Using Hand-Held Graphing Computers in College Mathematics

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The distinction between calculator and computer is no longer clear. In 1984, Brophy and Hannon wrote that, "In mathematics courses, computers offer an advantage over calculators in that they can express results graphically as well as numerically, thus providing a visual dimension to work with variables expressed numerically" (p. 61). This advantage of computers over calculators disappeared in early 1986 when Casio introduced the *fx-7000G*, a programmable scientific calculator with interactive graphics, that is, a hand-held graphing computer. Other similar yet more powerful and sophisticated machines soon followed: Casio's *fx-7500G*, and *fx-8000G*, Hewlett-Packard's *28C* and *28S*, and Sharp's *EL-5200*. In January 1986, only months before the introduction of the *fx-7000G*, at the Tulane conference on calculus reform, Tucker et al. (1986) had considered calculus curriculum revision based on the levels of technology required for various types of computational support. At that time numerical computation could be done on hand calculators, but interactive graphics and symbolic manipulation required micro or mainframe computers. Because of access problems, they shied away from recommending computer-based graphics and symbolic manipulation as a part of mainstream calculus. "The participants at that conference had no idea that a Casio *fx7000-G* [*sic*] or an *HP-28C* was looming on the horizon" (Tucker, 1987, p. 5).

Hand-held graphing computers combine the capabilities of a scientific calculator, a programmable computer, an interactive-graphics computer system, and in the case of the Hewlett-Packard calculators, a limited computer mathematics system that performs symbolic manipulation. These machines are powerful tools for mathematical experimentation and exploration. They are too small to lend themselves to typewriter-style keyboarding, and ultimately this may be the lone distinction that remains between hand-held and micro computers.

What Hand-Held Computers Can Do

The capabilities of these pocket-sized computers have been described in some detail elsewhere (see, e.g., Foley, 1987a, in press; Michel, 1987; Potter, 1987a; Tucker, 1987). Muciño (1988) has even provided a "buyer's guide" for these machines. This section provides a summary of the features of the Casio, Sharp, and Hewlett-Packard hand-held graphing computers as a reference for college mathematics faculty who are planning curriculum and instruction to take advantage of these versatile computational tools.

The hallmarks of these supercalculators are (a) large display screens, (b) interactive graphics, and (c) on-screen programming. When choosing to buy, use, or design a curriculum around one of these machines all three of these factors should be carefully considered. Durability and price are also important; so included below are some notes about the durability and lowest prices as of December 1988 for each model.

The Casio *fx-7000G*, *fx-7500G*, and *fx-8000G* all have eight-line text displays and graphics viewports that are 63 rows by 95 columns of pixels. They can readily produce the graphs of functions, and with some programming, the graphs of polar equations, parametric equations, conics, and even three-dimensional graphs. The viewing rectangle and the scaling units are set using the Range feature. The Trace command allows pixel-to-pixel movement along the most recently drawn function; the Casio displays the x- or y-coordinate associated with each pixel along the way. The automatic zoom feature or the Factor command can be used to zoom-in or zoom-out about a plotted or traced-to point, or as a default, about the center of the

This paper is an extension of a workshop with the same title presented on October 29, 1988 at the Conference on Technology in Collegiate Mathematics at The Ohio State University in Columbus. Preparation of this paper was supported in part by the *Mathematics Through Technology: Establishing Concepts and Skills of Graphing and Functions in Grades 9 through 12* grant from the National Science Foundation (TPE-8751353). This grant and others from British Petroleum (formerly, Standard of Ohio), the Ohio Board of Regents, and The Ohio State University have provided funding for the Calculator and Computer PreCalculus Project. The opinions and conclusions herein expressed are solely those of the author.

current viewing rectangle. Early versions of the *fx-7000G* did not have automatic zoom, but now all models have this important feature. Other capabilities include statistical features and binary, octal, and hexadecimal computation and conversion. Commands and programs can be selectively changed and re-executed. Casio syntax closely parallels standard algebraic syntax. The *fx-7000G* ($50) has only 0.4K memory. The *fx-8000G* ($70) has 1.4K regular memory plus an additional 1.9K for its file editor. It has an input buffer that saves the last prior command (this comes in handy if you accidently hit a wrong key). It can be linked to a printer or to a tape recorder to save programs externally. The *fx-7500G* ($65) has the fastest graphics and the most memory (4K) of the three.

The Sharp *EL-5200* ($75) has four lines of textual display and 32×96 pixels of graphics display. It has an extensive input buffer that saves several prior commands. The graphics are slower than on the Casios, and the user does *not* see the graph being drawn. In addition to all of the features of the Casio, the Sharp permits automatic setting of the y viewing-rectangle parameters and has a scrolling screen for tracing along a graph beyond the current viewing rectangle, plus it has built-in equation-solving and matrix capabilities. The programming is awkward compared to the Casio, but the memory capacity is larger (8K) even than the Casio *fx-7500G*. A drawback of the Sharp is that its right-hand keyboard is a touchboard connected to the machine's main circuitry by ribbon cables, which can be damaged by opening the calculator past the flat position.

