College Algebra in a Liberal Arts Curriculum

James E. Mann, Jr.
Wheaton College
Wheaton, Illinois 60187

It is well remarked that the mathematical skills of college students have declined over the past three decades. Frequently, the fact is stressed that students need mathematics for later life, that mathematical training produces clear thinking and that our scientific future is being threatened by this lack of mathematical training. Nevertheless, many students come to a selective liberal arts college, such as Wheaton, with extremely weak mathematical preparation. Many of these students would like to avoid mathematics altogether, but because mathematics is required for those who have low test scores, they are forced to take mathematics of some type. At Wheaton, there are three options available: improve the test score, or take finite mathematics, or take college algebra. I wish to address the content and technological support of the college algebra course.

Here are the premises for the discussion:

1) College algebra courses offer little that is not in a second year high school course.
2) Most students have had two years of high school algebra.
3) Students think they will not use the material in their academic or vocational life.

If premises (2) and (3) are true, then students who take college algebra feel they are being penalized by having to repeat material that is essentially useless to them. How then, can we offer a course that will cover algebra without that course being too hard or repetitious? The answer depends on whether they have actually had two years of high school algebra. If they have, then a similar course in college is bound to be boring and repetitious. We can also assume that they did not like it the first time. For this group, it is possible to design a course that emphasizes the use of functions, the graphs of functions and some of the ideas of calculus. The functions used in the course will be polynomials or rational functions, but these must be made the prototype of all functions. The idea is to give the student plenty of opportunity for manipulating polynomials and at the same time give them something new by introducing the idea of the derivative. Though many of the technical ideas from algebra such as factoring may not be useful to liberal arts majors, the idea of a functional relation between variables and the rate of change of such a relation is likely to be helpful to many.

Since the algebra and calculus are to be based in graphical and intuitive ideas, it is important to have some examples of functions that the students construct from their own environment, e.g. number of words written vs. day of the week or number of pages read vs. day of the week. After making graphs of a few everyday functions, the students will make a transition to polynomials. Graphing a polynomial is a tedious job which is made much easier by the technology which is the subject of this conference. However, I think that it is better not to give the students at this level the most advanced graphing capabilities that are available. It is important to emphasize the relation between ordered pairs and the graph by having the students actually locate the points on the graph paper. It is important to have students think about the scale of graph so that the points fit on the paper. If they think about these things, we will reinforce the idea that many everyday events can be thought of in graphical or functional form. If the student is given a graphing engine that takes a whole polynomial and makes a graph, the student gets a graphical picture of the function that he learns to associate with the whole polynomial formula rather than with the individual point pairs. As I want to give the students the latter rather than the former picture, I argue against the use of the highest technologic solution for graphing. Nevertheless, calculating many ordered pairs for a fifth degree polynomial by hand would be considered inhumane in this decade; the calculation of the points is tedious even with a simple calculator. I opt for an inexpensive, programmable calculator. I insist that everyone have the same type of calculator or that the student himself be responsible for programming his model. Currently, I recommend (or insist upon) the Texas Instruments TI-60 because of its low price and ability to retain a general program for evaluation of fifth degree polynomials.

I do not attempt to teach the students to write their own programs; rather I take each calculator and enter the program for the student. The program is written so that all the student has to do is enter the coefficients into the appropriate storage registers, enter the value of x, push the R/S button and have the value of y calculated. Each student is provided with a written version of the program so that if the stored one is lost in the calculator it can be re-entered. Some instruction is given on how to use the program and how the polynomial is evaluated (Horner's method).

The calculator is used in several ways. First, the student is expected to make modest sized tables of xy-pairs and plot the graph of polynomial. Second, the student is taught to calculate the slope of a secant line, by graphical and analytic techniques. The TI-60 calculator has so few registers that it is necessary to write the values on paper to make the slope calculations (this is a serious limitation of this particular calculator). Once students can calculate the slope of the secant, they calculate sequences of slopes for x-values closer and closer together. A third use of the calculator is to locate approximate zeros of the polynomial by interval halving methods. Most of the calculations mentioned would be too time consuming without at least a programmable calculator. I claim that this piece of equipment is at once the minimum that will do the job, that it easy to teach the student to use, that it reinforces the proper ideas, and that it is cheap enough even for those who will not go further in mathematics. Therefore, I claim that the programmable calculator is the optimum equipment for the task (optimal claims do not extend to the particular calculator chosen).

Let us now turn to what calculus can be covered in a short course. The first task is to get the students to infer the derivative of x^n for several values of x and a particular value of n from the calculations that they can make with their calculator. The motivation given is that they are finding the rate of change of the function. Once they have made several inferences, something of a theoretical demonstration of the correctness of their result is given. The result is generalized to a polynomial and to rational values of n. By multiplying polynomials and differentiating the result, the product rule can be derived. The chain rule can be done the same way. As students do these things, it is important to remember that they are supposed to be practicing the algebra of polynomials; carrying out the above verifications gives ample opportunity for practice. Once several rules have been found, we can turn to the idea of extreme values of polynomials. With more graphing, the student can locate maxima and minima of several polynomials. At this stage, it is helpful to have some examples of polynomials that it would be interesting to maximize (profit, cargo load, area enclosed, etc). After looking for the relative maximum on a graph, someone will surely notice that this maximum occurs at the point that the derivative is zero. This observation, when coupled with looking for the derivative of a cubic, provides a reason to develop the quadratic formula. At this point students have a reason for wanting to know the zeroes of polynomials. Thus they can be taught to use their calculators to find zeros by the interval halving or other methods.

Dealing with integral calculus is also easy in that the calculator makes possible the evaluation of approximations for different subdivisions of the domain. The fundamental theorem of calculus can be illustrated numerically and supported by reasonable theoretical arguments. The harder part of treating integral calculus is to give practical examples of the use of integration that involve polynomials only. After all, it is seldom that one is interested in the area of some figure bounded by polynomial curves. A tentative solution is to introduce first order differential equations (rate problems) for practical examples. Students are told that it is frequently easier to characterize the rate of change of a quantity than the quantity itself. The difficulty with differential equations is that we must find problems whose differential equations are separable and whose integrals are algebraic functions. Even though these constraints are limiting, there are enough problems to give students insight into the use of integration.

Conclusions:

College algebra can be taught from a numerical, graphical point of view. A programmable calculator makes teaching from this viewpoint practical. It is also possible to reinforce polynomial manipulation by doing the calculus of polynomials. The value of this approach is that it goes much farther intellectually than does algebra but it does not require the bag of tricks associated with a full calculus course.