on the side of caution, so did our Mathematics Department. Here, we would like to remind the reader of what happened to those "smart" students mentioned in the previous section when they wanted to shortcut the subtleties of the subject matter. It does not mean that that vision is impossible; we think that such a textbook might be a reality in the future only if further results indicate that it is the way to go.

5. Viva Mathematica

We are not presenting any statistical results, but our subjective impression is that our experiment is a success. The software is not easy to master, but it is doing us a great deal of good: it motivates the students, it turns around one of the major stumbling block in Calculus (plain old algebraic computations), it helps the students in visualizing some of their problems, and finally, it improves student performance.

The difference is that, before, we differentiate among the students as excellent, good, fair, poor, and failing; now that old structure has been altered in our Calculus classes. The excellent and good remain more or less the same, some of the fair students moved the good group, some of the poor moved to fair, and the failing completely vanished. In short, the technolgy either improves a student or tells him/her to get out fast!

One could ask whether similar results would not be obtained with another Computer Algebra System like Maple, for example, and why we selected Mathematica. The answer is that it does not depend on the platform; we chose Mathematica because the original NSF grant, obtained by Linda Boyd in 1993, specified the program to be used: it was then the most powerful software. After one year or so, the Mathematics Division surveyed the faculty again about the choice of a Computer Algebra System, and Mathematica got the most votes and was retained. Today, some of our teachers do use Maple and Scientific Notebook on the other campuses.

In conclusion: Given the size and the international dimension of our student body, the effect of the GPC experiment with is worldwide; it is possible that, one day, you will have one of our former students in your class who will expect to see our methods and procedures. So, with all humility, we respectfully submit the GPC experiment with Mathematica to be presented as a safe and efficient example for the introduction of a Computer Algebra System in a Calculus course sequence.

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