MATHEMATICS INTERNET RESOURCES

James R. Olsen
Western Illinois University
Mathematics Department
1 University Circle
Macomb, IL 61455
jr-olsen@wiu.edu

The Internet provides a huge storehouse of useable resources for the teaching of mathematics K-12. In this workshop/paper, I will demonstrate some of the resources I've found that are of high quality and usable in the 7-12 mathematics classroom. The computer and Internet can supplement what we are already doing with paper, pencil, chalk, and board, by providing us with new, powerful, dynamic representations of concepts which can significantly enhance student understanding. We will learn some fun, interactive, hands-on ways of learning mathematics (all of which are *free*).

Top Ten Reasons for Using Computers and Calculators to Help Students Learn Mathematics

My primary purpose is to provide you with an annotated list of web activities that can be used right away. As a structure for thinking about how we can use the Internet to help students learn mathematics, I've created a *Top Ten List*. Each *reason* in the Top Ten List is a pedagogical, or mathematical, reason why technology should be used in the classroom to enhance student learning. I think the process of thinking about general reasons for using technology is important for guiding us to make good classroom decisions. Otherwise, the Internet can often lead us astray with glitz, uniqueness, or novelty. I'll use my Top Ten List as the structure for this workshop/paper. I will provide specific examples of resources from the Internet for each of reasons #8 to #1 in the Top Ten List. (We could also provide examples of how *calculators* can be used to achieve these ends, but that is beyond the scope of this paper.)

Links to all the Internet resources can be found on my *Teaching Resources* page (http://www.wiu.edu/users/mfjro1/wiu/tea/page1-99.htm). It is a primary goal of this workshop/paper to take you directly to specific Internet activities. (If you are looking for general portals, I recommend the Eisenhower National Clearinghouse, NCTM, Geometry Center, and The Math Forum–also found on my *Teaching Resources* page.) *Hint:* Since my *Teaching Resources* page has grown quite large over the years, you may find it useful, once there, to use the Find-in-This-Page feature on your browser (usually Control-F) and type in a word (e.g., "Bon Voyage" or "Surfing") to jump down to the desired link.

I'll begin my *Top Ten Reasons for Using Computers and Calculators to Help Students Learn Mathematics* with Reason #10 and #9–as any good Top Ten List should–but you should

realize that these are the *wrong reasons* for using technology to help students learn mathematics.

Reason #10: They are faster and more accurate.

Reason #9: They are in the real world.

These are two of the main reasons the average person on the street (including parents and some school administrators) gives for why our children should be using computers, the Internet, and calculators in school. If these were the only reasons for using technology, I would not take valuable mathematics class time to integrate it! Now we move to the *real reasons* for using the Internet to enhance mathematics learning.

Reason #8: Students can focus on the problem and the overall process and not get bogged down by the calculations.

Example

Understanding the Least-Squares Regression Line - from the NCTM's electronic examples (standards.nctm.org/document/eexamples/index.htm). In this online applet, students can manipulate data points and a line to explore how well a linear model fits a set of data points (there are three methods for measuring the "fit"). Squares of distances actually appear as squares on the screen.

Reason #7: They lend themselves to student-centered, active learning.

Example

Combining Transformations - E-Lab Grade 8 Activity 21 from Harcourt School Publishers (www.harcourtschool.com/elab_subscription/activity.html?activity=21). Students use plane transformations (translations, rotations, reflections, and even dilations) to change the position (and size) of plane figures. There are four levels of play.

Reason #6: To see patterns.

Example

Tower of Hanoi - interactive game (www.cut-the-knot.com/Curriculum/Combinatorics/TowerOfHanoi.shtml). The Tower of Hanoi problem/game is so much better with technology than by hand! The computer counts your moves, it's easy to reset, and moves are faster, to name a few reasons. Of course, the main question is: What is the minimum number of moves needed to move a n disks? There are many other interesting generalizations to investigate: What should my first move be if I have an odd number of disks to move from pin A to pin B? What if there are an even number of disks? If I know how many moves are required to move a certain number of disks, how many moves will be needed if I add one more disk? Why is this? (This is inductive reasoning.) Patterns can be discussed at three levels: specific numbers; verbally, "The number of moves is 2 multiplied times itself the number of disks you have, then subtract 1."; or symbolically "2^n - 1."

Reason #5: To integrate content areas.

Example (follows on the next page)

An Atlas of the Universe - a view of near and distant stars (www.anzwers.org/free/universe/index.html). This site offers nine major maps, each one larger in scope than the previous one, to provide an encompassing picture of the galactic subdivision in which we find ourselves. Great for discussing measurement, large numbers, and scientific notation in a context. Now this one *is possible* to buy in book form, but most school district budgets do not allow the mathematics teacher to buy expensive resources for integrating math with other curriculum areas (and the convenience of the Internet may make the web site better than a printed book anyway!).

Reason #4: Students have access to more functions for investigation.

Example

Winplot - free graphing utility by Rick Parris at the Phillips Exeter Academy (math.exeter.edu/rparris/). Rick Parris's *Peanut Software* could very well be the best free mathematics software on the Internet! I have used **Wingeom** extensively and have found it can do most of the same things as Geometer's Sketchpad®. This software is of high quality, downloads and installs in seconds (about 25), and is updated regularly.