The Hewlett-Packard *HP-28C* (1.6K) is no longer manufactured and no longer available. It has been replaced by the *HP-28S*, that has 32K of memory. Both machines operate in essentially the same manner. Like the Sharp, the *HP-28* only has four lines of text, but the graphics viewport is a bit wider at 32×137 pixels, and graphs are shown as they are being drawn. The *HP-28* can solve equations and operate on matrices, and in addition, can find derivatives and definite integrals, generate Taylor series, determine antiderivatives of polynomials, and handle complex numbers. The *HP-28* has an operating logic based on Reverse Polish Notation, and its working memory is organized into a stack, or column of entries. The *HP-28* also permits the use of algebraic syntax. The machine is menu driven, and user-developed, stored programs are automatically added to the HP's extensive list of built-in functions. The programming is flexible and especially nice for experienced programmers, permitting BASIC-, FORTRAN-, and Pascal-like commands. It is the most powerful, and the most expensive ($170), of the machines described here. Customized, stored programs can powerfully personalize the *HP-28* to solve many mathematics problems with just a few keystrokes. Wickes (1988) provides much valuable information for those intending to make substantial use of the *HP-28*.

All of these hand-held graphing computers permit interactive experimentation. The Casios have the largest screen and best graphics. The *HP-28* is the most versatile and powerful. All of them can be used to make mathematics more oriented toward concept development and problem solving and less oriented toward paper-and-pencil computation. The use of hand-held computers can be applied to many areas of undergraduate mathematics, especially precalculus, calculus, and statistics. This paper focuses on the applications of these pocket computers to college algebra, trigonometry, and analytic geometry, especially the interactive graphing of functions and relations and the interpretation and use of the obtained graphs to solve problems.

The Ohio State C^2PC Project

The Ohio State University Calculator and Computer Precalculus (C^2PC) Project is a three-year field-based project aimed at developing a precalculus course that is rich in problems and takes full advantage of interactive computer graphics technology. The primary objectives of the project are (a) to create instructional materials that make effective use of computer- and calculator-based graphing to strengthen student problem solving skills; (b) to improve student understanding of functions, graphs, and analytic geometry–critical areas of mathematical deficiency in the current college preparatory curriculum; and (c) to increase significantly the number of students adequately prepared to pursue higher education in mathematics, science, and technical fields.

"Graphs of functions and relations are highly valued for their ability to display complex information visually. Yet students come to view graphing as a task to be completed rather than an interpretational aid" (Dick, 1989, p. 13). The interpretation and use of graphs is essential in today's world in which graphs are so widely used to present information. Moreover, these skills are especially important for students who intend to study calculus and to pursue scientific and technical careers. Yet, evidence from the Second International Mathematics Study shows that many 12th-grade precalculus students are weak in coordinate geometry, functions, and graphing, and in particular, they do not make a strong connection between a function and its graph (Chang & Ruzicka, 1985). For instance, most do not realize that the solution for a system of two equations corresponds to the intersection points of their graphs.

The C^2PC project materials (Demana and Waits, 1988c; Foley et al., 1988; Osborne and Foley, 1988; Waits and Demana, 1988b) are designed to shore up this sagging section of the curricular fabric. After one and a half years of piloting, these materials were field-tested in some 80 high schools and 40 colleges and universities across the nation during the 1988-89 academic year. Throughout the course, hand-held graphing computers and microcomputer software are used as tools for concept development, problem solving, and exploration. Most field-test teachers participated in summer inservice programs to gain familiarity with the materials and to help them prepare to create instructional environments in the spirit of the project. In order to give the reader a sense of the C^2PC instructional environment, some distinguishing characteristics of the project are outlined below. This general outline is followed by a set of illustrative examples.

Characterizing Features. C^2PC has three characteristics that together make it unique in establishing the concepts of functions and graphing:

1. *Interactive graphing.* Interactive computer graphing is used to provide a rich array of examples of graphs and functions for students to explore and examine. This gives the student many opportunities to form generalizations and to develop concepts about graphs, functions, and their characteristics.
2. *Problems as means.* Real world problems situations are used as the means to approach and teach concepts and skills instead of merely as exercises after the concept has been taught. Often a problem serves as the stimulus for a discussion of some new mathematics with the new mathematics serving as the conclusion for that discussion.
3. *Calculus topics without calculus.* The mathematics is organized differently from most texts. Many topics that are treated lightly or not at all in other precalculus texts are explored in depth. Limits, asymptotes, extrema, continuity, and other topics that foreshadow calculus are given a thorough treatment.

Classroom Arrangements. We have found that a key factor affecting success in using interactive computer graphing is the arrangement of the classroom. Graphs can and should serve as a stimulus for mathematical discussion, and students must be able to see what is being discussed. We have used a variety of arrangements, operating in one or a combination of the following modes:

1. *Graphing calculator mode* with each student using a hand-held graphing computer.
2. *Demonstration mode* with a single large monitor or with an overhead projection palette tied to a computer.
3. *Laboratory mode* with each student or pair of students stationed at a microcomputer.

Instructional Principles. The C^2PC course is organized around five methodological themes. These themes serve as threads that wind their way through the project materials. The themes are:

1. *Active involvement.* Students are actively involved in problem solving.
2. *Verbal interaction.* Students talk about the mathematics they are learning.
3. *Problem revisitation.* Important problem situations are revisited on a regular basis.
4. *Informal language.* The formal language of the mathematical topics is kept to a minimum and is not introduced until there is a need for it.
5. *Generalization.* Student learning is facilitated by encountering many instances from which to make generalizations.

