Curriculum changes in calculus for scientists and engineers.

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This discussion concerns the teaching of calculus to students majoring in science and engineering at a land grant university such as North Carolina State University. It is not concerned with the teaching of students majoring in mathematics.

In the excellent pamphlet "College Mathematics: Suggestions on How to Teach it", prepared in 1979 by the Mathematical Association of America, the Forward is taken from an article written in 1971 by Professor Peter Hilton. He suggests that the teaching of mathematics to future mathematicians is effective. Then he writes: "However, the main point to be made is that we are far less successful in teaching effectively those who are not destined to become professional mathematicians; and these, of course, constitute the vast majority of our clientele as teachers of undergraduate mathematics." Since that statement was made, computers and their accompanying technology have revolutionized many aspects of our lives; but the teaching of calculus has remained in a backwater; virtually untouched, except by a few people on an experimental basis. Apart from some cosmetic changes, our textbooks could have been written many years ago. It is time for radical change.

One of the consequences of today's technology is that those in control of the curricula of engineering students are demanding that the service courses in mathematics move more rapidly, while containing more material. My own department has aquiesced to such demands by simply concentrating further the traditional syllabus, with less than admirable consequences. Again, we need to change the syllabus.

The ingredients of the traditional syllabus, with the order in which they are addressed, and their accompanying "applications", congealed many years ago in an age when calculation, if it took place at all, was limited to the slide rule. The slide rule, physically, has gone. The mental attitudes that accompanied its use are still with us. How can we justify today such assignments as: "Solve an equation (non-linear) correct to one place of decimals", or "Use differentials to approximate the square root of 4.1"? Yet similar sentences appear in all of our contemporary textbooks. (And I have seen the approximation to the square root of 4.1 is described as "good".)

"Applications" form a large part of our syllabus. One response of authors to the need for change is to increase the numbers and range of the applications: this is particularly noticeable in introductory textbooks on ordinary differential equations. Presumably we teach applications to demonstrate how calculus can be actually applied. But some of my colleagues in engineering treat our "applications" with open contempt. I have heard these applications called "trivial", and unrelated to the material as it is used outside the calculus classroom; further, I have been told, they are presented by mathematics instructors who are unfamiliar with the subject matter. The engineers may have a valid point. For instance, many texts on differential equations, and nearly all texts on calculus include a discussion of orbital motion, including so-called "proofs" of Kepler's laws. I have yet to see a single presentation of this material that avoids fundamental error: an error that Newton himself corrected. It is clear to me that the authors are not competent to present this material, and that their textbooks are unusable for its presentation. At a time when the buzz word in curriculum review is "lean", it can be argued that "applications" represent fat.

Changes in the curriculum can be considered in various catagories:

1. Existing material can be dropped. Apart from many applications, some of the basic mathematics might go. Some examples: Most of the coverage of the convergence of infinite series could disappear without endangering the career of a single scientist or engineer. The same applies to the Wronskian. While "exactness" is important in calculus for future application elsewhere, "exact differential equations" are essentially unknown outside the classroom, and need not be taught. Many other similar assertions could be made.

2. Existing material can be modified. For instance, "curve sketching" is, in practice, mechanical drudgery for the student. But since curves can be graphed very easily, and almost instantaneously, on a computer monitor, visible to an entire class, or on a student's hand held calculator, we can now place our emphasis on the <u>interpretation</u> of curves rather than on their mechanical production. Newton's method for a single equation, which at present is applied with just two or three iterations, can be carried through to convergence to the full accuracy available: then qualities of the convergence can be observed and taught, and the method instantly compared with others, such as the secant method.

3. New material can be introduced. Newton's method for systems of equations, for example, or the method of steepest descent (easily implemented, and a beautiful application of the gradient vector). These topics cannot be tackled without at least a key-programmable calculator. Others will become accessible as programs for symbolic manipulation remove the drudgery from albegraic work. Power series are easy to manipulate on a computer, and can be applied to problems such as the numerical solution of non-linear differential equations. Other topics that are not part of calculus may be called for: those, for instance, traditionally found in the areas of difference

equations, linear algebra or finite mathematics.

4. The notion of "application" can be changed. The calculus and differential equations that we teach are, of necessity, so elementary, that only the most rudimentary physical systems can be discussed. Most non-trivial problems (especially if the field of differential equations) require computation. But if computational facilities, such as a PC are accessible, a freshman or sophomore can use calculus to discuss and solve problems that were, until recently, in the domain of graduate schools, and therefore never even seen by most students. Assuming that we teach applications to demonstrate how the mathematics can be usefully applied, we should use any available tool, and, especially, the computer.

5. The way in which we teach can change. Unless we ban calculators from the classroom, we must assume that students will have access to fundamental formulas through storage in calculators. So we must abandon putting a premium on the memorization of formulas. Instead, perhaps, we can concentrate more on technique and interpretation. Computers can be used in class to produce accurate figures. Three-dimensional figures, viewed with correct perspective, can ease the teaching and learning of multivariate calculus. Solutions of differential equations can be shown in animated form, emphasising the dynamic nature of differential equations. In all of these demonstrations, there can be interaction with the class, with students suggesting different parameters, such as the direction to view a three-dimensional surface, the height of a countour to be plotted, or initial conditions to a differential equation.

Curriculum revision requires experiment and argument. It also involves, ultimately, textbook revision, and this is a major problem, because of the crippling expense to the publisher. But perhaps the traditional type of textbook (all 1000+ pages of it) should become as obselete as the traditional curriculum: we should certainly consider alternatives. The chapters of a book cmight be published as separate modules, with options as to coverage, especially with regard to applications. If we can bear to dispense with glossy pictures, then a department, on the adoption of a text, would purchase rights to its reproduction from the appropriate diskettes. The material in a module could include software for computer aided instruction, supplementary exercises to be performed by student at the keyboard, and computer demonstrations that can be used in or out of class, with source code that can be understood and modified by the user.

For a presentation of this kind I can only apologise that the brashness of the comments cannot be balanced by more specific ideas. I have written further material in the following references:

"Learning Multivariate Calculus with the Help of the Computer." Collegiate Microcomputer, 1, 263-272, 1983.

"Computing Applications to Differential Equations." Prentice-Hall, 1985.

"Computer Applications to Differential Equations." From "Computers and Mathematics, MAA Notes, Number 9, pp. 73-78, 1988.