Using Numerical and Graphical Techniques to Teach Infinite Series

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To understand the fundamental ideas taught in the first-year calculus course, it is helpful for students to see these ideas from several points of view. For example, a student's understanding of the function concept is deepened by working with functions represented as graphs and tables of numbers in addition to functions represented as algebraic expressions. Similarly, by examining other calculus concepts using graphical and numerical techniques, as well as the traditional algebraic methods, students can gain a deeper understanding of these ideas and a better appreciation for their applicability. Although graphical and numerical techniques are part of every calculus course, the computational effort involved in using these techniques is so great that only a very small number of examples can be examined in this way. These techniques can be given greater prominence in the calculus course if students have access to a computer algebra system.

The mathematics department at St. Olaf is currently in the midst of a three-year curriculum development project in which students and faculty use a computer algebra system (SMP) in first-year calculus courses. Support for this project has come from the National Science Foundation's College Science Instrumentation Program and the U.S. Department of Education's Fund for the Improvement of Post-Secondary Education. Students use SMP on Sun-3 workstations to complete locally written homework assignments that emphasize the development of conceptual understanding in addition to manipulative skills.

Most students in first-year calculus courses learn to apply convergence tests to infinite series, but many fail to develop a real understanding of what convergence or divergence means. For example, the idea that a convergent infinite series is equal to a *number* doesn't really sink in. Computer algebra systems can be used to improve this situation. We have used the capabilities of a computer algebra system to allow students to view the sequence of partial sums of an infinite series graphically. We have also augmented our discussion of convergence tests with exercises that require students to compute numerical estimates of convergent infinite series. The result has been a substantial improvement in students' understanding of the definition of the sum of an infinite series and the development of a better intuitive understanding of the idea of rate of convergence.

In this talk I will show some examples that illustrate how we use graphical and numerical techniques to help students learn about infinite series.