Mathematical Emphases. The use of interactive graphics offers an opportunity to change emphases in the mathematical content of precalculus. Most calculus instructors have based much of their instruction through the years on the assumption that a picture or graph explains all. The assumption is that graphs intuitively provide a great deal of information about associated problem situations and algebraic representations. Our experience indicates that graphs become intuitive only after students have learned how to read and interpret the information they provide. That is, students must be *taught* what is contained in graphs before they can serve as an intuitive base for explanation. In large part, the content of the C^2PC course was selected to extend substantially students' ability to know what they are seeing when they encounter a graph and to establish firmly the connection between an equation and its graph. Following are some of the major content emphases that are exploited to build an understanding of graphs and functions.

1. *Viewing rectangles and scale.* With either paper-and-pencil graphing or a computer graphing utility, one only examines a portion of most graphs. A viewing rectangle specifies the portion of the plane within which a graph is to be examined; that is, the minimal and the maximal values of the x-coordinates and the y-coordinates. Students learn how to pick viewing rectangles to satisfy the purpose of the problem at hand; they learn how to zoom-in, zoom-out, and choose different scales for the two coordinate axes.
2. *Local behavior of functions.* The ability to change viewing rectangles permits students to examine closely the graphical behavior of functions. Students can zoom-in to see at close range such local features as extrema and intercepts.
3. *End behavior of functions.* Alternatively, students can zoom-out to obtain a global view of the graph of a function. The notion of asymptote comes alive for students in a new and richer way.
4. *Graphical solution algorithms.* Equations, inequalities, and systems of equations can be solved graphically through the use of a zoom-in procedure. This graphical approach is powerful. Equations involving any elementary functions can be solved by the zoom-in method; whereas their algebraic solutions require a myriad of paper-and-pencil methods.
5. *Parameters and functions.* Students can explore and discover the effects that equation parameters have on the graph of a function. For example, students can experiment with the equation $f(x) = ax^2 + bx + c$ to determine the effects that a, b, and c have on the graph of a quadratic function.
6. *Mathematical modeling.* Students investigate a wide variety of challenging problem situations. They create algebraic and geometric representations for a given situation and use these representations in the solution of the associated problem.

Examples

Three examples are given below to illustrate the C^2PC instructional approach and the interactive graphing capabilities of a hand-held graphing computer. The Casio is used in the examples because this is the machine used at most of the project field-test sites. The Range feature, which shows all six viewing-rectangle and scale parameters at once, and the fast, large-screen graphics make the Casio a suitable choice for the methods being illustrated. Other pocket graphers or computer graphing software could be used in a similar way to solve the example problems. Figures in this paper are given as they appear on the Casio graphics screen.

The first example is actually a set of examples intended to illustrate several aspects of the C^2PC approach. The example foreshadows calculus in its use of modeling, in its treatment of a function as the object of consideration, and in its inclusion of the extreme value concept. Few, if any, of the subproblems in the example would ever be considered in a traditional precalculus course. Parts (a), (b), and (c) – write an equation that models the problem, draw a complete graph of the equation, and identify the portion of the graph relevant to the problem – represent a typical sequence of steps stressed in the project materials. These steps are all that is asked of students on their first encounter with the full-fledged box problem. Earlier, students encounter the box problem in a restricted setting that leads to a quadratic equation. On subsequent *revisitations* question (d), then (e) is asked. In this way by the time students are faced with the extreme-value aspect of the problem, they can deal with this new and challenging issue in a familiar setting.

Example 1.

An open box is to be made from a rectangular piece of sheet metal 20 cm long and 15 cm wide by cutting and removing equal square pieces from each corner of the rectangular sheet and then bending up the sides.

(a) Write an equation that expresses the volume of the box as a function of the side length of the removed squares.

(b) Draw a complete graph of this equation.

(c) What part of the graph in (b) represents the problem situation?

(d) What size square must be cut and removed to form a box with a volume of $250\ \text{cm}^3$?

(e) What are the dimensions of the box with the largest volume? What is the maximum volume?

Solution.

(a) This step is nontrivial for most students when they first encounter a box problem of this sort; this step usually occurs in beginning calculus. In the C^2PC materials students face this problem only after they have had numerous experiences in modeling simpler problem situations. An equation for the volume function is $V(x) = x(15-2x)(20-2x)$.

(b) Graphing $y = x(15-2x)(20-2x)$ in the Casio default viewing rectangle of $[-4.7, 4.7] \times [-3.1, 3.1]$ yields the graph shown in Figure 1. Notice that the Casio graphics viewport does not include the top row and left-most column of pixels. The point $(0,0)$ has actually been graphed, but this is revealed only if the Trace command is used.