My primary point here is that if Algebra I students only work with linear and quadratic functions (the ones they can do by hand), they develop a narrow view and misconceptions about functions (see Leinhardt, Zaslavsky, Stein, 1990). With a graphing utility, students can investigate, by interacting with, a variety of mathematical functions (including those that they might not yet be able to handle by hand with paper and pencil). Research has shown that computer environments can broaden the view and reduce the misconceptions students have with functions (see Kwansik, 2000). There are numerous function graphing utilities available on the web.

Reason #3: Students have access to more real-world problems.

Example

Bon Voyage! - a fun activity involving foreign currency exchange rates by Evan Glazer (gbs.glenbrook.k12.il.us/Academics/gbsmat/Internet%20Projects/travel/Pages/assignment.html). Students plan a trip to two different countries and set up functions to analyze how their money will be exchanged and spent. I like this activity/project for a number of reasons. It's realistic and uses a real-time currency converter on the web (but students spend only moments on the computer to get their own numbers - a one-computer classroom may suffice). The study of functions is great (but they are basic, direct variation functions). Meaning of the input and output variables is key, going from country to country is actually *composition of functions*, and returning to America is an *inverse function*–the physical movement around the world can be thought of as a metaphor for, or representation of, composition and inverses! Other aspects of Social Studies can be integrated as the teacher sees fit.

Reason #2: Technology can create representations that are not possible on a chalkboard or paper.

Example (follows on the next page)

Ellipse Activity on ExploreMath.com - interactive applet in which the user manipulates quantities and see how the ellipse changes (www.exploremath.com/activities/Activity_page.cfm?ActivityID=3). Teachers have futilely tried hand-waving and colored chalk for years, but often have not been able to get across the ideas with static media (paper-pencil or chalkboard). This dynamic applet allows the user to manipulate the vertices, center, and foci of the ellipse on the graph and see how the quantities in the equation react (it is a *linked representation*, simultaneously showing the equation, individual quantities, and the graph). Conversely (as is the case with a good linked representation), the user can manipulate the quantities and see how the graph reacts. The applet may be used by students for exploration or by the teacher for demonstration purposes.

ExploreMath.com also has excellent applets for points in a plane, linear functions, the other conic sections, trigonometry, probability and more. It is my opinion that many of the interactive applets I've discussed here should be used *in the learning process*, but *not present in the assessment* of student learning step. Often the purpose of the technology is to plant a concept in the students' minds (e.g., what happens to the ellipse if the distance out to a focus is increased). After the learning has taken place and the concept has been planted, the student should be able to understand and visualize the idea *without the technology's assistance* (e.g., on a paper-and-pencil quiz).

And the number one reason for using technology to help students learn mathematics is so that...

Reason #1: Students can see change.

Understanding change is key if one is to comprehend, and making decisions in, our world. Textbooks and paper-pencil representations may reveal change to the expert, but the novice often doesn't see it. Dynamic web environments can show the change and let the novice see what is variant and what is invariant.

Examples

[From above] **Least-Squares Regression Line**–Change the line, points, method of measurement, and see what happens. **Combining Transformations**–Perform a transformation and see what is variant and what is invariant. **Wingeom**–manipulate the point on a parallel line, triangle, or circle and observe the changes. **Bon Voyage!**–If the value of the Japanese Yen drops, what does that mean for the tourist? **Ellipse Activity**–Have you ever seen the "string property" of an ellipse?

Surfing Derivatives - the user drags a tangent line along a curve and can see the tangent line (hence the derivative) change (www.ies.co.jp/math/products/calc/applets/doukan/doukan.html). I've tried hand-waving and rulers and string in calculus classes for most of my teaching career, but with the Surfing Derivatives applet I can teach the ideas of tangent lines, increasing and decreasing derivatives, and inflection points much better than I could before the Internet came along. This site also includes other great applets on calculus, trigonometry, and geometry (including neat geometric proofs of the Pythagorean Theorem).

Before making my concluding remarks I wish to lift up three more web sites which are useful, fun, learning environments for students.

Learning about Rate of Change - part of the NCTM's electronic examples (standards.nctm.org/document/eexamples/chap6/6.2/index.htm). Students analyze and interpret two interactive graphs relating cost per minute of use and total cost for a cellular-phone service provider.

Integrator - an applet which computes Riemann sums for user-defined functions and draws a graph of the function with the appropriate rectangles which approximate the integral (www.joma.org/more/more.html?id=12868). It's just one applet of the *Journal of Online Mathematics and its Applications* (JOMA) Mathlets Project.

Coin Problem - a fun game in which the student uses logic and a balance scale to find the lighter coin (matti.usu.edu/nlvm/nav/frames_asid_139_g_3_t_2.html). This is part of a large web site titled the *National Library of Virtual Manipulative for Interactive Mathematics*. The applets are nicely organized by grade level grouping and content standard from the NCTM's *Principles and Standards for School Mathematics* (2000).

Concluding Remarks

I believe that the Internet provides us with many excellent resources we can use to create interactive learning environments for our students and use for classroom demonstrations. I hope the web sites I've lifted up in this paper and others you can find my *Teaching Resources* web page will be useful to you. I feel that some of the best uses of the Internet are when we exploit the *dynamic* representations afforded by the technology, to illuminate mathematics concepts for our students.

I'll conclude with two reminders: (1) Murphy's Law of Technology: If anything can go wrong, it will!...Keep smiling and use technology anyway; and (2) More important than the teacher's expertise with the technology is the teacher's ability to (a) know a few basics, (b) enable the students to use technology, and (c) be a problem solver.

References

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