

## Technology in Differential Equations

In this presentation I want to share some of my background experiences, humble beginnings, some mistakes made, and some student response to the numerical solutions portion of a differential equations course. Then, I shall conclude with some report on directions I hope to move and what seems to work for my teaching style. If there is extra time, perhaps some persons present will share their use of technology in college teaching.

Over the past eight years, I have taught one introductory course in differential equations six different times. Not having foreseen the need for records of this course, I now wish that I had a more complete file of my experiences in differential equations classes.

### The Early Years

I taught in a community college in central Illinois until 1987. Back in 1979-1980, the differential equations course on that campus had been taught as totally analytic. When I first taught D.E. the following year, I introduced some numerical solutions. The students and I used hand-held calculators, usually TI-55 or comparable models, to do Euler and the Improved Euler Techniques on first order equations.

Our college had one computer terminal on a TSO (time-share option) arrangement. I wrote some BASIC programs which were used for demonstrating numerical solutions. With only one terminal available, student assignments were necessarily short. The next time I taught the course I had a few students with experience writing BASIC and/or Fortran programs. I required all students to input my programs or write their own. From this experience I inherited improved programs for future use (Exhibit "A"). Shortly thereafter, our time share agreement was terminated.

### Micro Computers

The community college established a lab of eight (8) TRS-80's (Radio Shack) computers. We adapted our BASIC programs to the TRS-80 set up. In a few years, we had a control center interfaced with the computers and three printers. At this stage the students no longer needed to input the programs since we could direct from the control station a program to various stations in the lab.

The interface and use of this lab for differential equations was often handled by one of my students who also worked in the computer lab. He was my first of a line of resident experts. [Each time that I have taught differential equations using technology--I have been blessed with a "resident expert" student. This quarter one of my

students is similarly gifted and industrious. I have his permission to include his work (Exhibit "C").

During one academic year, my students used TRS-80's interfaced with the control center, separate TRS-80's, and an Apple II computer housed in the Learning Resources Center at the college.

Several students in my class had Apple computers of their own or had access to them at their local high schools. The students had varied backgrounds. Some had experience with course work in programming while some had no "hands on" experience. Consequently, some were writing their own programs while others used only those programs provided by the instructor.

Soon the availability of textbooks including flow charts and/or sample programs showed that authors and publishers were responding to the modern technology needs of the student.

### Conduit's Differential Equations Program

Conduit's differential equations software package is user friendly. After working with this package for two classes over a period of two years, I have found this program helpful to students. The package is available for use on the Apple II line of computers. One of my more recent "resident experts" managed to use his software and programming ability to produce paper copy of some of the Conduit results (Exhibit "B"). I have become aware that Conduit has an "add on" hardware--Finger Print Plus--which allows printing of the screen images. The hardware is available for Apple II, IIe, and IIGS computers.

### Hand-Held Calculators

No matter what phase I have been working through this eight-year period, I have always included some hand-held calculator exercises for at least the Euler and Improved Euler techniques. These exercises help the student toward a better understanding of what goes into a numerical solution.

Students must perform numerical techniques on many differential equations--both those which are readily solved analytically and those which do not lend themselves to analytic techniques. From the former, students can compare their analytic solutions to their numerical solutions to begin to recognize the need for control of error. Those which are not readily done by analytic techniques, of course, demonstrate the power of technology in conjunction with numerical technique.

Some geometry helps the students appreciate their results. Stepping across the interval using subintervals and isoclines makes the solution more meaningful (Exhibit "C"). The Conduit Program provides graphs and, if desired, direction fields.

## Trends

With the availability of software packages, I no longer teach programming. Time is valuable, and I prefer to use the time on traditional content of differential equations.

By and large, my students have been sophomores in pre-engineering curriculum. They have transferred into programs at the University of Illinois or Southern Illinois University. Most of the students were successful at these schools.

Former students' feedback indicates the need for more numerical work and work with orthogonal trajectories. My own feeling is that I need to pay more attention to existence and uniqueness--before "running to the computer." Also, error control must be watched more closely in the future.

## Recommendations

Assign problems to be done numerically.

Suggest or require that students use a certain software package.

Spell out format to be used for hand-in work.

Require some problems be done with subinterval increments of  $n$  and  $n+1$  evaluations.

By comparing results students can get an idea of what truncation error will do to results. Of course, students should compare the results when increments are  $h = 0.1$ ,  $h = 0.05$ , and  $h = 0.01$ .

Students should compare various carefully selected problems using Euler, Improved Euler, Runge-Kutta methods, and some non self-starting predictor/corrector methods. (These comparisons are readily available in many recent differential equations textbooks.)

## Textbook

In selecting a textbook, John Van Iwarren, author of Conduit software package, has written a textbook specifically geared to the use of technology.

Other good texts with early introduction of numerical techniques could provide the base for an excellent course. [ N.B. In selecting a book, a publisher's claim of independent chapters must be verified by reviewing the content and working some problems.]

In conclusion, even with modest equipment and modest investment, one can have a successful unit on numerical solutions in his/her differential equations course.