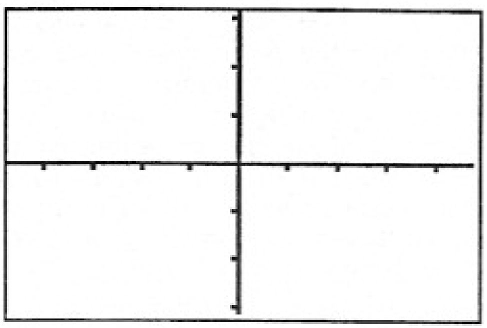

Figure 1. The graph of the volume function $y = x(15-2x)(20-2x)$ in $[-4.7, 4.7] \times [-3.1, 3.1]$

This graph sheds some light on one of the questions raised by Small, Hosack, and Lane (1986): "Should a student analyze a function to sketch the graph, or just call up a graphing program?" (p. 433). Some thought and experimentation is typically required to obtain a useful computer-generated graph of a given function. Here a *complete graph* is sought; that is, one that displays all key attributes of the function. This is a somewhat subjective and perhaps vague notion, but one that we have found to be pedagogically useful. Goldenberg's (1988) comment about interpreting graphs applies well to the task of obtaining a complete graph: "To interpret graphs correctly, we need mathematical knowledge and expectations, not just perceptual experience" (p.135). In this case a viewing rectangle of $[-5, 15] \times [-500, 1000]$ is suitable (see Figure 2).

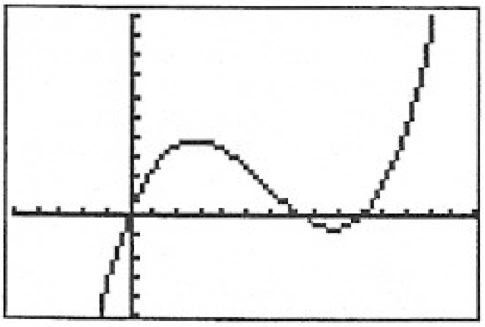

Figure 2. The graph of $y = x(15-2x)(20-2x)$ in $[-5, 15] \times [-500, 1000]$

(c) The only values of x that make sense in the problem situation are those between 0 and 7.5. The graph shown in Figure 3 extends just beyond these values.

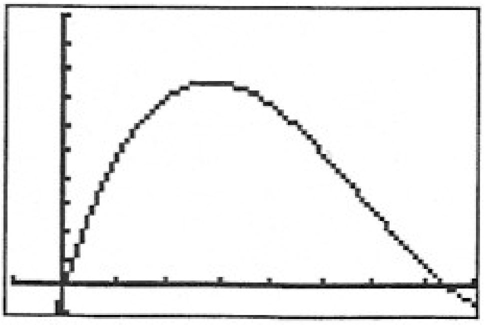

Figure 3. The graph of $y = x(15-2x)(20-2x)$ in $[-1, 8] \times [-50, 500]$

(d) To determine possible side lengths of removed squares that would yield a 250 cm^3 box, we overlay the graph of $y = 250$ and project down from the points of intersection to the x-axis (see Figure 4). The dashed downward pointing arrows do not appear on the Casio.

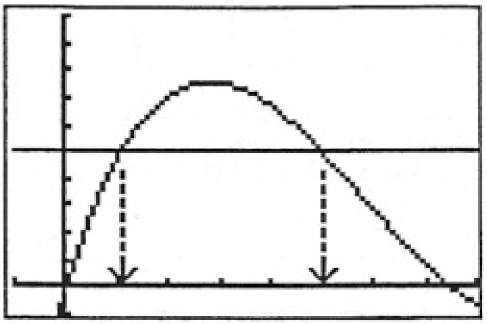

Figure 4. The graphs of $y = x(15 - 2x)(20 - 2x)$ and $y = 250$ in $[-1, 8] \times [-50, 500]$

It appears that an x-value of approximately 1.1 cm or 5.0 cm would yield a 250 cm^3 box. Substitution shows that $x = 5$ exactly satisfies this condition. Using traditional algebraic methods we can determine that the other exact solutions to the equation $x(15 - 2x)(20 - 2x) = 250$ are $x = \frac{25 \pm \sqrt{425}}{4}$, but only $x = \frac{25 - \sqrt{425}}{4}$ is between 0 and 7.5. The approximate value of this second solution to our problem is 1.10 cm.

(e) Using the graph of the volume function shown in Figure 3, we can use the Trace feature to approximate the maximum value at $y = 379.0377662$ (see Figure 5; notice that the Casio viewport is reduced to 55 x 95 pixels when the Trace function is being used), and then employ the $X \leftrightarrow Y$ command to obtain $x = 2.829787234$. Here x represents the height of the box in cm, and y represents the volume in cm^3. The other dimensions of the box would be 9.34 cm and 14.34 cm. The standard calculus solution is $x = \frac{35 - \sqrt{325}}{6} \approx 2.828707270$ and $y \approx 379.0378082$. This 10-decimal-place accuracy could be achieved by using zoom-in. Notice that our initial approximation for the volume has 7-place accuracy, but our initial approximation for the height has only 3-place accuracy. Why?

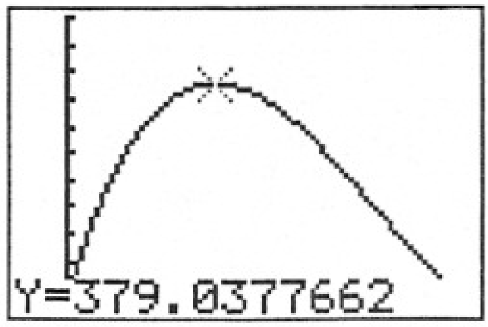

Figure 5. The graph of $y = x(15 - 2x)(20 - 2x)$ together with the readout of the relative maximum value on the interval $0 < x < 7.5$

The next example is a typical trigonometric equation. It illustrates the zoom-in method for equation solving, pointing out that zoom-in can be accomplished in four different ways on the Casio:

1. Key in Range setting parameters by hand.
2. Automatic zooming.
3. Use the Factor feature.
4. Setting the Range within a program.

