THE EMERGENCE OF A MATHEMATICS LABORATORY

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1. Beginnings. Three years ago my department was little involved with computation, although some of us were teaching computer science outside of the Department. This was the situation in spite of heroic efforts on the part of a few of my colleagues to increase our involvement, first with a connection to Dartmouth's Kiewit Center in the 1960's, later to the CYBER at the University of Massachusetts. Most of us, I think, were quite put off by the frequent exaggerations and misrepresentations concerning the uses of computers in classroom teaching.

For reasons which are difficult to recall, much less to document, a sudden change of attitude occured about four years ago. Certainly the conference in computer algebra, which was held at the Courant Institute in the spring of 1984, was influential. At about the same time, one of my colleagues produced a commercially distributed calculus package for classroom demonstrations and student experimentation. In any case, in the annual report to the President of the College, dated 10 July 1985, there appears the following:

> The Department as a whole and especially its more computer science oriented members would like to see computers become the laboratory instruments of mathematics.
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> On the surface, there is nothing very revolutionary in such an idea. Computers have been, and will continue to be, used extensively for mathematical computation. Our notion, however, is a pedagogical one which seems not to have been tried before: we would like all our students to be involved in experimentation to the same extent that a chemistry student might be. For our majors, and especially our thesis students, we would like to run a summer program which emphasizes computer experimentation.

In the course of just three years, these ideas have come to fruition, with a degree of success which we couldn't have anticipated. The ability to accomplish it derived from a wonderful diversity unusual in a mathematics department. Its faculty includes two applied statisticians, a physicist, and a mathematician who serves as chairman of the computer science program. It is also a department which is both research-active and innovative in its teaching, even at the most elementary level.

2. The Quantitative Reasoning Laboratory. "Case Studies in Quantitative Reasoning" was the first course to focus on computer experimentation. Developed with support from the Sloan Foundation through its New Liberal Arts program, it is an interdisciplinary course in data analysis and mathematical modelling. It was taught for the first time in the spring of

1987 and has been taught each semester since by a group of faculty whose membership changes from semester to semester. Presently, the course consists of of three modules:

I. "Salem Village Witchcraft" concentrates on data analysis in exploring the relationships between wealth and power as reflected in historical records from the seventeenth century.

II. "Aptitude and Achievement" concentrates on hypothesis testing, experimental design, and modelling. It uses blind data on recent graduates of Mount Holyoke, including aptitude and achievement test scores, grade point averages, and choices of majors.

III. "Population, Resources, and Disease" treats case studies drawn from the sciences, particulary ecology.

All students attend a large common lecture, smaller discussion sessions, and a weekly two-hour laboratory. The laboratory room houses ten MacIntosh Plus's, arranged along the walls of a modest-sized room. We have found this arrangement of the computers to be good for allowing for minimum distraction when students are using the computers, but forcing students to turn away from the machines when the lab instructor wants to have the attention of the class.

The first two units of the course use StatView software, which is an excellent package for hypothesis testing and statistical inference. The data sets are fairly large and are kept in files which can be pulled down from the software menu. Typical questions for study in these units are: "In New England in the period 1630-90, were women more likely than men to be convicted of witchcraft?" and "Is the mathematics SAT score a good predictor of performance in the freshmen year by students who subsequently elect science majors?" The fact that the data sets are real and lend themselves to questions in which our students have a natural interest contributes to the excitement of the course.

For the third unit, which deals with dynamical systems, we use Stella software. Here population growth is studied under a variety of assumptions. Typical instructions for the laboratory are: "Double the rate of food production and run the simulation. What happens to the carrying capacity of the environment? Change the death rate so that it rises more rapidly as food per individual declines and run the simulation. What happens to the population in the long term?"

The course serves a wide variety of students, most of whom will not subsequently study calculus and many of whom would otherwise have avoided coming to grips with quantitative issues. More than half are freshmen. Some will subsequently do further work in statistics, their motivations and abilities strengthened through the experience of this course. As is the case in many situations where much learning occurs, students consider the work of the course very demanding. However, some of the students who have completed the course regard it as being a turning point in their educational experience. One student commented: "It made all the difference in my study of biology and economics."