### Textbooks

Finizio and Ladas. Introduction to Differential Equations.  
Wadsworth, 1976.

Guterman and Nitecki. Differential Equations: A First Course, Second  
Edition. W. B. Saunders, 1988.

Hagin, Frank. A First Course in Differential Equations.  
Prentice-Hall, 1975.

Kells, Lyman. Differential Equations: A Brief Course,  
McGraw-Hall, 1963.

Van Iwaarden, John. Ordinary Differential Equations with Numerical  
Techniques. Harcourt, Brace, Jovanovich, 1985.

### Courseware

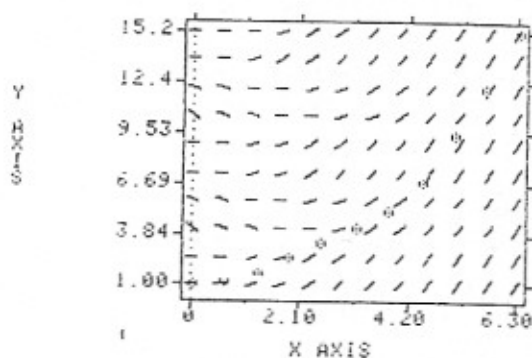
Van Iwaarden, John. Ordinary Differential Equations--Software  
Package. Conduit.

LOAD IMEUL  
LIST

"Improved Euler" Listing

```
10 DEF FN F(X) = - (1 - SIN (X) / 2) * Y
20 PRINT "ENTER LEFT AND RIGHT ENPTS,Y0,NMBIN,PSS"
30 INPUT LEFTX,RIGHTX,Y0,NMBIN,PSS
35 PR# 1
37 PRINT "USING IMEUL": PRINT
40 X = LEFTX
50 Y = Y0
55 Y1 = Y0
60 H = (RIGHTX - LEFTX) / NMBIN
70 PRINT "INTEGRATIONS:";NMBIN
80 PRINT "H:";H
90 PRINT "X", "Y"
100 PRINT X,Y
200 FOR I = 1 TO NMBIN
210 FO = FN F(X)
220 YP = Y + H * FO
230 X = X + H
235 Y = YP
240 ULUF = FN F(X)
250 Y = Y1 + .5 * H * (FO + ULUF)
255 Y1 = Y
260 IF INT (I / PSS) = I / PSS THEN GOTO 280
270 IF NMBIN < I THEN GOTO 290
280 PRINT X,Y
290 NEXT I
295 PR# 0
```

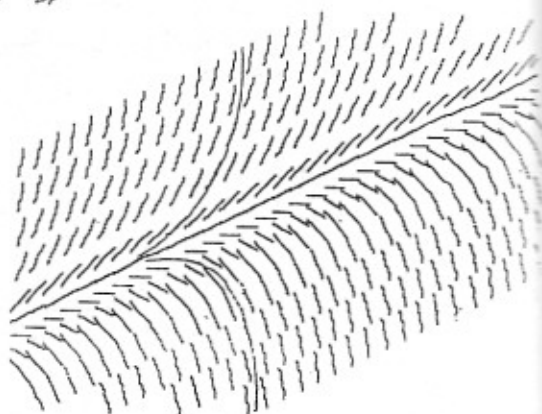
"A"



X	Y
0.0000	1.0000
0.6300	1.0803
1.2600	1.6341
1.8900	2.5172
2.5200	3.3660
3.1500	4.1800
3.7800	5.1588
4.4100	6.8430
5.0400	9.3881
5.6700	11.9629
6.3000	15.2135

$Y' = \sin X + X - 1$   
 $Y'(0) = 1.0$   
 Solution by IMEUL  
 on  $[0, 6.30]$  with  
 $h = 0.63$

$$\frac{dy}{dx} = 3x - 2x^2 \quad \begin{matrix} -2 < x < 2 \\ -2 < x < 2 \end{matrix}$$



LIST

```
70 TS = 1 / 8: OT = (4) / 279: OK =
(4) / 191: L = 10
90 MGR : MCOLR = 3
100 FOR U1 = - 2 TO 4: N1 = 2 *
U1: IF N1 = 3 THEN L = 14
101 IF U1 > 2 THEN N1 = 3: IF U
1 3 3 THEN N1 = 3
102 IF U1 < 7 THEN N1 = 0: U2 =
7: U1 = - 2
103 FOR U2 = 1 TO 2: N1 = N1: IF
U2 > 1 THEN N1 = - N1
104 IF N1 = 0 THEN U2 = 2
107 FLASH : PRINT "PM": NORMAL
110 FOR TP = - 2 TO 2 STEP TS:
T = 139 * TP / OT
120 XP = (N1 + 2 * TP) / 31F = 95
- XP / DX
120 CT = COS ( ATN ( N1) ) * L / 2
: CF = SIN ( ATN ( N1) ) * L /
2
125 PRINT "T": ...CT:CT: ...F:F:
...CF:CF
126 IF T - CT < 3 OR F - CF < 0
```

"C"

"B"