Zoom-in can also be used to solve inequalities and systems of equations and to locate relative extrema. The equation in Example 2, unlike many equations involving elementary functions, can be solved exactly by traditional methods; so a paper-and-pencil approach is sketched.

Example 2.

Solve $\cos x = \tan x, \quad 0 \le x \le \frac{\pi}{2}$.

Solution.

Using zoom-in. One approach to solving this problem is to graph the functions corresponding to the two sides of the equation on the same coordinate system and then to determine the x-coordinates of any points of intersection that lie between $x = 0$ and

$x = \frac{\pi}{2}$. On the Casio this can be done without any programming by entering and executing the dual command Graph Y = cos X : Graph Y = tan X, and then choosing a nested sequence of progressively smaller viewing rectangles each containing the relevant point of interest.

This zooming-in procedure can be accomplished by (a) using the Range feature or (b) using the Trace feature together with automatic zoom or the Factor command. Students unfamiliar with computer graphing can use the Range feature to set viewing rectangle parameters by hand until they gain some facility in making intelligent choices about picking their next view of a given situation. Next automatic zoom (Casio's instant factor function) with its relatively small magnification factor of 2 in each direction can serve as a prelude to the more versatile Factor command.

The following style of program, suggested by a colleague (Shumway, personal communication, 1987), is faster and more flexible than the automatic-zoom approach, and since it is stored in memory, you cannot lose it by hitting one wrong key while trying to zoom-in. It is simple, short, and easily modified.

```
"F =" ? → F
Factor F
Graph Y = cos X
Graph Y = tan X
```

Here's how it works: After entering the program we set the viewing rectangle to the Casio default of $[-4.7, 4.7] \times [-3.1, 3.1]$ with a scaling unit of 1 on each axis, and execute the program. The prompt F = ? will appear on the screen. It is asking for an F value, which will act as a magnification factor. In this case, to preserve our initial choice of viewing rectangle, we enter the value 1, and then continue the execution of the program by pressing EXE. The graph shown in Figure 6 will appear. Tracing along the graph of the tangent function to the pixel that best approximates the unique point of intersection between $x = 0$ and $x = \frac{\pi}{2}$ yields a readout of $x = 0.7$ (see Figure 7). We reexecute the program using $F = 10$, which yields Figure 8. Continuing in this manner, we obtain successive approximations of $x = 0.67$, shown in Figure 9, and then $x = 0.666$, shown in Figure 11. We could continue in this manner to obtain 10-decimal-place accuracy.

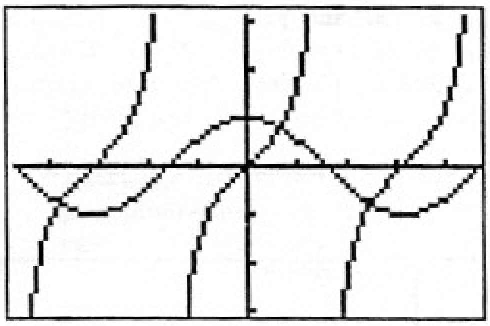

Figure 6. The graphs of $y = \cos x$ and $y = \tan x$ in $[-4.7, 4.7] \times [-3.1, 3.1]$

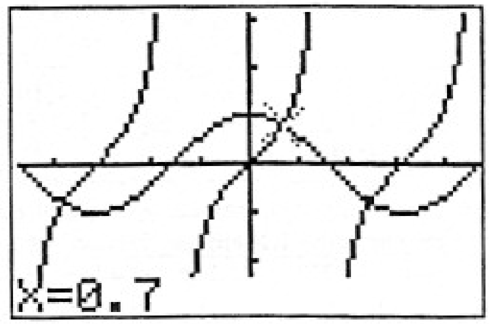

Figure 7. The x-coordinate readout for the pixel nearest the point of intersection in $[-4.7,\ 4.7] \times [-3.1,\ 3.1]$

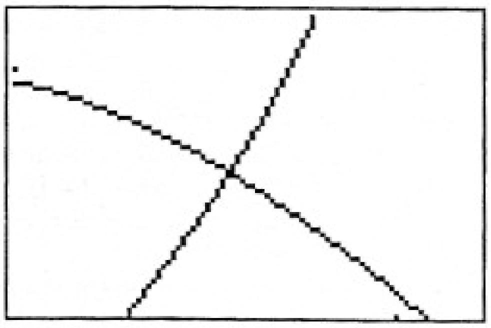

Figure 8. The graphs of $y = \cos x$ and $y = \tan x$ in $[0.23, 1.17] \times [0.49, 1.11]$

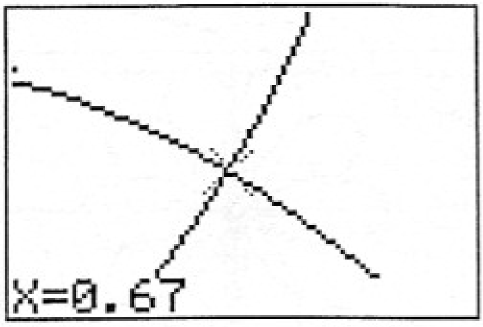

Figure 9. The x-coordinate readout for the pixel nearest the point of intersection in $[0.23, 1.17] \times [0.49, 1.11]$

