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A BETTER (Differential Equations) MOUSETRAP

As a teacher of mathematics I am primarily interested in educational technology as a source of tools that will enable me to better convey the concepts of my discipline. In one aspect of my differential equations course I have been able to develop a tool, a sequence of computer exercises, that has allowed me to go beyond anything I was able to accomplish using only words and chalk. Furthermore this success comes at no cost in class time, and at little cost in additional work for either myself or my students.

The course for which I developed the computer exercises is an introductory differential equations class; it has a year of calculus and a semester of linear algebra as prerequisites. The course serves about 35 to 40 students, in two sections, each spring. The students are sophomores, juniors, and seniors, with about two-thirds majoring in mathematics and the remainder drawn from physics, economics, etc. Most have not taken a college computer course. The text for the course is Boyce and DiPrima [1]; the course generally follows the text although with the linear algebra prerequisite I can do more with the linear operator point of view than the text does.

The specific aspect of differential equations that I addressed was the task of introducing students to the most basic ideas of the qualitative analysis of differential equations. In particular, I wanted a more effective way of teaching the material in Sections 2.6 and 7.5, 6, 7 of Boyce and DiPrima. This is a small part of my course, involving at most two or three lectures, and my expectations are correspondingly modest. I would like students to be able to sketch the direction field and the integral curves of a simple first order autonomous equation. Later in the course I want students to be able to draw the phase plane of a system of two equations with constant coefficients. Most of all, however, I want students to have an understanding of the meaning of the pictures. With only chalk and words I was not able to meet even these simple goals. Too many students became lost in the mechanics and details, and they never approached the conceptual understanding that I wanted.

Three years ago I turned to the computer for assistance, using a prototype program of uncertain antecedents that did what I required. The program (for a Macintosh) graphs the direction field of an entered equation or system, and draws in trajectories at the click of a mouse. Around this program I constructed two exercise sets that explore the mathematical behaviors of interest, and which bring in applications to give the results meaning. The sets are not perfect, even after several trials and many errors, but my main problem now is an embarassment of riches. The material has become quite easy for students. For example, I asked the following question on an exam last spring:

Let $\mathbf{X}' = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}\mathbf{X}$. The general solution is $\mathbf{X}(t) = c_1\begin{bmatrix} 1 \\ 2 \end{bmatrix}e^{-t} + c_2\begin{bmatrix} 2 \\ 1 \end{bmatrix}e^{2t}$.

Sketch a representative selection of trajectories in the phase space (i.e. the xy-plane). Label the trajectories with arrows in the direction of increasing t.

The exam was given in a standard setting, with only pencil and paper to use as tools. Every one of my 36 students answered this question correctly. Obviously what I need to do next is raise my expectations.

General Observations: The exercises I constructed are not models of ingenuity -- they aren't even particularly clever. They are, rather, very specific exercises based on a particular text and designed to work in concert with the text (and my lectures). The program I have is clever; it takes full advantage of the Macintosh capabilities to do exactly what I want it to do -- illustrate the graphs of solutions of a differential equation or a system of two differential equations. This is also all the program does, and the version I have has some serious flaws (for instance, attempting to print causes a system error). Thus the work I am reporting is not going to revolutionize education or even the teaching of differential equations; it is a limited experiment with limited relevance to other campuses.

The significance of my work, to me, is that of an existence proof -- it shows that "a better mousetrap" is possible. With technological assistance I have done a better teaching job than I was ever able to do before. I do not know how or to what extent this success can be duplicated in other areas, but at least now I have a rational basis for hope and some experience to go on.

An additional observation, learned through hard experience, I would like to share is that technology alone will rarely be a complete answer. When I first constructed my computer exercises I was not happy with the results. I felt the students had gained insight, but that insight did not show up on test scores. The problem, obvious in retrospect, is that there is limited transferability of knowledge and skills -- and I was giving the students computer work and then testing the results with pencil and paper questions. Since computer testing was not feasible, and since I was not interested in testing my students' facility with the graphing program anyway, the solution was to follow-up the computer work with pencil and paper homework and this combination was the successful one.

Finally, I would like to offer a few comments on educational technology, on what I would like to see, and on what I would use. The first comment is that, while revolutions may be exciting, revolutionary technology may not serve as intended. A classic example, one that no one knows what to do with, is the calculator. Simple numerical calculation is trivial these days, yet The Mathematics Report Card [2] found that only 26% of American high schools had calculators for use in 11th grade math classes. The public schools have an elaborate structure for teaching numerical calculations, and much energy is devoted to this task. Technology may have made (much of) this effort obsolete, yet it goes on with no end or even abatement in sight.

Personally, I am interested in revolutions but I am vitally involved in what I will be teaching next week -- and if technology is going to help me then there is where I would like the help. I want tools that will allow me to do my work better, or easier, or quicker. I want tools that require no training to use. My syllabii are already too full; I do not have time to teach the use of teaching tools, no matter how intrinsically interesting they may be. I also want specific suggestions and directions as to how to use the tools. I don't want a puzzle to figure out; my time is limited too. The appropriate model may be an exercise set. Give me a clear set of tools and let me select the combination to fit my work.

I don't know if such tools are economically feasible. This country has largely agreed on a single language, English, and textbooks can be marketed anywhere in the country. There are no such standards for hardware and software, so the educational technology market is fragmented. Obviously there are many problems, but there is also hope.

References

1. W. E. Boyce and R. C. DiPrima, Elementary Differential Equations and Boundary Value Problems, 4th Ed., John Wiley & Sons, 1986.

2. The Mathematics Report Card: Are We Measuring Up?, Educational Testing Service, June, 1988.