TEACHING CALCULUS WITH MACSYMA

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INTRODUCTION

In light of the recent developments in the area of applications of computer technology, from sophisticated computer algebra systems (CAS) to calculators capable of symbolic, as well as numerical computations, those concerned with the teaching of mathematics are faced with a serious question. This question is no longer whether or not computers should be used in the classroom, rather,it is how can the existing technology be most effectively used as an instructional tool.

The following is a brief account of how the CAS, MACSYMA, has been employed in the instruction of a three semester sequence of calculus. MACSYMA is a program which is capable of performing complex symbolic and numerical calculations and is available from Symbolics Inc. Computer Aided Mathematics Group.

FORMAT

Currently, MACSYMA is utilized in the instruction of one section of each part of the calculus sequence, which begins with the introduction of one variable calculus and ends with applications of multivariable calculus. There are four hours of lectures in calculus per week following a standard text. An additional one hour of discussion centers on the main theme of the topic in connection with the employment of the relevant capabilities of MACSYMA.

The computer sessions are usually conducted in a classroom which is equipped with a terminal and an overhead projector which is capable of projecting images from the monitor onto a large screen. Within this environment, the students are encouraged to fully participate in the process of experimentation by generating ideas and constructing interesting problems. Often the students' comments and questions bring about disscussions which help further explore the particular area of calculus being taught.
It is not uncommon for the instructor to be faced with questions concerning some of the capabilities of the CAS. However, direct answers to such questions may not be immediately apparent. Experience has shown that a teacher can frequently elicit worthwhile efforts from a large number

of students by posing the problem, with some guidance, as a challenge. This approach often leads to a continuation of the computer session discussion into the following lecture period, where students are eager to share their findings with the rest of the class and gain recognition for their efforts.

In addition to the usual quizzes and tests, short weekly assignments are given which concentrate on the main ideas of the topic under discussion. These assignments can be broadly divided into the following categories: 1) familiarity exercises--at the introductory stage of the course it is necessary to assign simple problems most of which can easily be solved without the use of the computer.

This will enable the students to achieve a basic understanding of the command structures of the CAS. For instance, one could require the use of appropriate commands in evaluating certain limits, derivatives or integrals. It has also proved interesting to give the students the task of constructing, within the limitations of the CAS, some elementary proofs which are given in the text. A typical case may be the proof of the quotient rule for differentiation. What plays a central role here is the process of translating a sequence of mathematical statements and operations, which are deductively related, from the language of the written text to the symbolism of the CAS. Inevitably, this process requires a firm understanding of the method of proof as opposed to the memorization of the intermediate steps. 2) Secondary problems-- among these one can include problems which may be conceptually easy to understand but cumbersome to compute (e.g."messy" inequalities or numerical integration with error calculations). This is where the students have to get their "hands dirty" with details and develop a sense for what is a "good" approximation in contrast to an exact solution. 3) Graphing and related applications-- under this category one can examine a variety of problems in relation to functions and their graphs. For instance, the salient features of graphs of large degree polynomials or of functions which exhibit rapid changes within a small interval (e.g. sin(1/x)) can be extracted with relative ease. 4) Simple programming problems-- sometimes a frequently required sequence of computations cannot be performed by a single command from the CAS. In such cases the students may be asked to write a short program which will perform all the desired calculations.

It may be noted here that by requiring the students to include appropriate comments in their assignment outputs, whether they have been successful or not in obtaining a correct or a meaningful solution, one can encourage and stimulate the process of critical thinking as opposed to what we usually do.

OBSERVATIONS

One of the most difficult problems in teaching an undergraduate mathematics course where the students have access to highly powerful symbolic manipulators is to clearly define and impose a set of constraints on such usage. As a result the task of designing problems which are suitable for an interactive session with the computer is a delicate one. The skill related problems found in most standard calculus text books can easily be viewed as trivial by the students. This could easily lead to a greater misunderstanding of mathematics as a trivial manipulation of symbols which a computer can perform much more efficiently than we can. On the other hand a problem which involves a large number of calculations as well as some relatively difficult concepts, for the average student, can be so overwhelming that the student may easily loose all confidence in his/her mathematical abilities. Aside from choosing suitable problems one can counter-balance such negative aspects by frequently discussing and sharing with the students the rationale for employing a CAS in various situations. Therein lies the major function of the teacher. The teacher must select problems and pose questions that enable the student to not only see the rationale for employing a CAS but also to build a conceptual framework of the concepts of calculus. The question of an appropriate skill level necessary will be an issue debated for several years.

CONCLUSION

In this project, the student interaction with the computer has been conducted separately from the standard part of the course. Consequently, the changes in the existing curriculum constitute relatively minor shifts of emphasis over certain topics. In this context the basic concepts and applications of calculus require considerably more time than the techniques of differentiation and integration. The reason for adopting a parallel approach is two fold. First, it is believed that until there is consensus within the mathematical community on the merits of utilizing a program such as MACSYMA the students enrolled in these courses should have the option of transferring to a standard calculus course without being at a disadvantage. Secondly, after completing one part of a calculus sequence with MACSYMA some students may have no alternatives, due to timetable constraints, but to study the next part without MACSYMA (or vice versa). This problem is clearly dependent on the lack of the required funds which limits the student access to the computer equipment. The present technology indicates that microsystems are more viable than mainframe in projects of this kind.

Successful implementation of an integrated approach with the appropriate curriculum changes requires the full support of the academic institution. Perhaps only then can the merits of such reforms in the instruction of calculus be fully assessed.