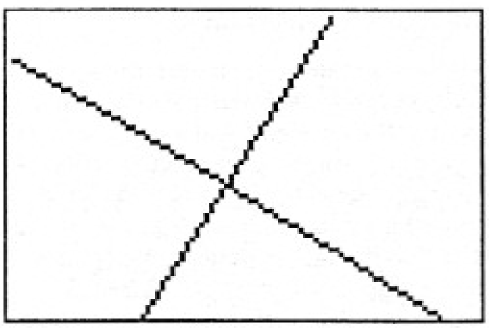

Figure 10. The graphs of $y = \cos x$ and $y = \tan x$ in $[0.623, 0.717] \times [0.759, 0.821]$

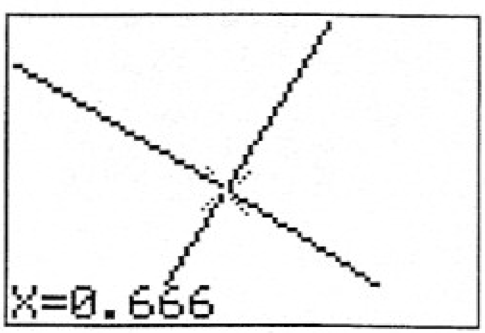

Figure 11. The x-coordinate readout for the pixel nearest the point of intersection in $[0.623, 0.717] \times [0.759, 0.821]$

Alternatively, we could use Vonder Embse's (1988) fancy 19-line zoom-in program. Vonder Embse's program, based upon Stickney's earlier version (1988, of these proceedings), allows the user to zoom-in on a point by tracing to each of two opposite corners of the next viewing rectangle. This emulates a feature of the *Master Grapher* software (Waits and Demana, 1988b) that we have found to be pedagogically advantageous throughout the C^2PC project; namely, letting the student see the new viewing rectangle within the old one. Goldenberg (1988) has also found this to be valuable: "When multiple scales are used to represent the same graph, graphing windows should contain internal frames . . . to help students recognize which portion of a distance [*sic*] view is being enlarged in a close-up view" (p. 171).

Traditional approach. The standard traditional method is to seek an exact solution by using trigonometric identities to obtain an equivalent equation that can be readily solved. Such an equation is $\sin^2 x + \sin x - 1 = 0$. Its one solution for $0 \leq x \leq \frac{\pi}{2}$ is $\sin^{-1}\left(\frac{-1+\sqrt{5}}{2}\right)$.

The exact solution gives rise to the following theorem: If one leg of a right triangle is in golden ratio to the hypotenuse, then the second leg is the geometric mean of the first leg and the hypotenuse, and conversely. We probably would have missed this relationship using the zoom-in method. Most traditional precalculus classes would miss it, too.

Moreover, the exact answer does not give the average student any idea of the size of the angle for which the cosine and tangent functions are equal. Many students would not even realize that is what the problem is asking for. To answer the question, "Which method is better?", we must first answer the question, "What is the educational goal of studying this problem?" Or perhaps, "What are the goals of the course, and how does this problem fit into the grand scheme?"

Exact answers tend to please our mathematical souls, yet they are rare. Zoom-in is a very general method that makes the solver of an equation think in terms of the functions involved and their graphical representations. Most students, with sufficient, carefully designed exposure to the zoom-in method, come to realize what it means to solve an equation, and they learn a great deal about functions and graphs in the process.

The final example comes from Larry Thursby, a high school student in the C^2PC project. After graphing ordinary rose curves in class one day, he went home and began exploring what I now call *generalized rose curves.* This is an example of what students can and will do on their own if put in the proper learning environment.

Example 3.

Graph $r = 6 \sin 2.5q$.

Solution.

This can be accomplished on the Casio by using the following program:

```
0 → T
Range -9, 9, 1, -6, 6, 1
Lbl 1
6 sin 2.5T → R
Rec (R, T)
Plot I,J
Line
T + 0.1 → T
T ≤ 4π ⇒ Goto 1
```

The portions of the program in **boldface** type may vary from problem to problem. Notice the variable T (used for the angle θ) takes on values from 0 to 4π in 0.1 size increments. The functional value R is computed for each of these values of T. These polar coordinates, (R,T) , are converted to rectangular coordinates; the point is plotted. Then that point is connected by a line segment to its immediate predecessor (except for the first time through the loop when there is no preceding point). The Line command is given in boldface because in some cases you may wish to delete this step to avoid connecting points. The Range parameters are chosen so that the entire graph will appear on the screen and to fit the 3:2 aspect ratio of the Casio screen.

When the program is executed the Casio flashes back and forth between the text window and the graphics window. At the end of execution, the Casio will show the text window, and the graph will be stored in the graphics window. The G ↔ T command will reveal the graph shown in Figure 12.

