A Special Calculus Course at UNL

by J.A. Eidswick

1. Background. Last spring it was decided that UN-L should better serve a special group of students: those entering the University after successful completion of one full year of high school calculus. In the past, such students had a difficult choice: enter at a lower level and risk being bored stiff or enter at a higher level and risk getting blown away.

It was also recognized that such students, given proper enlightenment, might develop into mathematics majors. At present, Nebraska's undergraduate mathematics program is generally perceived by the mathematics faculty as being close to nonexistent, so there's a definite need for attracting such students into mathematics.

It was thus decided to offer a two-semester "enriched" calculus course which would be tailored to the needs of this special class of students. Information and forms were mailed out late last spring to all Nebraska high schools involved in the teaching of calculus. In addition, records of incoming freshmen who had taken a year of calculus were checked for ACT/SAT scores, course grades, and other information. Ultimately, 123 students were invited and 21 signed up.

The course was billed as one in which enrollment would be limited to 25 (noting that our usual Calc II classes have as many as 120 students) and in which special emphasis would be placed on the concepts of calculus and the meaningfulness of those concepts. At the same time, good use would be made of scientific calculators, computers and other modern technology. A background in computers would be helpful but not necessary. One special feature of the course would be to introduce students to a "remarkable new calculator, the HP28S". Students could purchase a calculator for about $150, or if they did not wish to purchase one, they could use ones which would be on reserve in the math library.

From the beginning, we didn't know whether to call this an "honors course" because we didn't know if it would attract that kind of student. As it turned out, it did and it now has that distinction.

Sometime before all the work began, I was asked by the chairman of the department if I would like to teach the course. My response was that I would be delighted to do it, but that it would be time-consuming and I had a couple of requests. One was that I be given an assistant who could take the class occasionally and help with grading and other chores. That request was granted. The assistant that was assigned is a very able undergraduate mathematics major with a good deal of experience helping calculus students. A better choice could not have been made! [I had also requested appointment of an advisory committee which would have included a couple of key research people from the department. That request was denied because at least one of those key research people was known to regard activity such as this as counter to doing research.]

2. The starting point. The first problem was to determine the starting point for the course. This was solved (as well as it could be) by holding interviews. The entire second day of the semester was devoted to

that purpose and proved to be time well spent. Here's a summary of what was learned from the interviews and the resulting actions that were taken:

(i) Two of the original 21 students were found to be inadequately prepared. Their "calculus" courses had really been special topics courses in which some polynomial calculus had been taught. They were quickly transferred to a more appropriate class. All others, except one, had studied at least the equivalent of the first nine chapters of Howard Anton's "Calculus with Analytic Geometry". However, most felt a little rusty about the material and it was evident that a review would be welcomed. One student, who had grown up in Germany, had formally studied only the differential calculus but expressed confidence that he could pick up integral calculus on his own, and he was allowed to remain in the course.

It was decided that the course would officially begin with Chapter 10 of Anton. Chapters 1-9 would be regarded as review material and Chapter 12 (conic sections) would not be covered at all.

(ii) Computer backgrounds varied greatly. Two or three students were clearly in the "hot shot" category; others had not yet learned to appreciate wordprocessing. With the idea of keeping communication lines as open as possible, I had initially planned to have students keep "daily journals" of their reactions to the course. The idea was that they should keep a running record on a disk and submit a print-out every week. The plan was dropped when it was found that: (a) it would be a hardship for some to make daily trips to access a computer, (b) students seemed conducive to taking an unstructured course, and (c) all students indicated a willingness to be open about their feelings. More about this later.

(iv) A few of the students expressed a desire for longer, more challenging problems than the usual textbook problems. Because of this, I decided to offer a "smorgasbord" of special, nontraditional, problems and to require each student to solve and turn in at least two of them by the end of the semester. More about this later.

Of the 19 students remaining, 16 chose to purchase the HP28S calculator. (Two tried at first to get by on HP28C's but eventually found that the memory restriction of the HP28C was a serious handicap.) Two calculators and a printer are being kept on reserve in the math library exclusively for the use of the class.

3. <u>Calculus and calculators</u>. My plan was to carefully review the main concepts of Chapters 1-9 and quickly review the routine material (like differentiation and integration techniques). In addition, we would take whatever time was needed to explore what the class found interesting. We would take time for calculator activities whenever appropriate. From the beginning, I resisted the temptation to lecture explicitly about the calculator or about programming; when relevant activities were encountered, students were instructed to read designated portions of the "Owner's Manual".