3. **"Calculus in Context"**. This is an NSF-funded effort to improve the teaching of calculus. The project is in its infancy, so it would be premature to try to spell it out in much detail. I will simply quote from the Project Summary, as it appeared in the proposal:

> We will develop a calculus sequence in which the major mathematical concepts grow out of attempts to explore significant problems from the social, life, and physical sciences. Throughout the courses we will stress four mathematical themes: optimization, estimation and approximation, differential equations, and functions of several variables. The computer will enter as a basic conceptual device for structuring the way we think about problems and what it means to solve them.

Here we use software we created ourselves to meet our needs efficiently and precisely. As do StatView and Stella, it provides a set of tools which, like laboratory instuments, must be manipulated by the students in order to obtain measurements. It is menu-driven and runs on IBM-PC's with CGA graphics. But here, far from having any physical facility which could properly be called a laboratory, we are presently dependent on ten machines scattered across campus, two of which are in faculty offices.

Students begin the semester by obtaining plots of functions and estimating the location of local extrema. The next set of computer problems involves arc length calculuations, which is gives a nice introduction to the idea of approximation and paves the way technically for integration problems, which comprise the next topic.

I will try to give some idea of how integration works. We have control over two points, a "kept" point (X0,Y0) and a "variable" point (X,Y). Initially these coincide, but the location of the variable point is controlled by "L" (left) and "R" (right) keystrokes, which decrease or increase X by an amount H, which is also under our control. A "K" keystroke brings the kept point to the location of the variable point.

To integrate by the trapezoidal rule, we define an operator

$$0.5*(Y+Y0)*(X-X0) + AC,$$

where AC represents an "accumulator". An "O" keystroke performs the operation. Beginning with both X and X0 equal to the left endpoint of the interval of integration, we perform the succession of keystrokes R-O-K sufficiently many times to traverse the interval of integration. There is a also a programming feature which permits the succession of keystrokes to be done automatically.

Next we make estimates of derivatives, using central differences, and the inverse relationships between integration and derivation are explored, all the while computationally and visually.

The final topic in the first semester is differential equations, for which we have also developed software capable of handling systems of up to four equations. In particular, we are easily able to treat the logistic equation and the Lotka-Volterra equations in the first semester.

4. The Laboratory in Mathematical Experimentation. Another NSF-funded project, the Laboratory is intended to encourage students to come to grips with mathematical ideas through experiment, simulation, and conjecture. Subsidiary objectives are to ensure that students are competent in the use of computers; to foster precise thinking and writing; to introduce students to the rich interplay between technology and theory; and to interest more students in continuing their study of mathematics by awakening them to the intellectual liveliness and utility of the discipline.

The course is intended primarily for freshmen and sophomores and will be offered for the first time in Spring, 1989. Novices will be expected to learn enough Pascal to implement the algorithms. In the first offering, we plan to deal with the following topics: chromatic polynomials; the generation and the detection of randomness; distributions of primes; iteration of maps; the Mandelbrot set.

5. A Seminar in Software. In order to stimulate further innovation in computer experimentation, we are presently designing a weekly seminar, which will be organized cooperatively by Amherst, Hampshire, Holy Cross, and Mount Holyoke Colleges. Additional colleges and universities will participate. There will be weekly meetings and an annual conference with national participation, much in the spirit of the Nonlinear Dynamics Seminar at the University of Texas.

We will invite to the weekly meetings people who are doing interesting things in the way of curricular and software development. It is clear from our experience that there is no substitute for <u>seeing</u> what a software package can do, in the presence of someone who knows its capabilities. The seminar will provide a forum for the exchange of ideas on how computers can profitably be incorporated into the undergraduate curriculum, with high priority given to ways of making more mathematics accessible to students, especially at the most elementary level.

6. Why Experiment? One reason is that it's fun. Another is that the computer laboratory offers teachers of mathematics a nice way of bridging the enormous gulf between their training as mathematicians and the mathematical level of their students as they emerge from the high schools. Our experience so far seems to indicate the possibility of teaching mathematics to a vastly larger proportion of the college population. Professor James Yorke of the University of Maryland even foresees the possibility of a ten-fold increase in the number of students who study calculus.

At Mount Holyoke, we are in competition for majors with the other sciences. We found that the lack of a laboratory put us at a considerable disadvantage in certain important respects. For instance, laboratories are places where students spend time and where they socialize.

Also, computer laboratories provide a viable way of extending science education to a broader segment of our students, giving them a much better chance of being able to handle the mathematics which is necessary for science. Most important, computer laboratories are places where students can become actively involved in doing mathematics.