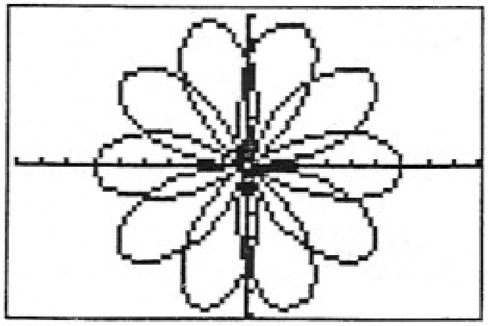

Figure 12. The graph of $r = 6 \sin 2.5\theta$ in $[-9,9] \times [-6,6]$

Discussion and Conclusion

Hand-held graphing computers with their interactive graphics capabilities have profound implications for what we can and should teach and how we should teach it. Graphs of functions and relations can be quickly drawn and explored. Interactive graphical methods, such as zoom-in, can be used to develop mathematical connections and to solve realistic problems. The Sharp *EL-5200* goes beyond interactive graphics with its built-in equation-solving and matrix capabilities. For instance, the Sharp's SOLVE key makes short work of Example 2. We can subtract $\tan x$ from each side of the original equation to obtain $f(x) = \cos x - \tan x$, set x-min $= 0$ and x-max $= 1$, and then graph f using the AUTODRAW feature, which picks the y-min and y-max for us automatically. This yields the graph shown in Figure 13. After graphing the function, we need only press the SOLVE key. After a brief wait the solution $x = 0.666239433$ will appear on the screen (see Figure 14; notice that the Sharp viewport is reduced to 24×96 pixels when the SOLVE key is used).

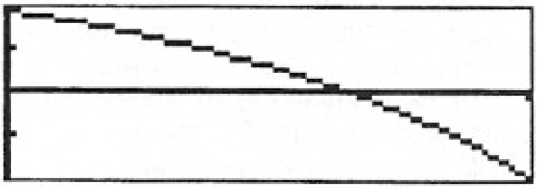

Figure 13. The graph of $y = \cos x - \tan x$ in $[0, 1] \times [-1.0711, 1]$

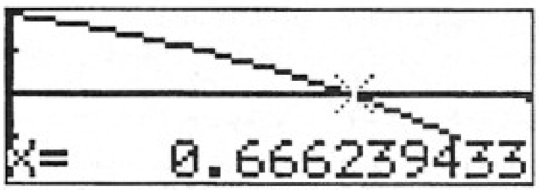

Figure 14. The x-intercept readout obtained by using the SOLVE key

The *HP-28* can solve Example 2 without even drawing a graph. Using the Solve menu, we can store the equation in its original form, provide the HP with an initial guess, say $x = 0.5$, and then in a matter of seconds have the 12-place solution 0.666239432493 with a message that says, "Sign Reversal," indicating an approximate solution.

The C²PC materials do not include automatic equation solving because that is not in keeping with the project's goal of developing a strong, stable connection between algebraic and geometric representations of functions and relations. When the instructional focus is not on solving the equation or gaining geometric intuition but on using the solution, the automatic solving routines of the Sharp and the *HP-28* are appropriate. This should be the case by the time a student studies calculus, if not before. Hand-held graphing computers, especially the symbol-manipulating *HP-28*, offer many powerful, sometimes controversial new approaches to the teaching and learning of college mathematics. Many more questions can be asked than answered. This article has offered a set of examples that illustrate how these machines can be used in the context of a course in college algebra, trigonometry, and analytic geometry designed to prepare students for calculus. We should all reflect on how this new breed of calculators can be used to improve the instruction in the courses we teach.

References [1]

Brophy, J., & Hannon, P. (1985). On the future of microcomputers in the classroom. *The Journal of Mathematical Behavior, 4,* 47-67.

Chang, L.-C., & Ruzicka, J. (1985). *Second international mathematics study: United States technical report I.* Champaign, IL: Stipes Publishing.

Demana, F., & Waits, B. K. (1987). Problem solving using microcomputers. *College Mathematics Journal, 18,* 236-241.

Demana, F., & Waits, B. K. (1988a). The Ohio State University Calculator and Computer Precalculus Project: The mathematics of tomorrow today! *The AMATYC Review, 10*(1), 46-55.

Demana, F., & Waits, B. K. (1988b). Pitfalls in graphical computation, or why a single graph isn't enough. *College Mathematics Journal, 19,* 236-241.

Demana, F., Waits, B. K. (Principal Authors), Osborne, A., & Foley, G. D. (1988c). *Precalculus mathematics, a graphing approach* (preliminary ed., 2 vols.). Reading, MA: Addison-Wesley.

Dick, T. P. (1989, January). From ends to means: Using graphing calculators in mathematics instruction [abstract of paper presented at the Graphing Calculators contributed paper session at the meeting of the Mathematical Association of America, Phoenix, AZ]. In *Program: Phoenix Joint Mathematics Meetings* (p. 13).

Dick, T. P., & Shaughnessy, J. M. (1988). The influence of symbolic/graphic calculators on the perceptions of students and teachers toward mathematics. In M. J. Behr, C. B. Lacampagne, & M. M. Wheeler (Eds.), *Proceedings of the Tenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 327-333). DeKalb: Northern Illinois University.

[1] In addition to the sources cited in the body of this article, the bibliography includes other works on using hand-held graphing computers and microcomputer graphing to enhance the teaching of mathematics.

Educational Technology Center. (1988). *Making sense of the future: A position paper on the role of technology in science, mathematics, and computing education.* Author, Harvard Graduate School of Education, Cambridge, MA.

Fiske, M. B. (1988, October). Teaching linear regression using the Casio fx-7000G calculator. In this volume.