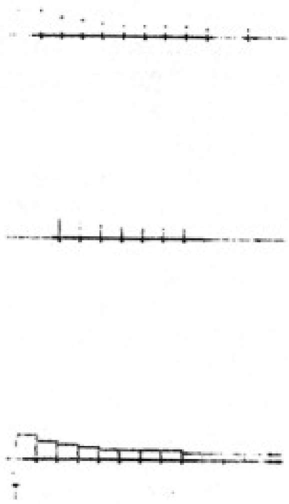
The calculator got into the act right away -- and has been there ever since -- mainly because of its excellent graphing capabilities and because of the close connection between calculus and properties of graphs. These machines are truly remarkable, and wonderous discoveries are being made on almost a daily basis.

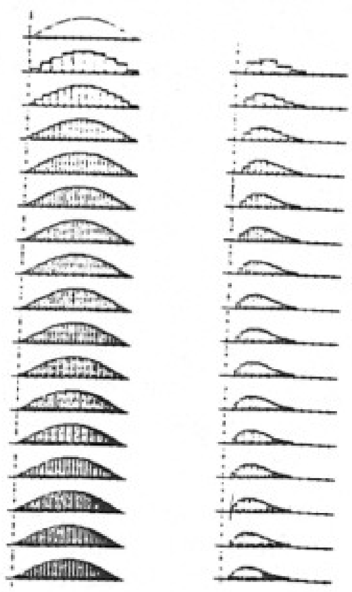
After only 8 weeks, the following four very effective ways of utilizing these machines have emerged:

(1) To obtain approximate solutions. The two most apparent areas of application: (i) analysis of graphs, including finding zeros, extreme points, intervals of monotonicity, intervals of concavity, and inflection points and (ii) evaluation of definite integrals (the ease at which these machines solve arc length problems is especially impressive).

(2) As a pedagogical tool. A moving picture is worth a thousand words -- especially if you can hold it in your hand!

EXAMPLES: Geometric interpretations of sequences, series, and convergence of Riemann sums.





(3) To make conjectures.

EXAMPLES:

1. $1 + 1/2 + 1/4 + 1/8 + \dots = ?$

2. $\lim x_n = ?$, where $x_1 = \sin a$ and $x_{n+1} = \sin x_n$ $(n = 1, 2, \dots)$

3. $\lim y_n = ?$, where $y_n = n \cdot x_n^2$

4. $\int_0^{\pi/2} [(7x^2 - 3\pi x) \ln(2\sin(x))]\, dx = ?$

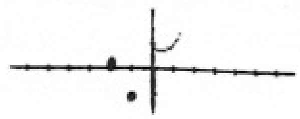
(4) <u>From limitations to concepts</u>. No matter how sophisticated calculators and computers get, there will always be limitations. The appropriate answer to the inevitable question, "what went wrong?" is that the machine simply didn't understand the concepts well enough. Conceptual understanding plus machine capability is a hard combination to beat!

EXAMPLES.

1. $\int_0^\infty e^{-t} t^{-1/2}\, dt = ?$ 2. $\int_0^{2/\pi} \sin(1/x)\, dx = ?$

3. $\int_0^2 [\text{IFTE } (x < .5 , 0 , .5) + \text{IFTE } (x > .5 , 0 , .5)]\, dx$

4. F(x) = (1-cosx^6)/x^12 ?? 5. G(x) = x^x



What are the questions?

6. H(x) = .5 + (1-(1-.5*x^100))/x^100 ??

4. <u>Other technology</u>. Limited use in the course has been made of a computer/Data Display/graphing software combination. One interesting story: In an experiment with a graphing program, it was noted that the fifth derivative of x^x resembled a straight line. Obviously, the sixth derivative would look like a horizontal line. Wrong! Due to an inflection point that was hard to detect, the sixth derivative actually looked much more like a parabola than a horizontal line.

Limited use also has been made of MACSYMA to solve initial value problems which arise from pursuit problems.

5. <u>Adjustments</u>. About four weeks into the semester, it became clear that communication lines between me and the students were <u>not</u> as open as they should have been. Some students were apparently having doubts about their abilities, and tensions and frustrations seemed to be building up. So the decision was made to go back to the "daily journal" idea, this time allowing pencil and paper journals. The decision was the right one.

One source of anxiety turned out to be the "special problems". Even though these problems were within striking distance for most students, lack of success on one or two problems apparently had a rippling effect and resulted in general discouragement for some.

6. <u>Future plans</u>. Next semester UN-L will offer a calculator enhanced linear algebra course for students who have excelled in calculus. Next fall, we are planning a repeat of the present course plus, possibly, a special differential equations class.