Using Computer Algebra Systems in Calculus

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Kenyon College is in its second year of using Computer Algebra Systems, MACSYMA and Maple, in the entire calculus sequence. After studying reports [1 and 2] on the need for calculus reform, the mathematics department agreed to use MACSYMA, and later, Maple, in the three semester calculus sequence. Our goal was to use the software package to compute limits, derivatives, and integrals that made up the bulk of the calculus curriculum. Now, the bulk of instructional time is devoted to teaching the concepts of calculus. We want our students to understand the meaning of limit, derivative, integral and how to set up and analyze the computations related to these concepts. Students view what instructors say and what instructors spend the majority of instructional time on as important. Therefore, now our students are learning that calculus is not merely a bag of computational tricks.

The syllabus has to change, not so much in the topics that are presented, but in the methods used to present the topics. The majority of lecture time is spent exploring definitions and theorems through graphs, applications, examples, and counter-examples in which the ideas are emphasized and the computer is used as the tool to carry out the computations. In Figure 1 is a list of traditional first year calculus topics that have been omitted, that receive reduced attention, or receive additional attention. Kenyon uses Stein's Calculus and Analytic Geometry textbook. Notice that many of the techniques of integration have been deleted. The time saved by not teaching these computations allows us to delve into certain other topics.

Deleted topics:
- Integration of rational functions
- Integration by partial fractions
- Integration of powers of trigonometric functions
- Integration by trigonometric substitutions

Deemphasized topics:
- Computation of limits
- Product, quotient, and chain rules for derivatives
- L'Hopital's rule

Added/expanded topics:
- Bisection method (supported by computer graphics)
- Applied max/min problems
- Newton's method
- Deriving Simpson's method
- Estimating definite integrals

Figure 1. Changes in the Kenyon College Calculus Courses

Currently, the calculus courses are using Maple, Version 4.1, on the time sharing VAX 8600 running VMS. MACYSMA is also available on the system, but using VMS, it can only support a few simultaneous users. Maple is our system of choice for class use. A number of terminals are available around campus and in the dormitories. A terminal with an overhead display device is in each mathematics classroom. Each mathematics faculty office has a terminal.

A lab session in the Computer Center introduces beginning calculus students to Maple at the start of the semester. The Computer Center has 27 terminals so that each individual gets hands-on work with Maple as the program is being demonstrated. The student is also provided a Maple Card, a mini-manual of Maple detailing how to use the program and commands. Students who have never used a computer find Maple easy to learn and use. Programming is not required.

Exams are given in the Computer Center. Maple is used as a super-calculator during these exams. Although it would be foolish to have numerous computations on the exam, some are included to determine how well students use Maple. The bulk of the exams are comprised of conceptual questions. Below are some sample problems taken from final exams and worksheets in the first year calculus course:

1. Let $T(t)$ represent the temperature in degrees at time t (in hours) on a certain day. Explain the behavior of the temperature given the following information:

$$T(4) = 45, \quad T'(4) = 0, \quad T''(4) = 3,$$

$$\frac{\int_0^4 T(x)\,dx}{4} = 30$$

2. Let $f(t) = \int_{-1}^{t} [\cosh(x^2+1) - x - 2]\, dx$

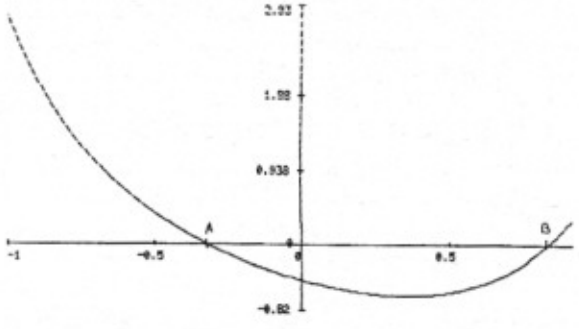
Find the value(s) of t on $[-1, 1]$ so as to maximize $f(t)$ and to minimize $f(t)$. (WHAT FOLLOWS (Figure 2) IS A MAPLE SOLUTION)

> $y := \cosh(x\hat{}2+1) - x - 2$:

(CONSIDER THE INTEGRAND)

$y := \cosh(x^2 + 1) - x - 2)$

> plot $(y, -1..1)$; (PLOT THE INTEGRAND, NOTING THE ZEROS, A AND B)



> fsolve $(y = 0, x)$; (NUMERICALLY COMPUTES A ZERO)

-.3244352253

> $b :=$ fsolve $(y = 0, x, 0.5..1)$; (COMPUTE THE OTHER ZERO BY SPECIFYING AN INTERVAL IN WHICH IT OCCURS)

.8390836331

(BY THE GRAPH IT'S OBVIOUS THAT THE MAXIMUM OCCURS AT POINT A. IT IS NOT SO CLEAR IF THE MINIMUM OCCURS AT THE POINT B OR AT THE ENDPOINT $t = -1$.)

> int $(y, x = -1..b)$;

$$\text{int}(\cosh(x^2+1) - x - 2, x = -1..8390836331)$$

(MAPLE WAS NOT SUCCESSFUL IN FINDING AN ANTIDERIVATIVE)

> evalf (");

(NUMERICALLY EVALUATES THE PREVIOUS DEFINITE INTEGRAL)

.1953158959

THE MINIMUM OCCURS AT THE ENDPOINT $t = -1$

Figure 2. A Maple Solution for Problem 2

3. Let the sketch in Figure 3 depict the graph of $f(t)$ on $(-\infty, \infty)$

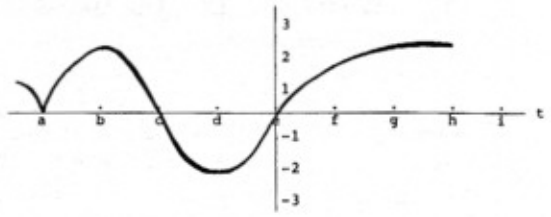


Figure 3. The Graph of $f(k)$

a. Compute $\lim_{t \to a} \dfrac{f(t) - f(a)}{t - a}$.

b. Where does the Mean Value of the Derivative occur on the interval $[c, e]$? EXPLAIN

c. Where does the Mean Value of the Integral occur on the interval $[a, e]$? EXPLAIN

Students are permitted to use Maple on all homework assignments and exams. They soon begin to realize which problems lend themselves better to mental or to pencil-and-paper computations (i.e., differentiating a polynomial). Worksheets, developed by the faculty, supplement the text by extending examples especially suited to the use of Maple.

Several students have had calculus in high school and find the CAS-calculus "different, but not necessarily more difficult." They comment on how much better they now understand calculus.

Although devising more conceptual exams, lectures, and assignments takes some creativity,

the payoff is clear from the improvement of understanding demonstrated by the students.

Computer Algebra Systems such as Maple and MACSYMA are becoming more powerful and cheaper each day. Hand-held calculators with computer algebra are already on the market for less than $200. We do our students a disservice by not introducing them to the latest computational devices, especially students in majors where calculus is used as a tool.

References

1. Douglas, R. G. (Ed.). (1986). *Toward a lean and lively calculus.* (MAA Notes Number 6). Washington, DC: The Mathematical Association of America.

2. Steen, L. A. (1987a). *Calculus for a new century: A pump, not a filter.* (MAA Notes No. 8). Washington, DC: The Mathematical Association of America.