Foley, G. D. (1987a). Future shock: Hand-held computers. *The AMATYC Review, 9*(1), 53-57.

Foley, G. D. (1987b). Reader reflections: Zoom revisited [response to Montaner, 1987]. *Mathematics Teacher, 80*, 606.

Foley, G. D. (1988a). Let us teach exploration. *The AMATYC Review, 10*(1), 63-66.

Foley, G. D. (1988b). Timeless and timely issues in the teaching of calculus. *The AMATYC Review, 9*(2), 55-60.

Foley, G. D. (in press). Using hand-held computers in calculus and analytic geometry. Essay to appear in the revision of A. Mizrahi & M. Sullivan (Eds.), *Calculus and analytic geometry* (2nd ed.). Belmont, CA: Wadsworth.

Foley, G. D., Edwards, A., Demana, F., Waits, B. K., & Wern, L. A. (1988). *Computer graphing laboratory manual* to accompany *precalculus mathematics, a graphing approach* (preliminary ed.). Reading, MA: Addison-Wesley.

Frantz, M. E. (1986). Interactive graphics for multivariable calculus. *College Mathematics Journal, 17*, 172-181.

Goldenberg, E. P. (1988). Mathematics, metaphors, and human factors: Mathematical, technical, and pedagogical challenges in the educational use of graphical representations of functions. *The Journal of Mathematical Behavior, 7*, 135-173.

Heid, M. K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. *Journal for Research in Mathematics Education, 19*, 3-25.

Michel, F. (1987). La HP-28C. *Mathématique et Pédagogie*, No. 62, 21-26.

Montaner, F. R. (1987). Use of the zoom in the analysis of a curve. *Mathematics Teacher, 80*, 19-28.

Muciño, M. (1988). Buyer's guide to graphics calculators. *Mathematics Teacher, 81*, 705, 707-708.

Nievergelt, Y. (1987). The chip with the college education: The HP-28C. *American Mathematical Monthly, 94*, 895-902.

Nievergelt, Y. (1988). The HP-28S brings computations and theory back together in the classroom. *Notices of the American Mathematical Society, 35*, 799-804.

Osborne, A., Foley, G. D. (Principal Authors), Demana, F., & Waits, B. K. (1988). *Instructor's manual* for *Precalculus mathematics, a graphing approach* (preliminary ed.). Reading, MA: Addison-Wesley.

Potter, D. (1987a, August). Seven ounces of pocket graphics: Casio's new fx-8000G and friends. *Pico: The Magazine of Portable Computing*, pp. 22-23.

Potter, D. (1987b, August). Casio's fx-8000G goes to high school. *Pico: The Magazine of Portable Computing*, pp. 23-24.

Shumway, R. J. (1988). Graphics calculators: Skills versus concepts. In J. de Lang & M. Doorman (Eds.), *Senior Secondary Mathematics Education* (pp. 136-140). Utrecht, The Netherlands: University Press.

Small, D., Hosack, J., & Lane, K. (1986). Computer algebra systems in undergraduate instruction. *College Mathematics Journal, 17*, 423-433.

Smith, D. A., Porter, G. J., Leinback, L. C., and Wenger, R. H.. (Eds.). (1988). *Computers and mathematics: The use of computers in undergraduate instruction* (MAA Notes No. 9). Washington, DC: Mathematical Association of America.

Steen, L. A. (1987, October 14). Who still does math with paper and pencil? *The Chronicle of Higher Education*, p. A48.

Stickney, A. (1988, October). "Zooming In" On a Graphing Calculator. In this volume.

Tall, D. O. (1985, March). Understanding the calculus. *Mathematics Teaching*, No. 110, 49-53.

Tucker, T. (1987, January-February). Calculators with a college education? *Focus: The Newsletter of the Mathematical Association of America*, pp. 1, 5.

Tucker, T. (Chair), Goldstein, J., Lax, P., Raphael, L., Rodi, S., van der Vaart, R., Van Valkenburg, M. E., & Zorn, P. (1986). Calculus syllabi: Report of the Content Workshop. In R. G. Douglas (Ed.), *Toward a* *lean and lively calculus: Report of the conference/workshop to develop curriculum and teaching methods for calculus at the college level* [MAA Notes No. 6] (pp. vii-xiv). Washington, DC: Mathematical Association of America.

Vonder Embse, C. (1988). *Graphing calculators: Programming on the Casio.* Unpublished manuscript, Central Michigan University, Department of Mathematics, Mount Pleasant.

Waits, B. K., & Demana, F. (1987). Solving problems graphically using microcomputers. *UMAP Journal, 8*(1), 1-7.

Waits, B. K., & Demana, F. (1988a). Manipulative algebra–the culprit or the scapegoat? *Mathematics Teacher, 81*, 332- 334.

Waits, B. K., & Demana, F. (1988b). *Master grapher* [Computer software]. Reading, MA: Addison-Wesley.

Waits, B. K., & Demana, F. (1988c). The Ohio State University Calculator and Computer Precalculus (C^2PC) Project. *Mathematics Teacher, 81*, 759, 775.

Waits, B. K., & Demana, F. (1989). Computers and the rational-root theorem, another view. *Mathematics Teacher, 82*, 124-125.

Wickes, W. C. (1988). *HP-28 insights: Principles and programming of the HP-28C/S.* Corvallis, OR: Larken Publications.