LINEAR ALGEBRA USING SPREADSHEETS

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This paper briefly describes a first course in linear algebra which was taught using a spreadsheet as basic software with no matrix calculation facilities built-in to start with. The following topics were handled using the spreadsheet:

1) Gaussian Elimination
2) Arithmetic operations on matrices
3) Calculation of inverse
4) Calculation of determinant
5) calculation of eigenvalues and eigenvectors
6) Gram-Schmidt Process

Each topic was passed through three stages. First, the students had to work typical problems by hand. This is to enable them to learn the principles. Secondly, they solved similar problems using the spreadsheet. Here they had to know the appropriate formulas but the spreadsheet will do the arithmetic and the repetitive portions of the task. This highlights to the student the advantage of using the computer to solve the problem. The third step was the creation of a template for solving any problem of a particular type. The use of this template was restricted in the following way: a template for a problem of type X should not be used to solve a stand-alone problem of the same type. But if a problem of type Y involves the solution of a problem of type X, the template can be applied to the problem of type X and the solution used in the Y type problem. For instance, the template for Gaussian Elimination is not allowed to be used to solve a problem on systems of equations. But it can be used to solve the systems of equations which arise when calculating eigenvectors.

The class met once a week in the computer lab where they used the computer to solve problems which they did by hand during the other two meetings in the regular lecture room. Each test was of two parts (i) a classroom part involving definitions, theorems and problems of small magnitude (ii) a computer lab part involving a larger

number of problems of bigger magnitude.

The spreadsheet was tried as the basic software for this course due to the following reasons:

1) The software will only do the arithmetic and the repetitive tasks. The student will have to know the principles and the formulas. The software is not a "black box" facility.

2) The student can see all the steps of the solution on the screen including the formulas used. These formulas can be modified any time if necessary without having to start the problem from scratch.

3) There are very few rules to be learnt in order to use the software for the purpose of solving linear algebra problems. Further, the software has a lot of other built-in mathematical functions. Therefore it can be used in other courses like calculus and discrete mathematics too.

4) Spreadsheets like Lotus 123 have a Pascal-like language facility which can be used to write programs which will reside in the particular workfile and aid calculation or other required tasks. Writing such programs which are called "macros" is more easily learned than regular programming languages like Pascal or Basic.

5) In many cases, solving one problem of a particular type automatically creates a template for solving all problems of that type and this template can be used again and again.

Course Material

We give below two computer lab instructions as samples of course material prepared - one for Gaussian Elimination and the other for calculating a determinant. These are based on the spreadsheet Lotus 123, but can be modified easily to fit any other spreadsheet.

Gaussian Elimination

Problem: Solve the system $2x-2y+z=3$
$3x+y-z=7$
$x-3y+2z=0$

Procedure: Enter the augmented matrix in the cells b13 to e15.

2	-2	1	3
3	1	-1	7
1	-3	2	0

To reduce this, we start by making the entry in b13 equal to 1 by dividing the whole row by 2. To do this, go to b20 and type the formula +b13/2. Then, press the keys

/, c, <enter>, -->, ., -->, -->, <enter> ...(1)

This has the effect of dividing the whole row by 2. The idea is that the formula is applied to the first element in the row and just copied to the other elements. For all row operations, the same procedure can be applied. This saves time and the possibility of arithmetic mistakes. We will signify the sequence of keystrokes (1) by "replicate" henceforth. The following table describes the further procedure.

To do row operation	go to cell	enter the formula	do
r2	b21	+b14	replicate
r3	b22	+b15	replicate
r1	b24	+b20	replicate
r2-3*r1	b25	+b21-3*b20	replicate
r3-r1	b26	+b22-b20	replicate
r1	b28	+b24	replicate
r2/4	b29	+b25/4	replicate
r3	b30	+b26	replicate
r1+r2	b32	+b28+b29	replicate
r2	b33	+b29	replicate
r3+2*r2	b34	+b30+2*b29	replicate

r1	b36	+b32	replicate
r2	b37	+b33	replicate
r3/0.25	b38	+b34/0.25	replicate
r1+(0.125)r3	b40	+b36+(0.125)*b38	replicate
r2+(0.625)r3	b41	+b37+(0.625)*b38	replicate
r3	b42	+b38	replicate

This produces the matrix

$$\begin{matrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{matrix}$$

From this you can read the solution x = 2, y = 0, z = -1.

Determinant of a 3 by 3 matrix

Enter the matrix

$$\begin{matrix} 1 & 2 & -1 \\ 3 & -1 & 0 \\ 2 & 5 & 1 \end{matrix}$$

in the cells b5 to d7.

To calculate the determinant of this matrix, enter the following formula in cell b10 :

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+b5*(c6*d7-c7*d6)-c5*(b6*d7-b7*d6)+d5*(b6*c7-b7*c6)
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The determinant is displayed immediately. Now whenever you want to find the determinant of a 3 by 3 matrix, just enter the matrix in the location b5 through d7. Then the determinant is automaticlly calculated and the result is displayed in cell b10.

Exercise: Make a similar template for calculating the determinant of a 4 by 4 matrix.